

Notes on the AISC 360-16 Provisions for Slender Compression Elements in Compression Members

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ABSTRACT

Compression member strength is controlled by the limit states of flexural buckling, torsional buckling, and flexural-torsional buckling, as applicable. These compression members may buckle globally or locally, depending on the overall column slenderness and the local plate element slenderness for the plates that make up the shape. If any of the plate elements will buckle at a stress lower than that which would cause the column to buckle globally, the local buckling of the plate will control the overall column strength. When this occurs, the column is said to be composed of slender elements.

This paper will briefly discuss past specification provisions for slender element compression members and introduce the new provisions in the 2016 AISC *Specification*. It will present a simplification that reduces the number of constants that must be used and will present the specification requirements in an alternate format. Because the 2016 requirements result in different strengths than the 2010 requirements, figures are provided to illustrate the overall impact of these changes on column strength.

Key Words: compression members, plate buckling, slender elements, AISC *Specification*.

INTRODUCTION

Compression member strength is controlled by the limit states of flexural buckling, torsional buckling and flexural-torsional buckling, as applicable (AISC, 2010). These compression members may buckle globally or locally, depending on the overall column slenderness and the local plate element slenderness for the plates that make up the shape. If any of the plate elements will buckle at a stress lower than that which would cause the column to buckle globally, the local buckling of the plate will control the overall column strength. When this occurs, the column is said to be composed of slender elements.

This paper briefly discusses past specification provisions for slender element compression members and introduces the new provisions in the 2016 AISC *Specification*. It will present a simplification that reduces the number of constants that must be used and will present the specification requirements in an alternate format. Because the 2016 requirements result in different strengths than the 2010 requirements, figures are provided to illustrate the overall impact of these changes on column strength.

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HISTORICAL PERSPECTIVE

The AISC *Specification* approach for determining the element slenderness at which local buckling begins to control column strength has evolved over the years. Prior to the 1961 AISC *Specification*, a simple, maximum, width-to-thickness ratio was specified. For instance, in the 1949 *Specification*, the projecting elements of single-angle struts had a limiting width-to-thickness ratio of 12. In the 1961 *Specification*, the provisions were revised to include recognition that new materials with different yield strengths were being used and that yield strength of the material then played a role in determining at what stress level local buckling should be considered. The limit was changed to $2,400/\sqrt{F_y}$, where F_y was taken in pounds per square inch. In 1969, the limit was essentially unchanged but was presented as $76.0/\sqrt{F_y}$ with F_y now taken in kips per square inch. In order to convert the 1993 LRFD *Specification* to metric units, the 1994 Metric LRFD *Specification* set the limit as a unitless equation by restoring the variable E in the limit. Thus, this same limit became $0.45\sqrt{E/F_y}$. Over that same period of time, several new elements were defined. For the 2010 *Specification*, there were nine cases defined in Table B4.1a for the limiting width-to-thickness ratios for compression elements in members subject to axial compression. However, the actual limits were essentially the same as they had been since 1961.

During this same period, the approach to account for the influence of elements that exceeded these limitations also evolved. Prior to the 1969 *Specification*, the practice was to remove the width of the plate that exceeded the limitation. This approach required the section properties to be recalculated based on this new geometry, a cumbersome and

uneconomical approach. With the 1969 *Specification* (AISC, 1969), a new approach was introduced that followed the approach used in the 1969 *AISI Specification for the Design of Cold-Formed Steel Structural Members* (AISI, 1969). A reduction factor, Q , was defined as the ratio of the local buckling stress to the yield stress for members with slender elements. In the column strength equations, F_y was replaced by QF_y . Two separate approaches were used for determining Q . One was for unstiffened elements, which were assumed to reach their limit state when the element reached its local buckling stress. The other was for stiffened elements, which made use of their post-buckling strength. For unstiffened elements, Q was directly determined through specification equations based on material and geometric properties of the elements. For stiffened elements, an effective width was determined, and the ratio of the effective area to the gross area was used to establish Q . This approach was based on the actual stress in the member under the buckling load rather than the yield stress as was used for unstiffened elements. The provisions in the 2016 *AISC Specification* use the effective width approach for both stiffened and unstiffened elements following the practice used by AISI for cold-formed members since 2001 (AISI, 2001).

2016 SLENDERNESS PROVISIONS

To determine if one must even consider element slenderness in determining column strength, there needs to be some value against which the element width-to-thickness ratio can be compared. As has been the case since the 1961 *Specification*, when F_y was introduced as part of the limiting ratio, the assumption used to determine that limit is that the member can be uniformly stressed to the yield stress even though compression members are rarely stressed to this level. This limit, when exceeded, is used to direct the designer to Section E7, "Members with Slender Elements," of the *Specification* (AISC, 2010). This assumption caused some designers difficulty when they subsequently determined, after following all the requirements of Section E7, that the section strength was not reduced due to element slenderness. This can be understood by recognizing that the member is not stressed to the yield stress, as originally assumed to direct the designer to these provisions, so the element is less likely to buckle. Although the limits shown in Section E7 for 2016 now include the critical stress for the column determined without consideration of slender elements, it is still the width-to-thickness limit based on F_y from *Specification* Table B4.1a that tells the designer to consider the slender element provisions.

The 2016 provisions are written in a unified form for both stiffened and unstiffened elements using the effective width formulation for all but round HSS. This change is not so

much the result of new research as it is a reinterpretation of the foundational work of von Kármán et al. (1932), Winter (1947), and Peköz (1987), as summarized in Ziemian (2010). The effective widths are used to determine the effective area, and that area is multiplied by the critical stress, determined without consideration of slender elements, to obtain the nominal compressive strength. The 2016 provisions, except for round HSS, are given as:

$$(a) \text{ When } \lambda \leq \lambda_r \sqrt{\frac{F_y}{F_{cr}}}$$

$$b_e = b \quad (2016 \text{ Spec. Eq. E7-2})$$

$$(b) \text{ When } \lambda > \lambda_r \sqrt{\frac{F_y}{F_{cr}}}$$

$$b_e = b \left(1 - c_1 \sqrt{\frac{F_{el}}{F_{cr}}} \right) \sqrt{\frac{F_{el}}{F_{cr}}}$$

$$(2016 \text{ Spec. Eq. E7-3})$$

where b is the element width, b_e is the element effective width, and F_{cr} is the critical stress determined in accordance with Section E3 or E4 without consideration of slender elements.

The limiting slenderness, λ_r , is taken from Table B4.1a and, in all cases, is a function of $\sqrt{E/F_y}$. The width-to-thickness ratio, λ , is, according to Table B4.1a, b/t , d/t or h/t , depending on the element being considered. Thus, the widths in Equations E7-2 and E7-3 will also be taken as b , d or h , depending on the element being considered.

The elastic local buckling stress, F_{el} , from classic plate buckling theory (Ziemian, 2010) is

$$F_{el} = k \frac{\pi^2 E}{12(1-\nu^2) \left(\frac{b}{t} \right)^2} \quad (1)$$

which is written in the 2016 *Specification* as

$$F_{el} = \left(c_2 \frac{\lambda_r}{\lambda} \right)^2 F_y$$

$$(2016 \text{ Spec. Eq. E7-4})$$

The constant c_1 is the empirical correction factor associated with imperfection sensitivity and c_2 is a constant determined by c_1 alone and used only for convenience. The constants c_1 and c_2 , given in 2016 *Specification* Table E7.1, are

Case	Slender Element	c_1	c_2
(a)	Stiffened elements except walls of square and rectangular HSS	0.18	1.31
(b)	Walls of square and rectangular HSS	0.20	1.38
(c)	All other elements	0.22	1.49

Table B4.1a Case	Table E7.1 Case	k_c	c_1	c_2	c_3	c_4	c_5	Appendix A Equation Number
1	(c)	1.0	0.22	1.49	0.56	0.834	0.184	A-3
2	(c)	k_c	0.22	1.49	0.64	0.954	0.210	A-5
3	(c)	1.0	0.22	1.49	0.45	0.671	0.148	A-9
4	(c)	1.0	0.22	1.49	0.75	1.12	0.246	A-7
5	(a)	1.0	0.18	1.31	1.49	1.95	0.351	A-11
6	(b)	1.0	0.20	1.38	1.40	1.93	0.386	A-15
7	(a)	1.0	0.18	1.31	1.40	1.83	0.330	A-13
8	(a)	1.0	0.18	1.31	1.49	1.95	0.351	A-11

2016 SLENDERNESS PROVISIONS —SIMPLIFIED

Because these provisions require the use of the tabulated limiting slenderness ratio from Table B4.1a and the constants c_1 and c_2 from Table E7.1 each time a particular type element is considered, it may be helpful for the user to combine them all one time and then use this new equation. To accomplish this simplification, the limits from Table B4.1a are taken as $\lambda_r = c_3 \sqrt{\frac{k_c E}{F_y}}$, so that the resulting equation can be used for all cases covered in that table except for round HSS. The variable k_c is taken as 1.0 for all cases in Table B4.1a, except Case 2 (flanges of built-up I-shaped sections and plates or angles projecting from built-up I-shaped sections), where it can vary from 0.35 to 0.76 (no change from earlier *Specifications*). Thus, the limit on application of Equation E7-3 becomes

$$\frac{b}{t} = \lambda > \lambda_r \sqrt{\frac{F_y}{F_{cr}}} = c_3 \sqrt{\frac{k_c E}{F_y}} \sqrt{\frac{F_y}{F_{cr}}} = c_3 \sqrt{\frac{k_c E}{F_{cr}}} \quad (2)$$

Then determine F_{el} in terms of c_3 . Thus,

$$F_{el} = \left(c_2 \frac{\lambda_r}{\lambda} \right)^2 F_y = \left[c_2 \left(\frac{c_3 \sqrt{\frac{k_c E}{F_y}}}{b/t} \right) \right]^2 F_y = \left(\frac{c_2 c_3 \sqrt{\frac{k_c E}{F_y}}}{b/t} \right)^2 F_y \quad (3)$$

Substituting F_{el} from Equation 3 into Equation E7-3 yields

$$b_e = b \left(1 - c_1 \sqrt{\frac{F_{el}}{F_{cr}}} \right) \sqrt{\frac{F_{el}}{F_{cr}}} \\ = b \left[1 - c_1 \sqrt{\frac{\left(\frac{c_2 c_3 \sqrt{\frac{k_c E}{F_y}}}{b/t} \right)^2 F_y}{F_{cr}}} \right] \sqrt{\frac{\left(\frac{c_2 c_3 \sqrt{\frac{k_c E}{F_y}}}{b/t} \right)^2 F_y}{F_{cr}}} \quad (4)$$

which simplifies to

$$b_e = c_2 c_3 t \sqrt{\frac{k_c E}{F_{cr}}} \left[1 - \frac{c_1 c_2 c_3}{(b/t)} \sqrt{\frac{k_c E}{F_{cr}}} \right] \quad (5)$$

Combining the constants in Equation 5 yields

$$b_e = c_4 t \sqrt{\frac{k_c E}{F_{cr}}} \left[1 - \frac{c_5}{(b/t)} \sqrt{\frac{k_c E}{F_{cr}}} \right] \quad (6)$$

where $c_4 = c_2 c_3$ and $c_5 = c_1 c_2 c_3$.

For all cases in Table B4.1a, except for round HSS for which the specification provisions are different and remain essentially unchanged from 2010, the constants are as tabulated in Table 1.

Table 2. Description of Shapes Used to Develop Figures				
Figure	Shape	F_y , ksi	Element Slenderness	Limiting Slenderness for Local Buckling
1	W30×90	50	$\frac{h}{t_w} = 57.5$	$1.49 \sqrt{\frac{E}{F_y}} = 35.9$
2	HSS16×4× $\frac{3}{16}$	46	$\frac{h}{t} = 89$	$1.40 \sqrt{\frac{E}{F_y}} = 35.2$
3	WT15×45	50	$\frac{d}{t_w} = 31.5$	$0.75 \sqrt{\frac{E}{F_y}} = 18.1$
4	L5×3× $\frac{1}{4}$	36	$\frac{d}{t} = 20$	$0.45 \sqrt{\frac{E}{F_y}} = 12.8$
5	Built-up I-shape (slender flange) Flange: 24 in. × $\frac{1}{2}$ in. Web: 24 in. × $\frac{3}{4}$ in.	50	$\frac{b_r}{2t_f} = 24$ $\frac{h}{t_w} = 32$	$0.56 \sqrt{\frac{E}{F_y}} = 13.5$ $1.49 \sqrt{\frac{E}{F_y}} = 35.9$
6	Built-up I-shape (slender flange and web) Flange: 24 in. × $\frac{1}{2}$ in. Web: 24 in. × $\frac{1}{2}$ in.	50	$\frac{b_r}{2t_f} = 24$ $\frac{h}{t_w} = 48$	$0.56 \sqrt{\frac{E}{F_y}} = 13.5$ $1.49 \sqrt{\frac{E}{F_y}} = 35.9$

Thus, for webs of doubly symmetric rolled I-shaped sections—Case 5 in Table B4.1a and Case (a) in Table E7.1—the following constants are determined:

$$\begin{aligned} c_1 &= 0.18 \\ c_2 &= 1.31 \\ c_3 &= 1.49 \\ c_4 &= 1.95 \\ c_5 &= 0.351 \end{aligned}$$

and Equation E7-3 becomes, from Equation 6,

$$b_e = 1.95t \sqrt{\frac{E}{F_{cr}}} \left[1 - \frac{0.351}{(b/t)} \sqrt{\frac{E}{F_{cr}}} \right] \quad (7)$$

This effective width equation is very close to Equation E7-17 from the 2010 *Specification*, with the constants only slightly different. In addition, F_{cr} here is the same as f in Equation E7-17. The 2016 provisions are rewritten using Equation 6 and presented in full in this paper's Appendix.

The same comparison to the 2010 *Specification* cannot be made for unstiffened elements because the effective width approach in the 2016 *Specification* is a new approach for those elements.

IMPACT OF 2016 PROVISIONS

It is the intent of these new 2016 provisions to reduce the complex nature of the previous slender element provisions and to present a unified approach for both stiffened and unstiffened elements. In some instances, the changes implemented for 2016 will have little to no impact on the strength of slender element compression members, while in other instances, they may yield a significant increase in predicted strength. Where significant strength increase is seen with the 2016 provisions, the overly conservative nature of the previous provisions has been reduced.

Figures 1 through 6 illustrate the nominal strength for several slender element compression members, showing the results of the 2010 provisions and those of the 2016 provisions. As an aid to understanding the overall significance of slender elements on reducing column strength, the nominal strength, with the reduction for slender elements ignored, is also shown. The shapes used for Figures 1 through 6 and their element slenderness values are tabulated in Table 2.

In each of the cases presented, the rolled shape was selected because it is the one with the most slender element for that shape. The built-up shape was selected as an extreme case to illustrate the significance of the new provisions for

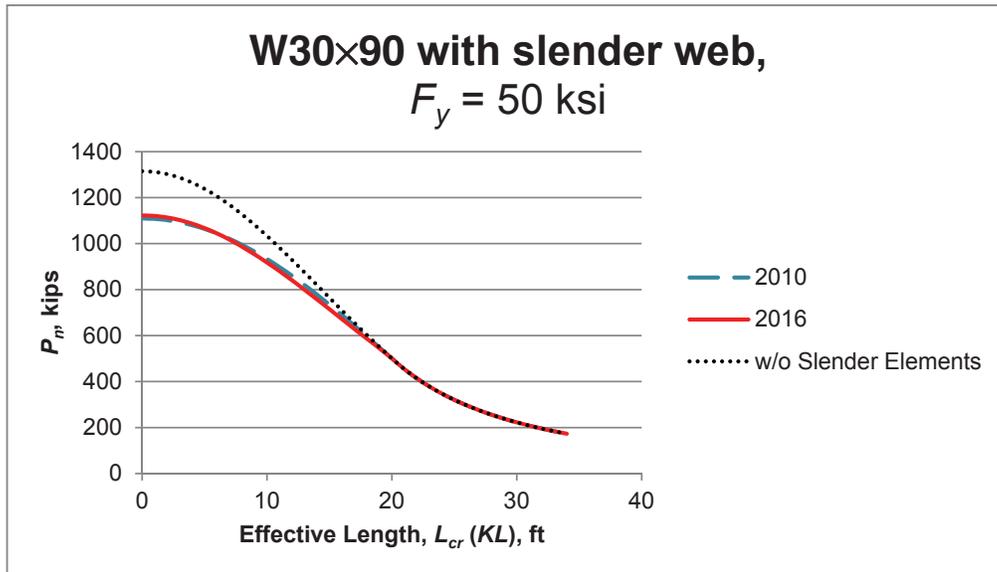


Fig. 1. Comparison of 2010 and 2016 slender element column strength, W30×90, $F_y = 50$ ksi.

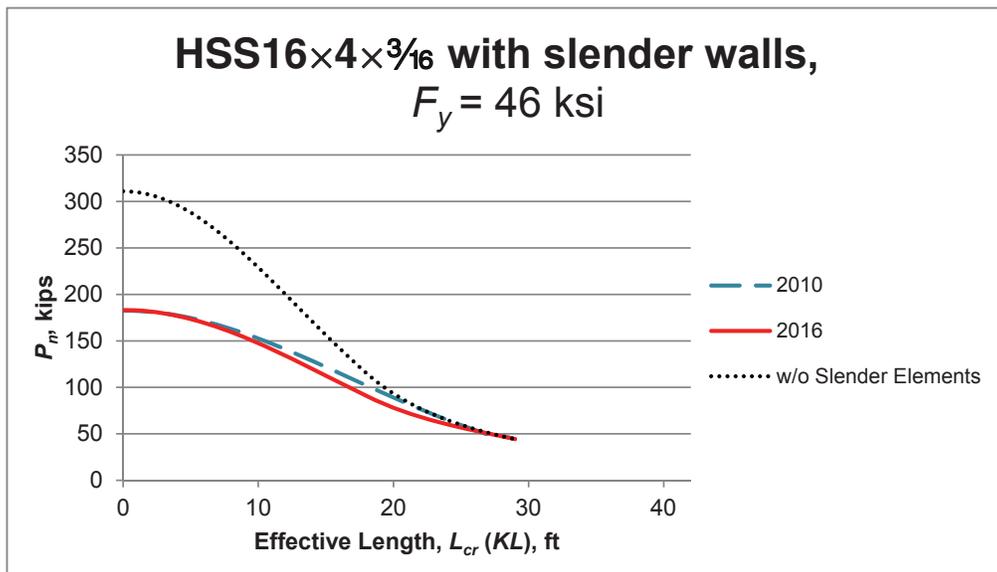


Fig. 2. Comparison of 2010 and 2016 slender element column strength, HSS16×4× $\frac{3}{16}$, $F_y = 46$ ksi.

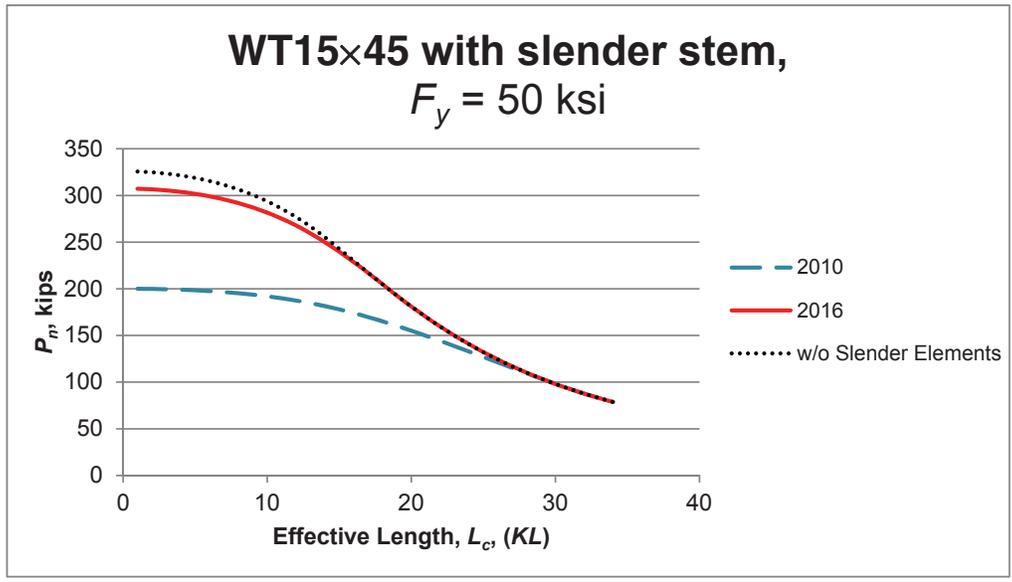


Fig. 3. Comparison of 2010 and 2016 slender element column strength, WT15×45, $F_y = 50$ ksi.

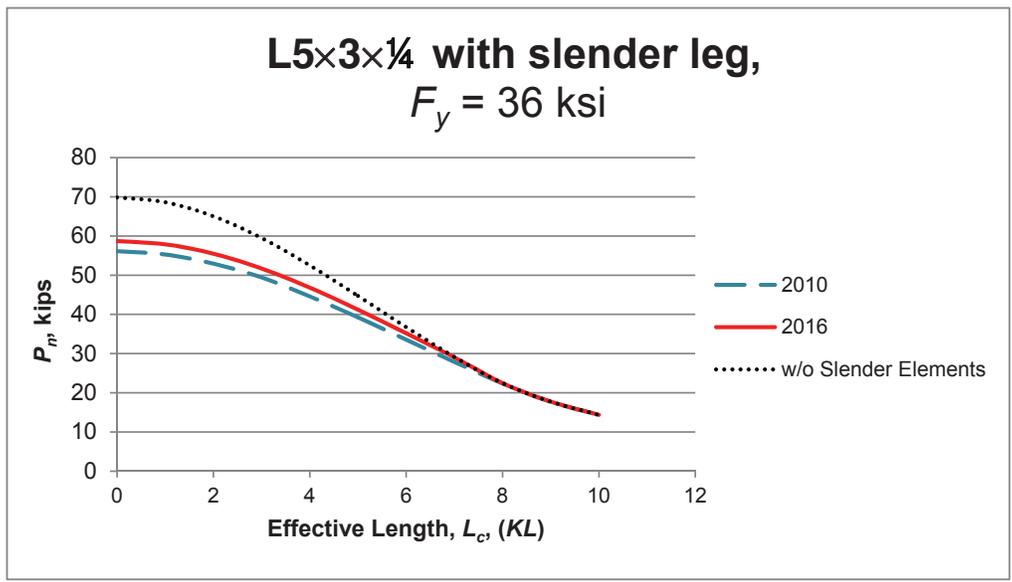


Fig. 4. Comparison of 2010 and 2016 slender element column strength, L5×3×¼, $F_y = 36$ ksi.

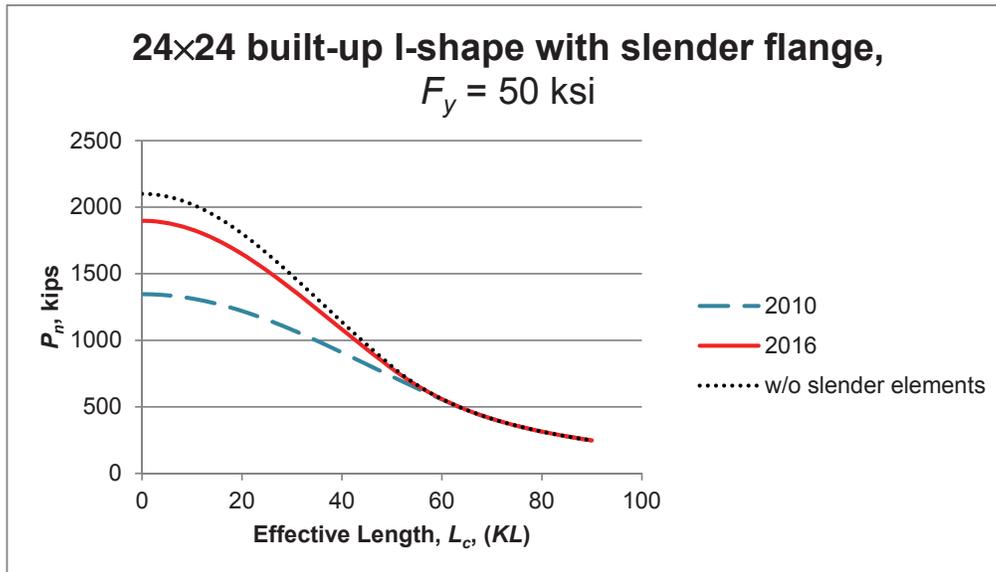


Fig. 5. Comparison of 2010 and 2016 slender element column strength, 24x24 built-up I-shape, $F_y = 50$ ksi.

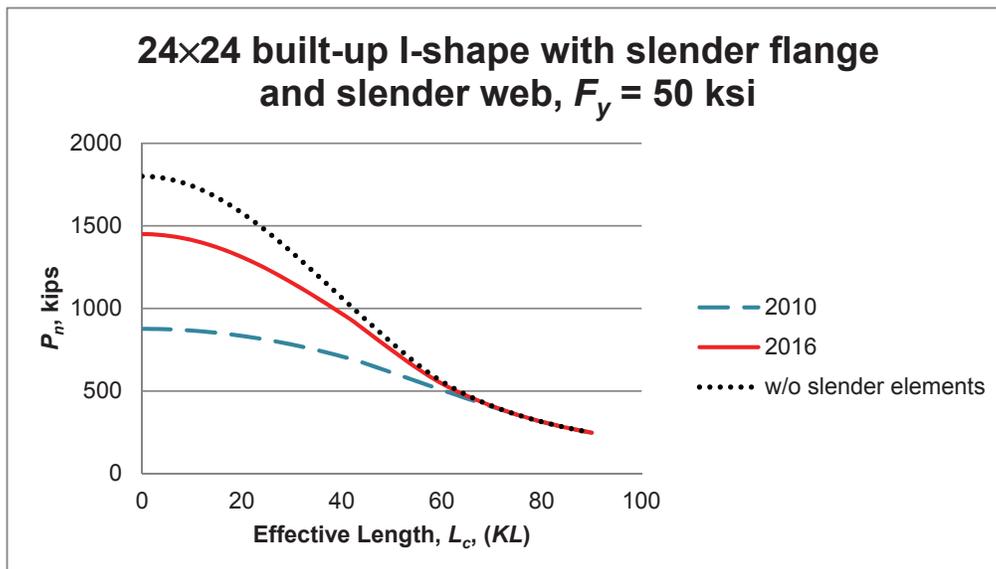


Fig. 6. Comparison of 2010 and 2016 slender element column strength, 24x24 built-up I-shape, slender web and slender flange, $F_y = 50$ ksi.

both slender flanges and slender webs. The shapes that show the most significant change are the built-up I-shape, WT and angle. These are all members with unstiffened slender elements. The W-shape and the HSS show less change, illustrating the relatively minor impact on columns with slender stiffened elements.

CONCLUSIONS

The 2016 *Specification* provisions for slender compression elements in compression members treats stiffened and unstiffened elements in a similar fashion through the same governing equation. It also accounts for the fact that columns are not designed to be stressed to the yield stress, so limiting width-to-thickness ratios need not be based on a limit established using the yield stress.

A comparison between the 2010 *Specification* and 2016 *Specification* for six slender element members shows that the change in strength can be significant for members with slender unstiffened elements. Two alternate approaches have been presented that produce the same results as the new 2016 *Specification*. Equation 6, with the constants given in Table 1, may be used for all slender element members except round HSS, or the expanded presentation given in the Appendix may be used.

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APPENDIX

This presentation reorganizes Section E7 of the 2016 AISC *Specification* with specific equations given for each case, similar to the 2010 *Specification*. The constants from Table E7.1 and Table 1 have been included in the equations. With the 2016 *Specification*, each time a particular shape is considered, the same constants will need to be used and the same equation will eventually result. Thus, writing out the equations once for each case, as done here, may be a simplification useful to the designer.

E7. MEMBERS WITH SLENDER ELEMENTS

This section applies to slender-element compression members, as defined in Section B4.1 for elements in uniform compression.

The nominal compressive strength, P_n , shall be the lowest value based on the applicable limit states of flexural buckling, torsional buckling, and flexural-torsional buckling.

$$P_n = F_{cr} A_e \quad (A-1)$$

where

A_e = summation of the effective areas of the cross-section based on the reduced effective width, b_e or d_e , in.² (mm²), or as given by Equations A-16 or A-17

F_{cr} = critical stress determined in accordance with Section E3 or Section E4, ksi (MPa)

1. Slender Unstiffened Elements

The effective width, b_e or d_e , for slender unstiffened elements is determined as follows:

- (a) For flanges, angles and plates projecting from rolled columns or other compression members:

$$(i) \text{ When } \frac{b}{t} \leq 0.56 \sqrt{\frac{E}{F_{cr}}}$$

$$b_e = b \quad (A-2)$$

$$(ii) \text{ When } \frac{b}{t} > 0.56 \sqrt{\frac{E}{F_{cr}}}$$

$$b_e = 0.834t \sqrt{\frac{E}{F_{cr}}} \left[1 - \frac{0.184}{(b/t)} \sqrt{\frac{E}{F_{cr}}} \right] \quad (A-3)$$

- (b) For flanges, angles and plates projecting from built-up I-shaped columns or other compression members:

$$(i) \text{ When } \frac{b}{t} \leq 0.64 \sqrt{\frac{k_c E}{F_{cr}}} \\ b_e = b \quad (A-4)$$

$$(ii) \text{ When } \frac{b}{t} > 0.64 \sqrt{\frac{k_c E}{F_{cr}}} \\ b_e = 0.954t \sqrt{\frac{k_c E}{F_{cr}}} \left[1 - \frac{0.210}{(b/t)} \sqrt{\frac{k_c E}{F_{cr}}} \right] \quad (A-5)$$

where

$k_c = \frac{4}{\sqrt{h/t_w}}$ and shall not be taken less than 0.35 nor greater than 0.76 for calculation purposes

(c) For stems of tees:

$$(i) \text{ When } \frac{d}{t} \leq 0.75 \sqrt{\frac{E}{F_{cr}}} \\ d_e = d \quad (A-6)$$

$$(ii) \text{ When } \frac{d}{t} > 0.75 \sqrt{\frac{E}{F_{cr}}} \\ d_e = 1.12t \sqrt{\frac{E}{F_{cr}}} \left[1 - \frac{0.246}{(d/t)} \sqrt{\frac{E}{F_{cr}}} \right] \quad (A-7)$$

(d) For single angles, double angles with separators, and all other unstiffened elements:

$$(i) \text{ When } \frac{b}{t} \leq 0.45 \sqrt{\frac{E}{F_{cr}}} \\ b_e = b \quad (A-8)$$

$$(ii) \text{ When } \frac{b}{t} > 0.45 \sqrt{\frac{E}{F_{cr}}} \\ b_e = 0.671t \sqrt{\frac{E}{F_{cr}}} \left[1 - \frac{0.148}{(b/t)} \sqrt{\frac{E}{F_{cr}}} \right] \quad (A-9)$$

where

b = width of unstiffened compression element, as defined in Section B4.1, in. (mm)

d = depth of tee, as defined in Section B4.1, in. (mm)

t = thickness of element, as defined in Section B4.1, in. (mm)

2. Slender Stiffened Elements

The effective width, b_e , for slender stiffened elements is determined as follows:

(a) For all shapes except cover plates, diaphragm plates, walls of square and rectangular HSS and round HSS:

$$(i) \text{ When } \frac{b}{t} \leq 1.49 \sqrt{\frac{E}{F_{cr}}} \\ b_e = b \quad (A-10)$$

$$(ii) \text{ When } \frac{b}{t} > 1.49 \sqrt{\frac{E}{F_{cr}}} \\ b_e = 1.95t \sqrt{\frac{E}{F_{cr}}} \left[1 - \frac{0.351}{(b/t)} \sqrt{\frac{E}{F_{cr}}} \right] \quad (A-11)$$

(b) For cover plates and diaphragm plates:

$$(i) \text{ When } \frac{b}{t} \leq 1.40 \sqrt{\frac{E}{F_{cr}}} \\ b_e = b \quad (A-12)$$

$$(ii) \text{ When } \frac{b}{t} > 1.40 \sqrt{\frac{E}{F_{cr}}} \\ b_e = 1.83t \sqrt{\frac{E}{F_{cr}}} \left[1 - \frac{0.330}{(b/t)} \sqrt{\frac{E}{F_{cr}}} \right] \quad (A-13)$$

(c) For walls of square and rectangular HSS:

$$(i) \text{ When } \frac{b}{t} \leq 1.40 \sqrt{\frac{E}{F_{cr}}} \\ b_e = b \quad (A-14)$$

$$(ii) \text{ When } \frac{b}{t} > 1.40 \sqrt{\frac{E}{F_{cr}}} \\ b_e = 1.93t \sqrt{\frac{E}{F_{cr}}} \left[1 - \frac{0.386}{(b/t)} \sqrt{\frac{E}{F_{cr}}} \right] \quad (A-15)$$

(d) For round HSS, the effective area is determined as follows:

(i) When $\frac{D}{t} \leq \frac{0.11E}{F_y}$

$$A_e = A_g \quad (\text{A-16})$$

(ii) When $\frac{0.11E}{F_y} < \frac{D}{t} < \frac{0.45E}{F_y}$

$$A_e = \left[\frac{0.038E}{F_y(D/t)} + \frac{2}{3} \right] A_g \quad (\text{A-17})$$

where

D = outside diameter of round HSS, in. (mm)

t = thickness of wall, in. (mm)