Technical Note

Notes on Determining Required Connector Strength in Built-up Compression Members

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INTRODUCTION

TC-4, the Member Design task committee of the AISC Committee on Specifications, was asked to provide guidance to the profession on determining the required strength of connectors in built-up compression members. The following, presented on behalf of the task committee, is based on material presented to the committee by Todd Helwig and derived in large part by Joseph Yura.

Connections between individual components of a built-up compression member carry no force when the member is perfectly straight in the unbuckled configuration under load. Yet AISC *Specification for Structural Steel Buildings* (AISC, 2016), hereafter referred to as the AISC *Specification*, Section E6.1, requires that the end connection of these built-up members be welded or connected by means of pretensioned bolts with Class A or B faying surfaces. Nothing is provided in the AISC *Specification* or *Commentary* to help the designer determine the required strength of these connectors. This paper suggests one way of determining these required strengths and provides two LRFD examples. Other assumptions may be used to derive different, but equally acceptable, required strengths.

REQUIRED CONNECTOR SHEAR FORCE

To determine the required connector shear force, the built-up compression member will be assumed to be out-of-straight by an amount equivalent to that for a rolled compression member and in the form of a sine curve. That out-of-straightness will be amplified to account for second-order effects and the shear force distribution will be determined based on that amplified out-of-straightness.

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Figure 1 shows a built-up, double-angle compression member with an out-of-straightness, Δ, and an axial load, P. Figure 1(b) is a free-body diagram consisting of the left half of the member. For equilibrium in the deformed configuration, a moment, M, is necessary at the mid-point of the member. If the deformed shape is assumed to follow a sine curve, the moment equation taken about the deformed position may be written as

$$M_x = P\Delta \sin\frac{\pi x}{L} \tag{1}$$

The moment diagram is shown in Figure 1(c). Shear along the member is determined by taking the derivative of the moment so that

$$V_x = \frac{dM_x}{dx} = P\Delta\frac{\pi}{L}\cos\frac{\pi x}{L} \tag{2}$$

and the shear diagram is given in Figure 1(d). The maximum shear is at the end where $x = 0$. Thus, $V_{max} = P\Delta\pi/L$. The shear force per unit length at the interface of the connected angles is determined as

$$v_x = \frac{V_x Q_y}{I_y} = \frac{Q_y}{I_y}P\Delta\frac{\pi}{L}\cos\frac{\pi x}{L} \tag{3}$$

where Q_y is the first moment of the area of one angle about the y-axis of symmetry of the double-angle member, and I_y is the moment of inertia of the double-angle member about the y-axis.

Thus, the total shear force to be resisted between the mid-point of the compression member and its end is determined by integrating the shear force per unit length, Equation 3, from zero to $L/2$, giving

$$\begin{aligned} V_{Total} &= \int_0^{L/2}\frac{V_x Q_y}{I_y}dx \\ &= \frac{Q_y}{I_y}P\Delta\frac{\pi}{L}\int_0^{L/2}\cos\frac{\pi x}{L}dx \\ &= \frac{Q_y}{I_y}P\Delta\frac{\pi}{L}\Big|_0^{L/2}\frac{L}{\pi}\sin\frac{\pi x}{L} \\ &= \frac{Q_y P\Delta}{I_y} \end{aligned} \tag{4}$$

If there were connectors at the ends only, this would be the shear force these end connectors must be designed to resist.

However, if there are also intermediate connectors, these end connectors may not be required to provide as much resistance. Consider intermediate connectors at the third points of the length. The total shear force between midpoint and third point can be determined by integrating Equation 3 from $L/3$ to $L/2$.

$$\begin{aligned} V_{L/3} &= \int_{L/3}^{L/2} \frac{V_x Q_y}{I_y} dx \\ &= \frac{Q_y}{I_y} P\Delta \frac{\pi}{L} \int_{L/3}^{L/2} \cos\frac{\pi x}{L} dx \\ &= \frac{Q_y}{I_y} P\Delta \frac{\pi}{L} \left|_{L/3}^{L/2} \frac{L}{\pi} \sin\frac{\pi x}{L} \right. \\ &= \frac{Q_y P\Delta}{I_y}\left(\sin\frac{\pi}{2} - \sin\frac{\pi}{3}\right) \\ &= 0.133\frac{Q_y P\Delta}{I_y} \end{aligned} \tag{5}$$

Therefore, the connector at the one-third point only carries 13% of the total shear, and the end connector carries 87% of the total shear. For simplicity, the required strength of the end connectors is taken as the total shear, regardless of the number of intermediate connectors. Therefore,

$$V_r = V_{Total} = \frac{Q_y P\Delta}{I_y} \tag{6}$$

The displacement at the column midpoint, Δ, can be estimated by amplifying the initial out-of-straightness, Δ_o, by the second-order amplification factor, B_1, so that

$$\Delta = B_1 \Delta_o \tag{7}$$

If the applied load is taken as the required strength, P_r, and Equation 7 is substituted into Equation 6, the required shear strength becomes

$$V_r = \frac{Q_y P\Delta}{I_y} = B_1 \Delta_o P_r \frac{Q_y}{I_y} \tag{8}$$

The out-of-straightness tolerance for a rolled compression member, as given in ASTM A6-14 (ASTM, 2014), may be taken as $\Delta_o = 0.001L$, where L is the length of the member. Thus, Equation 8 becomes

$$V_r = 0.001 B_1 P_r \frac{L Q_y}{I_y} \tag{9}$$

The amplification factor, as given in AISC *Specification* Appendix 8, Equation A-8-3, is

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{e1}} \tag{10}$$

If the required strength is exactly equal to the available strength in the elastic region, using LRFD,

$$P_r = \phi P_n = 0.9(0.877 P_e) \tag{11}$$

Since $P_{e1} = P_e$, C_m is conservatively taken as 1.0, and $\alpha = 1.0$ (LRFD) or 1.6 (ASD), the amplification for LRFD becomes

$$B_1 = \frac{1}{1 - \dfrac{1.0(0.9)(0.877 P_e)}{P_e}} = 4.75 \tag{12}$$

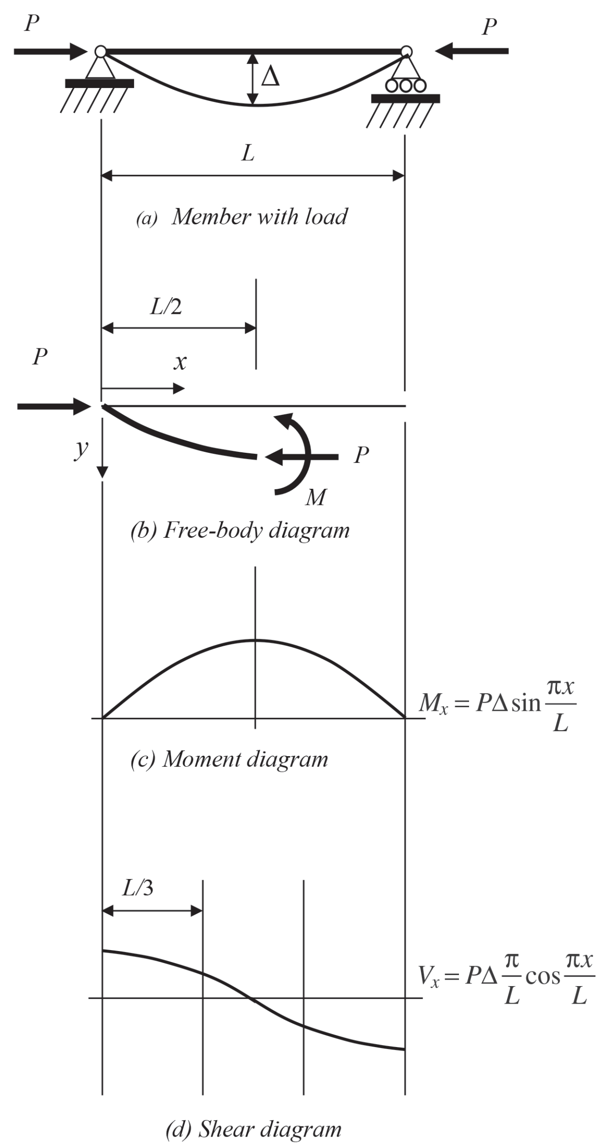

Fig. 1. Double-angle built-up compression member.

Thus, Equation 9 becomes

$$V_r = 0.001(4.75)P_r\frac{LQ_y}{I_y} = 0.00475P_r\frac{LQ_y}{I_y} \qquad (13)$$

Because of the conservatism associated with the use of α to convert from ASD loads to ultimate loads for consideration of second-order effects, the amplification when required strength is exactly equal to available strength using ASD increases to B_1 = 6.26. Example 2 will show that it is not necessary to use these extreme B_1 values when the actual required strength is known and a better measure of second-order effects can be determined.

EXAMPLE 1

Given:

Determine the required end connectors for an ASTM A36 2L5×3×7⁄16 LLBB (3⁄8-in. separation) double-angle compression member, with length of 10 ft, using maximum second-order amplification. Use 3⁄4-in.-diameter Group A slip-critical bolts in standard holes. The required axial force is P_u = 100 kips (LRFD) and P_a = 66.7 kips (ASD).

Solution:

From the AISC *Steel Construction Manual* (AISC, 2017) Table 2-4, the material properties are as follows:

ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L5×3×7⁄16 (single angle)
$A = 3.31$ in.^{2}
$I_y = 2.29$ in.^{4}
$\bar{x} = 0.722$ in.

The available strength in axial compression is taken from the bottom (*y-y* axis) portion of AISC *Manual* Table 4-9:

LRFD	ASD
$\phi_c P_n = 117 \text{ kips} > 100 \text{ kips}$ **o.k.**	$\frac{P_n}{\Omega_c} = 78.0 \text{ kips} > 66.7 \text{ kips}$ **o.k.**

For buckling about the *y-y* axis (axis of symmetry), the connectors will be in shear; thus, the geometric properties for the *y-y* axis are required.

$$I_{y\,double} = \sum(I_y + Ad^2)_{single}$$
$$= (2)\left\{2.29 \text{ in.}^4 + (3.31 \text{ in.}^2)\left[0.722 \text{ in.} + \left(\frac{3/8 \text{ in.}}{2}\right)\right]^2\right\}$$
$$= 10.1 \text{ in.}^4$$

The first moment of the area of one angle about the *y*-axis of symmetry of the double-angle member, where $\bar{x}$ is the distance to the centroid for a single angle measured from the axis parallel to the axis of symmetry, is:

$$Q_y = A\left(\bar{x} + \frac{s}{2}\right)$$
$$= \left(3.31 \text{ in.}^2\right)\left[0.722 \text{ in.} + \left(\frac{3/8 \text{ in.}}{2}\right)\right]$$
$$= 3.01 \text{ in.}^3$$

Determine the required shear strength of the end connectors:

LRFD	ASD
From Equation 13: $$V_r = 0.00475 P_r \frac{LQ_y}{I_y}$$ $$= 0.00475(100\text{ kips})\frac{(10\text{ ft})(12\text{ in./ft})(3.02\text{ in.}^3)}{10.1\text{ in.}^4}$$ $$= 17.0\text{ kips}$$	From Equation 9, with $B_1 = 6.26$: $$V_r = 0.001 B_1 P_r \frac{LQ_y}{I_y}$$ $$= 0.001(6.26)(66.7\text{ kips})\frac{(10\text{ ft})(12\text{ in./ft})(3.02\text{ in.}^3)}{10.1\text{ in.}^4}$$ $$= 15.0\text{ kips}$$

Determine the required number of ¾-in.-diameter Group A slip-critical bolts in standard holes:

LRFD	ASD
From *Manual* Table 7-3: $$\phi r_n = 9.49\text{ kips}$$ The required number of bolts is: $$\frac{V_r}{\phi r_n} = \frac{17.0\text{ kips}}{9.49\text{ kips}} = 1.79$$	From *Manual* Table 7-3: $$\frac{r_n}{\Omega} = 6.33\text{ kips}$$ The required number of bolts is: $$\frac{V_r}{r_n/\Omega} = \frac{15.0\text{ kips}}{6.33\text{ kips}} = 2.37$$
Use two ¾-in.-diameter Group A slip-critical bolts at each end in standard holes with a Class A or B faying surface.	Use three ¾-in.-diameter Group A slip-critical bolts at each end in standard holes with a Class A or B faying surface.

EXAMPLE 2

Given:

Reconsider the problem of Example 1 if the second-order amplification is determined based on the actual required strength, Equation 10, rather than the maximum strength in the elastic region, as was done in Example 1.

Solution:

The elastic critical buckling strength is determined from AISC *Specification* Appendix 8, Equation A-8-5. From Example 1, $I_{y\,double} = 10.1\text{ in.}^4$

$$P_{e1} = \frac{\pi^2 EI}{L_c^2} \qquad \text{(from \textit{Spec}. Eq. A-8-5)}$$

$$= \frac{\pi^2(29{,}000\text{ ksi})(10.1\text{ in.}^4)}{\left[(10\text{ ft})(12\text{ in./ft})\right]^2}$$

$$= 200\text{ kips}$$

The amplification factor, as given in AISC *Specification* Appendix 8, Equation A-8-3, is

LRFD	ASD
$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{e1}} = \frac{1.0}{1 - \frac{1.0(100\text{ kips})}{200\text{ kips}}} = 2.00$$	$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{e1}} = \frac{1.0}{1 - \frac{1.6(66.7\text{ kips})}{200\text{ kips}}} = 2.14$$

Determine the required shear strength of the end connectors using Equation 13:

LRFD	ASD
$$V_r = 0.001 B_1 P_r \frac{LQ_y}{I_y} = 0.001(2.00)(100\text{ kips})\frac{(10\text{ ft})(12\text{ in./ft})(3.02\text{ in.}^3)}{10.1\text{ in.}^4} = 7.18\text{ kips}$$	$$V_r = 0.001 B_1 P_r \frac{LQ_y}{I_y} = 0.001(2.14)(66.7\text{ kips})\frac{(10\text{ ft})(12\text{ in./ft})(3.02\text{ in.}^3)}{10.1\text{ in.}^4} = 5.12\text{ kips}$$

Determine the required number of ¾-in.-diameter Group A slip-critical bolts in standard holes:

LRFD	ASD
From *Manual* Table 7-3: $$\phi r_n = 9.49\text{ kips}$$	From *Manual* Table 7-3: $$\frac{r_n}{\Omega} = 6.33\text{ kips}$$
The required number of bolts is: $$\frac{V_r}{\phi r_n} = \frac{7.18\text{ kips}}{9.49\text{ kips}} = 0.757$$	The required number of bolts is: $$\frac{V_r}{r_n/\Omega} = \frac{5.12\text{ kips}}{6.33\text{ kips}} = 0.809$$
It would be acceptable to use one ¾-in.-diameter Group A slip-critical bolt at each end in standard holes with a Class A or B faying surface.	It would be acceptable to use one ¾-in.-diameter Group A slip-critical bolt at each end in standard holes with a Class A or B faying surface.

If the intermediate bolts at the one-third points only carry 13% of the total shear, it is clear that any high-strength bolt will be acceptable. Although these intermediate bolts appear to resist very little force in these calculations, they are critical in determining the overall column strength because of their impact on determining the built-up member effective length as specified in AISC *Specification* Section E6.1.

When the end connection of a built-up compression member is also required to transfer the full compressive load to the remainder of the structure, such as a double-angle compression member in a truss, the end connection may be designed to transfer the load through a bearing connection but the bolts must be pretensioned.

CONCLUSIONS

The 2016 AISC *Specification* requires that the connectors at the ends of built-up compression members be designed as welds or slip-critical bolts with Class A or B faying surfaces. However, it does not provide guidance on the required strength of those connectors. This paper provides one approach to determining the required strength of these end connectors based on the initial out-of-straightness specified in ASTM A6-14 and the second-order amplification given in AISC *Specification* Appendix 8. It further recommends that the end connectors be designed for the entire shear force, regardless of the number of intermediate connectors because these intermediate connectors take only a small portion of the total shear force.

Other assumptions may lead to different required shear strength, but because the number of required connectors is fairly small, a significant change in the number of slip-critical bolts is unlikely.

REFERENCES

AISC (2016), *Specification for Structural Steel Buildings*, ANSI/AISC 360-16, American Institute of Steel Construction, Chicago, Ill.

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