Determination of Second-Order Effects and Design for Stability Using the Drift Limit

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ABSTRACT

Buildings for which second-order effects are significant are often governed by drift limits. Amplifier-based approximate second-order analysis, as presented in AISC *Specification* Appendix 8 (2016), typically utilizes factors based on first-order drift, for which a preliminary design and an analysis are required. This paper derives equations for the amplifier used in approximate second-order analysis, B_2, based on the second-order drift. Upper-bound values of amplifiers based on the drift limit can thus be determined in advance of design, eliminating the need for iteration and simplifying the design process; these values are not excessively conservative for drift-governed designs.

Keywords: stability design, drift limit, second-order analysis, amplifier.

INTRODUCTION

This paper presents methods for utilizing information known in advance of member selection (loading, frame geometry, and drift limits) to determine upper-bound values of the B_2 amplifier used in approximate second-order analysis [defined in of the AISC *Specification* Appendix 8 (2016)]. The paper defines a second-order stability index that can be determined based on the drift limit and that can be used to calculate the B_2 amplifier. Two examples applying the second-order stability index are presented. The first is design example, consisting of member selection to meet both drift and strength criteria. The second is a hand calculation to confirm the validity of the results of a computer second-order analysis of a multi-story building.

SECOND-ORDER DRIFT

Many structures (especially moment frame structures) are drift controlled, meaning that the governing consideration in member selection is achieving sufficient system stiffness to meet a drift limit or limit deformations to prevent damage to key building components such as cladding or partitions. Such buildings are generally only as stiff as necessary to meet these requirements. Second-order effects tend to be significant for such flexible, drift-governed buildings, and many engineers use a limit such as 1.5 on second-order effects to ensure designs are not overly sensitive to loading and modeling assumptions. Because the second-order drift is the expected drift under a given set of load conditions, it is the appropriate quantity to limit in order to achieve acceptable performance.

Second-order effects reduce the stiffness of structures and thus increase the drift for a given applied lateral load. This reduction in system stiffness depends on the vertical load present, and at service levels, the effect is much smaller than under strength-level vertical loads. Nevertheless, there are many cases in which second-order effects are significant at service-load levels (LeMessurier, 1977; Griffis and White, 2013). Additionally, for seismic design (and for drift-sensitive safety conditions in wind design), drift under full design loads must be determined.

The ASCE/SEI-7 standard explicitly requires consideration of the second-order effect for seismic design (ASCE, 2016), and the ASCE *Prestandard for Performance-Based Wind Design* (ASCE, 2019) requires it for wind. The principles of mechanics require consideration of second-order effects, regardless of explicit treatment in building codes. It is recommended that second-order effects always be included in the calculation of drifts unless they can reliably be discounted.

Judgment-based drift limits for wind have historically been used in conjunction with first-order drift to achieve acceptable performance. This practice precedes the advent of reliable second-order analysis with finite-element analysis programs. Such an evaluation using first-order deformations may result in acceptable performance for buildings with a low to moderate second-order magnification, but for buildings with significantly higher second-order effects, it is effectively a much more permissive criterion and may

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lead to unacceptable performance. Additionally, the first-order drift may not be sufficiently accurate for comparison to the quantified strain capacity of cladding systems, interior partitions, or other drift-sensitive building components. AISC Design Guide 3, *Serviceability Design Considerations for Steel Buildings,* (West et al., 2003) provides guidance on such drift-sensitive components.

In this study, second-order drifts are compared to drift limits, and therefore the phrase "drift limit" should be understood as such; the concept of a first-order drift limit is not adopted. Using second-order drifts and deformations for comparison to drift limits and deformation capacities of cladding and partitions will provide a more consistent criterion across the full range of second-order effects (Griffis, 1993; Aswegan et al., 2015).

Many engineers find the use of amplifiers for approximate second-order analysis expedient and appropriate for their structures. Methods of determining force and displacement amplifiers based on first-order drift may give very approximate results or require iteration. Determination of amplifiers in advance of design using the drift limit can eliminate the need for iteration, simplifying the design process. Such a process is illustrated in the design example in Appendix C of the paper by Sabelli (2020); the equations used in that example have been further refined in Sabelli and Griffis (2021).

While the equations and methods developed are applicable to structures with braced frames and mixed (dual) systems, the considerations addressed are most significant for moment-frame structures. Importantly, the amplifiers determined using the drift limit are reasonable, upper-bound estimates for drift-governed buildings (as is common for moment frames) but become unreasonably conservative for buildings much stiffer than required.

For buildings in which the deformation imposed on deformation-sensitive elements does not correlate to story drift, application of drift-based methods such as proposed here may be impractical.

AMPLIFIERS FOR APPROXIMATE SECOND-ORDER ANALYSIS AND DESIGN FOR STABILITY

Second-Order Effects and First-Order Displacements

Second-order effects can be expressed using a system "stability index," which relates the geometric and mechanical stiffness. AISC Design Guide 28, *Stability Design of Steel Buildings,* (Griffis and White, 2013) defines the stability index, Q (shown here as Q_1, indicating that it uses first-order displacement):

$$Q_1 = \frac{P_{story}\Delta_1}{R_M HL} \tag{1}$$

where

H = first-order shear, kips

L = story height, in.

P_{story} = total gravity load, $P_{mf} + P_{lean}$, at LRFD level, kips

Q_1 = first-order stability index

R_M = stiffness-reduction coefficient to account for member P-δ influence on structure P-Δ

Δ_1 = first-order story drift corresponding to load H (Δ_H in the *Specification*), in.

and where

P_{lean} = gravity load on non-moment-frame columns, kips

P_{mf} = gravity load on moment-frame columns, kips

The AISC *Specification* ASD/LRFD adjustment factor α is omitted from the gravity-load definitions for brevity.

It should be noted that the stability index Q_1 defined here follows Griffis and White (2013) and includes the R_M coefficient; other literature has not consistently included this coefficient. AISC *Specification* Appendix 8, Equation A-8-8, for R_M is:

$$R_M = 1 - 0.15\frac{P_{mf}}{P_{story}} \tag{2}$$

See Sabelli and Griffis (2021) for a more accurate equation for R_M.

The force amplifier as presented in AISC *Specification* Appendix 8 utilizes the same quantities as does the stability index Q_1:

$$B_2 = \frac{1}{1 - \dfrac{P_{story}\Delta_1}{R_M HL}} \tag{3}$$

where

B_2 = amplifier for second-order effect

This amplifier can be expressed as a function of the stability index defined previously by combining Equations 1 and 3:

$$B_2 = \frac{1}{1 - Q_1} \tag{4}$$

Determination of amplification using Equation 4 requires a first-order drift, which typically is determined from a preliminary design and an analysis. AISC *Steel Construction Manual* Part 2 (2017) provides the "simplified method," whereby the first-order drift is assumed to be equal to the drift limit, and thus an amplifier can be obtained prior to design (Carter and Geschwindner, 2008), although iteration may be required.

Second-Order Effects and Second-Order Displacements

While the simplified method of utilizing the drift limit as the first-order drift results in a reasonable, liberal estimate of the amplifier, a more accurate estimate can be obtained using methods based on second-order drift. Prior to design, the structure can be assumed to be exactly stiff enough to meet the drift limit, and the amplification can be determined by setting the target second-order drift equal to that drift limit. This method is elaborated below, following work done by Statler et al. (2011).

Sabelli and Griffis (2021) present the force amplifier B_2 as a function of the second-order drift based on the equilibrium in the deformed condition:

$$B_2 = 1 + \frac{P_{story}\Delta_2}{HL} \tag{5}$$

where

Δ_2 = second-order story drift

Additionally, Sabelli and Griffis show that Equations 3 and 5 and have as a corollary the following expression for drift amplification:

$$\frac{\Delta_2}{\Delta_1} = \frac{B_2}{R_M} \tag{6}$$

Sabelli and Griffis note, however, that Equation 6 requires use of a more accurate equation for R_M than Equation 2. For structures with low to moderate second-order effects, the value of R_M (determined per Sabelli and Griffis) is close to 1.0, and drift amplification can be approximated by:

$$\frac{\Delta_2}{\Delta_1} \approx B_2 \tag{7}$$

For convenience, a stability index Q_2 based on the second-order drift is defined here:

$$Q_2 = \frac{P_{story}\Delta_2}{HL} \tag{8}$$

where

Q_2 = second-order stability index

This index differs from Q_1 because Q_2 uses the second-order drift, Δ_2, in lieu of the first-order drift, Δ_1. Additionally, the coefficient R_M (which accounts for member P-δ influence on structure P-Δ) does not directly figure into Q_2, but contributes to the reduced stiffness that results in the displacement Δ_2 by means of inclusion of member P-δ effects in the analysis.

The force amplifier can be expressed thus by combining Equations 5 and 8:

$$B_2 = 1 + Q_2 \tag{9}$$

Equation 9 can thus be used to determine the force amplifier based on the second-order drift, which can be assumed to equal the drift limit for approximate analysis of drift-governed buildings.

Combining Equations 4 and 9, the two indices are related by the force amplifier:

$$Q_2 = B_2 Q_1 \tag{10}$$

Design for Stability Using Drift Limit

Incorporating the preceding methods, the "indirect analysis method" (IAM) (Sabelli, 2020) may be used to permit design for stability based on the drift limit. In this method, second-order analysis lateral-load effects are further amplified by a factor B_3 that addresses stiffness-reduction effects (including member imperfections and inelasticity as well as uncertainty in member stiffness, similar to the 0.8 stiffness-reduction factor in the direct analysis method). The IAM amplifier for stiffness reduction can be computed using the B_2 amplifier and the flexural stiffness reduction parameter τ_b (taking the smallest value for τ_b at each story):

$$B_3 = \frac{0.8\tau_b}{1-(1-0.8\tau_b)B_2} \tag{11}$$

where

τ_b = flexural stiffness reduction parameter based on column axial force from AISC *Specification* Section C2.3

The parameter τ_b is equal to 1.0 for braced-frame columns and for moment-frame columns with axial force not exceeding 50% of the yield force, and thus the parameter τ_b can be taken as 1.0 for the majority of real buildings. In such cases, Equation 11 simplifies to:

$$B_3 = \frac{4}{5 - B_2} \tag{12}$$

Thus, if B_2 can be determined based on the drift limit, so too can B_3. With these two amplifiers the upper-bound of the lateral-load effect can be determined in advance of design and analysis.

These amplifiers so determined can be utilized directly in the design process or may be used in a simple hand calculation to confirm the results of a computer second-order analysis (incorporating Equation 12 for a second-order analysis with direct-analysis stiffness).

RECOMMENDED DESIGN APPROACH FOR DRIFT-GOVERNED BUILDINGS

The design of most moment-frame buildings is governed by the need for sufficient stiffness to control drift and deformation demands, rather than the need for strength, regardless of whether wind or seismic loads govern the member selection. (This is also true of some braced-frame and dual-system buildings.) In such cases, the designer can streamline the design process by selecting member sizes to maintain a target story drift considering second-order effects directly and, subsequently verifying adequate strength, using the appropriate combination of vertical and lateral loads for each evaluation. The methods presented here utilize a drift limit to estimate these second-order effects and are thus applicable to conditions in which such a drift limit applies (whether by code or as a means to limit damage to deformation-sensitive elements) and for which the drift limit is a governing criterion.

Application of this approach to seismic design is complicated by the dependency of the loading on the building period (ASCE, 2016), which is a function of the system lateral stiffness. Incorporation of that dependency into the required stiffness is beyond the scope of this paper.

Example 1: Design Example

To illustrate application methods based on second-order drift, a design example is presented, based on Carter and Geschwindner (2008), as shown in Figure 1. The example shows member selection for column A to meet a drift limit and confirmation of adequate strength, including design for stability. The example has drift limits corresponding to both serviceability and strength evaluations.

Given:

Similar to many (if not most) building structures, the example structure has no sway under gravity loads, and thus the lateral restraint force, R_{nt}, is zero. This permits the application of amplifiers B_2 and B_3 to the lateral loads or to the lateral-load effects, rather than to the effect of lateral loads plus R_{nt}. (See AISC *Specification* Appendix 8 Commentary for additional information regarding the determination and use of R_{nt}.)

Different loads and drift limits are used in the example for LRFD and serviceability evaluations. Loads for the LRFD evaluation are taken from Carter and Geschwindner (2008). The drift limits and serviceability loads are assumed here. The drift limits have been selected such that design for drift requires the member sizes from Carter and Geschwindner. Members are selected based on the minimum moment of inertia that limits second-order drift to the drift limit. The drift is based on the second-order displacement, which is approximated here by applying the amplifier B_2 to the first-order cantilever displacement:

$$\Delta_2 \leq \frac{B_2 H L^3}{3EI} \tag{13}$$

where

E = modulus of elasticity, ksi

I = moment of inertia of cantilever column, in.^{4}

For simplicity, shear deformations are not included in the analysis.

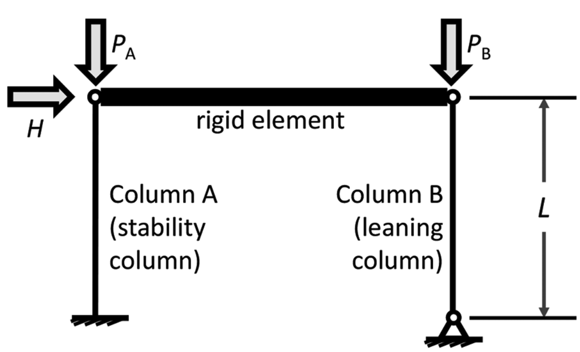

Fig. 1. Example frame.

Table 1. Loading and Design Information		
	Service	**Strength (LRFD)**
H (kips)	12	20
P_A (kips)	125	200
P_B (kips)	125	200
Drift limit	1.00 in. = $L/180$	1.80 in. = $L/100$

The required moment of inertia is:

$$I \geq \frac{B_2 H L^3}{3E\Delta_2} \tag{14}$$

Loads for the frame are shown in Table 1, along with the drift limits. The height L is 15 ft (180 in.).

Solution:

Determine Coefficient RM

From AISC *Specification* Appendix 8, Equation A-8-8, the R_M coefficient is determined using Equation 2:

$$\begin{aligned} R_M &= 1 - 0.15\frac{P_{mf}}{P_{story}} \qquad (2) \\ &= 1 - 0.15\frac{(200 \text{ kips})}{(400 \text{ kips})} \\ &= 0.925 \end{aligned}$$

Because the ratio of P_{mf} to P_{story} in this example does not change with the level of loading, this value of R_M applies to both serviceability and strength evaluations.

Service-Level Member Selection (Drift Only)

Using the service-level drift limit as the second-order drift, the upper bound of the amplifier B_2 is determined using Equation 5. This is used to determine the required moment of inertia of the cantilever column. Once member selection is made based on this service-level second-order drift limit, the first-order drift can be computed for recalculation of the force amplifier B_2 based on the actual system stiffness.

Equation 5 is used to determine the amplifier B_2 for the serviceability evaluation based on:

$$\begin{aligned} \frac{P_{story}}{H} &= \frac{(125 \text{ kips} + 125 \text{ kips})}{(12.0 \text{ kips})} \\ &= 20.8 \end{aligned}$$

$$\begin{aligned} \frac{\Delta_2}{L} &= \frac{(1.00 \text{ in.})}{(180 \text{ in.})} \\ &= 0.00556 \end{aligned}$$

The second-order stability index Q_2 is:

$$\begin{aligned} Q_2 &= \left(\frac{P_{story}}{H}\right)\left(\frac{\Delta_2}{L}\right) \qquad (8) \\ &= (20.8)(0.00556) \\ &= 0.116 \end{aligned}$$

The amplifier B_2 is:

$$B_2 = 1 + Q_2 \qquad (9)$$
$$= 1 + 0.116$$
$$= 1.12$$

The error in neglecting the second-order effect is 12% unconservative in this serviceability evaluation. If the drift limit is used as a first-order drift (including the factor R_M of 0.925), the value of B_2 obtained using Equation 4 is 1.14. (The error associated with use of the drift limit as a first-order drift is tabulated at the end of this example.)

The required moment of inertia is:

$$I \geq \frac{B_2 H L^3}{3E\Delta_2}$$
$$\geq \frac{(1.12)(12.0 \text{ kips})(180 \text{ in.})^3}{3(29{,}000 \text{ ksi})(1.00 \text{ in.})}$$
$$= 901 \text{ in.}^4$$

A W14×90 (I = 999 in.^{4}) will be used. Note that neglecting the second-order effect would result in the selection of a smaller member (W14×82, I = 881 in.^{4}), which would in turn result in not meeting the drift limit. With the selected member the first-order drift is:

$$\Delta_1 = \frac{HL^3}{3EI}$$
$$= \frac{(12.0 \text{ kips})(180 \text{ in.})^3}{3(29{,}000 \text{ ksi})(999 \text{ in.}^4)}$$
$$= 0.805 \text{ in.}$$

$$\frac{\Delta_1}{L} = \frac{(0.805 \text{ in.})}{(180 \text{ in.})}$$
$$= 0.00447$$

The first-order stability index from Equation 1 is:

$$Q_1 = \left(\frac{1}{R_M}\right)\left(\frac{P_{story}}{H}\right)\left(\frac{\Delta_1}{L}\right) \qquad (1)$$
$$= \left(\frac{1}{0.925}\right)(20.8)(0.00447)$$
$$= 0.101$$

Using this value of Q_1 with Equation 4 gives:

$$B_2 = \frac{1}{1 - Q_1} \qquad (4)$$
$$= \frac{1}{1 - 0.101}$$
$$= 1.11$$

Thus, the value of B_2 determined from Equation 5 using the target second-order drift limit is effective in determining the required member size directly without the iteration that would be required using Equation 3.

LRFD-Level Evaluation (Drift and Strength)

Amplifiers Determined Prior to Analysis and Design

Next, the stability index for the strength evaluation is determined using the appropriate vertical load. Using the LRFD load level drift limit as the second-order drift, the stability index Q_2 is used to determine the amplifiers B_2 and B_2B_3 for stability design according to the IAM. The stability index Q_2 is based on:

$$\frac{P_{story}}{H} = \frac{(200 \text{ kips} + 200 \text{ kips})}{(20.0 \text{ kips})} = 20.0$$

$$\frac{\Delta_2}{L} = \frac{(1.80 \text{ in.})}{(180 \text{ in.})} = 0.0100$$

The second-order stability index is:

$$Q_2 = \left(\frac{P_{story}}{H}\right)\left(\frac{\Delta_2}{L}\right) = (20.0)(0.0100) = 0.200 \tag{8}$$

The amplifier B_2 is:

$$B_2 = 1 + Q_2 = 1 + 0.200 = 1.20 \tag{9}$$

If the drift limit is used as a first-order drift (Q_1 = 0.216 using Equation 1, including the factor R_M), the value of B_2 obtained from Equation 4 is 1.28.

The amplifier B_3 depends on the axial force in moment-frame columns. The axial force to yield force ratio for the cantilever column is below 0.5 (justifying use of Equation 12):

$$\frac{\alpha P_r}{P_s} = \frac{\alpha P}{F_y A} = \frac{(1.0)(200 \text{ kips})}{(50 \text{ ksi})(26.5 \text{ in.}^2)} = 0.151 \le 0.50$$

The amplifier B_3 is:

$$B_3 = \frac{4}{5 - B_2} = \frac{4}{5 - 1.20} = 1.05 \tag{12}$$

The product B_2B_3 is:

$$B_2B_3 = (1.20)(1.05) = 1.26$$

The required moment of inertia is:

$$I \geq \frac{B_2 H L^3}{3E\Delta_2}$$
$$\geq \frac{(1.20)(20.0\text{ kips})(180\text{ in.})^3}{3(29{,}000\text{ ksi})(1.80\text{ in.})}$$
$$= 894\text{ in.}^4$$

A W14×90 (I = 999 in.[4]) satisfies the strength-level drift limit. At this point the member has been selected to meet the LRFD load level drift limit, and the strength evaluation can proceed using either the approximate values of B_2 and B_3 determined earlier, or more precise values determined based on the calculated drift from the selected member. While the former approach is more expedient, for purposes of comparison the latter approach will be used.

Amplifiers Determined Based on Analysis of Selected Member

The first-order drift for the selected member under LRFD loading is:

$$\Delta_1 = \frac{HL^3}{3EI}$$
$$= \frac{(20.0\text{ kips})(180\text{ in.})^3}{3(29{,}000\text{ ksi})(999\text{ in.}^4)}$$
$$= 1.34\text{ in.}$$

$$\frac{\Delta_1}{L} = \frac{(1.34\text{ in.})}{(180\text{ in.})}$$
$$= 0.00744$$

The first-order stability index, Q_1, for the LRFD strength evaluation is:

$$Q_1 = \left(\frac{1}{R_M}\right)\left(\frac{P_{story}}{H}\right)\left(\frac{\Delta_1}{L}\right) \tag{1}$$
$$= \left(\frac{1}{0.925}\right)(20.0)(0.00744)$$
$$= 0.161$$

$$B_2 = \frac{1}{1 - Q_1} \tag{4}$$
$$= \frac{1}{1 - 0.161}$$
$$= 1.19$$

$$B_3 = \frac{4}{5 - B_2} \tag{12}$$
$$= \frac{4}{5 - 1.19}$$
$$= 1.05$$

$$B_2 B_3 = (1.19)(1.05)$$
$$= 1.25$$

Note that this value of the product $B_2 B_3$ is 99% of the value obtained using the drift limit.

Table 2. Summary of B_2 Values				
Loading Level	(1) Correct Value Using Calculated First-Order Drift (Equation 1)	(2) Approximation Using Drift Limit as First-Order Drift (Equation 1)	(3) Approximation Using Target Drift Limit as Second-Order Drift (Equation 8)	(4) Ignoring Second-Order Effects (Not Recommended)
Service	1.11 (0%)	1.14 (+2.8%)	1.12 (+0.3%)	1.00 (–11.2%)
Strength	1.19 (0%)	1.28 (+7.0%)	1.20 (+0.1%)	1.00 (–19.2%)

From Carter and Geschwindner (2008), the design strengths are:

$$\phi P_c = 1{,}000 \text{ kips}$$

$$\phi M_n = 573 \text{ kip-ft}$$

The required flexural strength is determined using the B_2B_3 amplifier on the first-order load effect:

$$\begin{aligned} M_u &= B_2B_3HL \\ &= \frac{(1.25)(20.0 \text{ kips})(180 \text{ in.})}{12 \text{ in./ft}} \\ &= 375 \text{ kip-ft} \end{aligned}$$

The interaction check from AISC *Specification* Equation H1-1a using LRFD is:

$$\begin{aligned} \frac{P_r}{\phi P_c} + \left(\frac{8}{9}\right)\frac{M_u}{\phi M_n} &= \frac{(200 \text{ kips})}{(1{,}000 \text{ kips})} + \left(\frac{8}{9}\right)\frac{(375 \text{ kip-ft})}{(573 \text{ kip-ft})} \\ &= 0.782 \end{aligned}$$

Note that this is slightly lower than the value of 0.796 obtained by Carter and Geschwindner (2008) for the DM and by Sabelli (2020) for the IAM because the value of R_M was calculated in this example, as is permitted by the 2016 AISC *Specification* (AISC, 2016), rather than taken as 0.85 per the 2005 AISC *Specification* (AISC, 2005). Using $R_M = 0.85$, the first-order stability index would be $Q_1 = 0.175$; the corresponding demand-to-capacity ratio is 0.796, matching the previous studies.

Example Summary

Table 2 summarizes the values of B_2 obtained for both service-level and strength-level evaluations, including B_2: (1) determined using the calculated drift, (2) approximated using the drift limit as the first-order drift, and (3) approximated using the target drift limit as the second-order drift (as recommended in this paper). The latter two also show the percent error compared to the first. The last column (4) shows the error if second-order effects are not considered at all, which is not recommended.

Use of the target drift limit as the first-order drift is acceptable for purposes of damage control but is not recommended because of the conservative error shown in Table 2 (column 2), and the ease of utilizing the more accurate second-order methods presented herein (column 3). Ignoring second-order effects in drift determination (column 4) is not recommended due to the unconservative error, and the method's inaccuracy potentially leading to damage in cladding, partitions, and other building components.

Use of the drift limit as the second-order drift from Equations 5, 8, and 9 (as proposed in this paper) results in negligible overestimates for drift-governed designs as compared with using Equations 1, 3, and 4, which would normally require iteration. For cases utilizing the latter approach in which the selected members result in second-order drifts significantly below the drift limit, the use of the refined analysis with the selected member stiffness (using the first-order stability index and further iterations on member size) may permit refinement of the design but with more effort.

Table 3. Summary of Eight-Story Frame Loading, Criteria, and Analysis Results										
General		Drift Loading and Criteria			Drift Evaluation	Strength Loading		Strength Evaluation		
Level	L (in.)	P_{story} (kips)	$H_{service}$ (kips)	$\Delta_{allowable}$ (in.)	$\frac{\Delta_2}{\Delta_{all}}$	P_{story} (kips)	H (kips)	Δ_1 (in.)	Δ_2 (in.)	$\frac{\Delta_2}{\Delta_1}$
8	180	2,000	20.0	0.450	0.61	2,400	30	0.39	0.49	1.25
7	180	4,400	40.0	0.450	0.96	5,600	60	0.59	0.79	1.35
6	180	6,800	60.0	0.450	0.89	8,800	90	0.54	0.74	1.36
5	180	9,200	80.0	0.450	0.95	12,000	120	0.57	0.79	1.38
4	180	11,600	100.0	0.450	0.93	15,200	150	0.56	0.78	1.39
3	180	14,000	120.0	0.450	1.00	18,400	180	0.60	0.84	**1.41**
2	180	16,400	140.0	0.450	0.93	21,600	210	0.56	0.78	1.38
1	**180**	18,800	**160.0**	**0.450**	0.72	**24,800**	240	0.45	0.59	1.32

Example 2: Computer-Analysis Review Example

While Example 1 illustrates the application to a simple structure designed by hand, in current practice, computer analysis is utilized for the majority of structures of any significant complexity. Nevertheless, engineers should be equipped to critically evaluate the results of such analyses using simple methods in order to prevent errors.

In this section, the methods described earlier are utilized to evaluate the results of the Appendix C example from Sabelli (2020). That example presents the design and analysis (including second-order analysis) of an eight-story building with two-bay moment frames on the perimeter. For simplicity, an evaluation is made based on loads at the bottom story here, and a hand-calculated amplifier B_2 is compared to the drift amplification from a second-order analysis. The hand calculation does not rely on any analysis results, although it assumes the building drift is equal to the drift limit.

Given:

Selected design criteria, loading, and analysis results for the eight-story building are presented in Table 3; all information is taken from the Appendix C example from Sabelli (2020). Values used in the example are shown in bold. Readers are referred to Sabelli (2020) for more information.

Computation of Amplifier B_2

The required system effective stiffness based on the drift-design criteria is:

$$\frac{H}{\Delta_2} = \frac{(160\text{ kips})}{(0.450\text{ in.})}$$
$$= 356\text{ kips/in.}$$

The second-order stability index, Q_2, for the strength evaluation (using the strength-level vertical loads) is:

$$Q_2 = \left(\frac{P_{story}}{L}\right)\left(\frac{\Delta_2}{H}\right) \tag{8}$$
$$= \left(\frac{24{,}800\text{ kips}}{180\text{ in.}}\right)\frac{1}{356\text{ kips/in.}}$$
$$= 0.388$$

The corresponding amplifier B_2 is:

$$B_2 = 1 + Q_2 \\ = 1 + 0.388 \\ = 1.39 \tag{9}$$

Table 3 shows the ratio of second-order drift to first-order drift ranging from 1.25 to 1.41 for the strength evaluation, with the highest value corresponding to the floor with the drift exactly at the drift limit for the drift evaluation. [Note that the ratio of second-order drift to first-order drift only approximates the force amplification, per Equation 7, and as discussed in Sabelli and Griffis (2021).] The simple hand calculation above confirms the second-order analysis, giving the engineer higher confidence in the results. A similar hand calculation could be made incorporating Equation 12 to verify the direct analysis method results.

CONCLUSIONS

Equations for the force amplifier (B_2) are presented that utilize a second-order stability index, which can be based on a drift limit. Two examples are presented. A design example shows the application of the methods, determining the amplifiers prior to design based on the target drift limit. That example confirms that the methods result in very close approximations of the amplifiers determined after member selection for the simple, drift-governed design presented. A second example illustrates the use of the second-order stability index to estimate the magnitude of the second-order effect prior to member selection and building analysis. The amplification value so determined is compared to the second-order effect from a computer analysis, providing confirmation of that analysis.

SYMBOLS

B_2 Amplifier for second-order effect (AISC *Specification* Appendix 8)

B_3 IAM amplifier to account for stiffness reduction due to inelasticity (Sabelli, 2020)

E Modulus of elasticity, ksi (AISC *Specification*)

H First-order shear at the load level under consideration, kips

I Moment of inertia, in.^{4}

L Story height, in.

P_{lean} Load on leaning columns, kips

P_{mf} Load on moment-frame columns, kips

P_{story} Story gravity load, kips

Q_1 First-order stability index (Q in Design Guide 28)

Q_2 Second-order stability index

R_M Coefficient to account for member P-δ influence on structure P-Δ

R_{nt} Lateral reaction of frame restrained from translation used in approximate second-order analysis, kips (AISC *Specification* commentary)

Δ_1 First-order story drift, in. (Δ_H in the AISC *Specification*)

Δ_2 Second-order story drift, in. (Design Guide 28)

τ_b Flexural stiffness reduction parameter (AISC *Specification* Section C2.3)

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