Cross-Section Strength of Circular Concrete-Filled Steel Tube Beam-Columns

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ABSTRACT

Closed-form expressions for the cross-section strength of steel-concrete composite beam-columns according to the plastic stress distribution method are tabulated in the AISC *Seismic Design Manual* and the AISC *Design Examples*. Approximations have been used in the derivation of these formulas, most of which do not significantly affect the accuracy of the results. However, an approximation in the equation for the axial strength of circular, concrete-filled steel tubes that are simultaneously subjected to flexure at one of the key points on the interaction curve (designated as Point E) leads to results that are unconservative. The derivation of the equation is reviewed and a more accurate expression is proposed.

Keywords: steel-concrete composite, concrete-filled steel tube, beam-column strength, cross-section strength.

INTRODUCTION

The AISC *Specification for Structural Steel Buildings* (2010) allows for the cross-section strength of compact composite columns to be computed by the plastic stress distribution method. In this method, the steel components are assumed to have reached the yield stress, F_y, in either tension or compression, and the concrete components are assumed to have reached a stress of $0.85f_c'$ in compression, where f_c' is the concrete compressive strength. A higher stress, $0.95f_c'$, is permitted for circular, concrete-filled steel tubes to account for confinement. As a design aid, closed-form expressions defining several anchor points, labeled Points A through E on the interaction diagram (Figure 1), are tabulated in the AISC *Seismic Design Manual* (AISC, 2012) and the AISC *Design Examples* (AISC, 2011). Approximations were made in the derivation of some of the closed-form expressions, typically resulting in negligible variation from the exact solution. In this case, "exact" refers to axial load and bending moment pairs that have been precisely calculated from the assumed plastic stress distribution and assumed cross-sectional geometry. However, the equation for the axial strength of circular concrete-filled steel tubes at Point E (Equation 4) tabulated in the AISC publications (2011, 2012) produces results that are unconservative due to an assumption made in its derivation.

DERIVATION

Point E is defined by a plastic neutral axis location a distance h_E from the centroid, where h_E is the average of h_n, the location of the plastic neutral axis for pure bending (Point B) and the distance to the inside face of the steel tube. The assumed stress distribution at Point E is shown in Figure 2. Based on this stress distribution, the axial compression, P_E, is defined by Equation 1:

$$P_E = F_y A_{sc} - F_y A_{st} + 0.95 f_c' A_{cc} \tag{1}$$

where
  A_{cc} = area of concrete in compression, in.^{2}
  A_{sc} = area of steel in compression, in.^{2}
  A_{st} = area of steel in tension, in.^{2}

The equation can be revised to relate P_E to the axial strength under pure compression, P_A:

$$P_A = F_y A_s + 0.95 f_c' A_c \tag{2}$$

$$P_E = P_A - 2F_y A_{st} - 0.95 f_c' A_{ct} \tag{3}$$

where
  A_c = total area of concrete, in.^{2}
  A_s = total area of steel, in.^{2}
  A_{ct} = area of concrete in tension, in.^{2}

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The current formula for P_E from Table 7-11B of the AISC *Seismic Manual* is as follows:

$$P_E = P_A - \frac{1}{4}\left[F_y\left(d^2 - h^2\right) + \frac{0.95 f_c'}{2} h^2 \right]\left(\theta_2 - \sin\theta_2\right) \qquad (4)$$

where
  d = outside diameter of steel tube, in.
  h = inside diameter of steel tube, in.

The angle between two lines extending from the center of the cross-section to the intersections of the plastic neutral axis and the inside face of the steel tube is calculated as follows (see Figure 2):

$$\theta_2 = \pi - 2\arcsin\left(\frac{2h_E}{h}\right) \qquad (5)$$

Through an examination of Equations 3 and 4, the assumed area of concrete in tension and area of steel in tension can be deduced as Equations 6 and 7, respectively:

$$A_{ct} = \frac{h^2}{8}\left(\theta_2 - \sin\theta_2\right) \qquad (6)$$

$$A_{st} = \frac{d^2 - h^2}{8}\left(\theta_2 - \sin\theta_2\right) = \frac{d^2}{8}\left(\theta_2 - \sin\theta_2\right) - A_{ct} \qquad (7)$$

Both A_{ct} and A_{st} (Equations 6 and 7) are based on circular segment geometry as shown in Figures 3a and 3c. The expression for A_{ct} exactly represents the shape of the cross-section and given stress pattern; however, the use of θ_2 for the circular segment in the expression for A_{st} leads to

an underestimation of the area of steel in tension and thus an overestimation of P_E. An exact expression for A_{st} would make use of circular segment geometry with θ_{2s} in lieu of θ_2 (Equations 8 and 9) as shown in Figure 3b:

$$A_{st} = \frac{d^2}{8}\left(\theta_{2s} - \sin\theta_{2s}\right) - A_{ct} \qquad (8)$$

$$\theta_{2s} = \pi - 2\arcsin\left(\frac{2h_E}{d}\right) \qquad (9)$$

Alternatively, an approximation that is more accurate than the current formula and does not increase the complexity of the calculation by introducing a second angle can be obtained by assuming a circular-sector geometry (Figure 3d), where A_{st} is computed by Equation 10. Most importantly, the circular-sector geometry matches the geometric assumption used in the formula for the bending moment at Point E, M_E (Geschwindner, 2010). The final formula for P_E, using Equation 10 for the area of steel in tension, is Equation 11. This expression is recommended to replace the current formula (Equation 4) in the design tables.

$$A_{st} = \frac{d^2 - h^2}{8}\theta_2 \qquad (10)$$

$$P_E = P_A - \frac{1}{4} F_y\left(d^2 - h^2\right)\theta_2 - \frac{1}{8} 0.95 f_c' h^2\left(\theta_2 - \sin\theta_2\right) \qquad (11)$$

An example illustrating the accuracy of the proposed equation is presented in the following section.
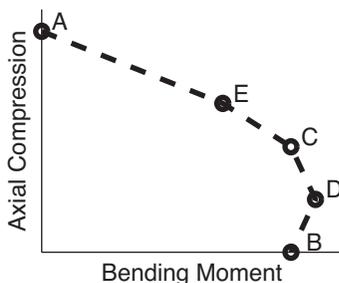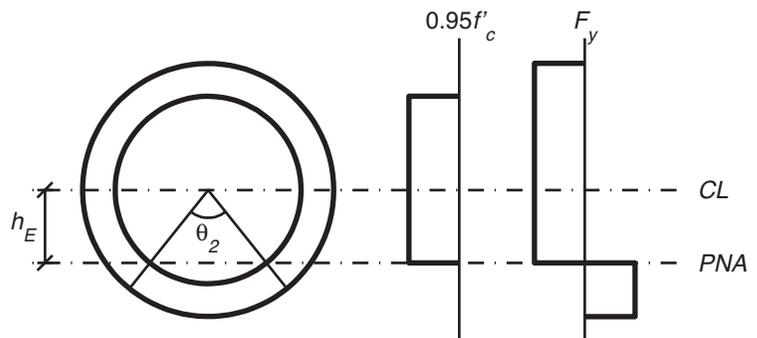

Fig. 1. Schematic interaction diagram.

Fig. 2. Assumed stress distribution.

DESIGN EXAMPLE

Given:

Determine the axial compression and bending moment at Point E for an ASTM A500 Grade C HSS10.750×0.250 composite compression member filled with 5-ksi concrete ($f'_c = 5$ ksi).

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C
$F_y = 46$ ksi

From AISC *Manual* Table 1-13, the HSS geometric properties are as follows:

HSS10.750×0.250
$D = 10.750$ in.
$t = 0.233$ in.

Solution:

Determine cross-sectional properties:

$$d = D$$
$$= 10.750 \text{ in.}$$

$$h = d - 2t$$
$$= 10.75 \text{ in.} - 2(0.233 \text{ in.})$$
$$= 10.3 \text{ in.}$$

$$A_s = \pi \left(dt - t^2 \right)$$
$$= \pi \left[(10.75 \text{ in.})(0.233 \text{ in.}) - (0.233 \text{ in.})^2 \right]$$
$$= 7.70 \text{ in.}^2$$

$$A_c = \frac{\pi}{4} h^2$$
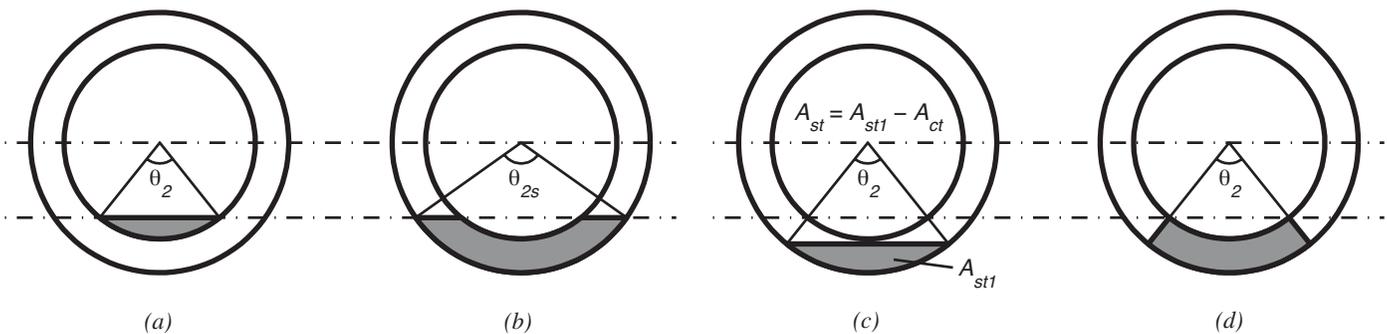$$= \frac{\pi}{4} (10.3 \text{ in.})^2$$
$$= 83.3 \text{ in.}^2$$

Fig. 3. Schematic of cross-sectional areas: (a) A_{ct}; (b) A_{st}, exact; (c) A_{st}, current; (d) A_{st}, proposed.

Equation 2 is used to determine P_A:

$$P_A = F_y A_s + 0.95 f_c' A_c$$
$$= (46 \text{ ksi})(7.70 \text{ in.}^2) + 0.95(5 \text{ ksi})(83.3 \text{ in.}^2)$$
$$= 750 \text{ kips}$$

From AISC *Seismic Manual* Table 7-11B, Point B equations, h_n is calculated as follows:

$$K_c = f_c' h^2$$
$$= (5 \text{ ksi})(10.3 \text{ in.})^2$$
$$= 530 \text{ kips}$$

$$K_s = F_y \left(\frac{d-t}{2} \right) t$$
$$= (46 \text{ ksi}) \left(\frac{10.75 \text{ in.} - 0.233 \text{ in.}}{2} \right) (0.233 \text{ in.})$$
$$= 56.4 \text{ kips}$$

$$\theta = \frac{0.0260 K_c - 2K_s}{0.0848 K_c} + \frac{\sqrt{(0.0260 K_c + 2K_s)^2 + 0.857 K_c K_s}}{0.0848 K_c}$$

$$= \frac{0.0260(530 \text{ kips}) - 2(56.4 \text{ kips})}{0.0848(530 \text{ kips})} + \frac{\sqrt{[0.0260(530 \text{ kips}) + 2(56.4 \text{ kips})]^2 + 0.857(530 \text{ kips})(56.4 \text{ kips})}}{0.0848(530 \text{ kips})}$$

$$= 2.34 \text{ rad}$$

$$h_n = \frac{h}{2} \sin \left(\frac{\pi - \theta}{2} \right) \leq \frac{h}{2}$$
$$= \left(\frac{10.3 \text{ in.}}{2} \right) \sin \left(\frac{\pi - 2.34 \text{ rad}}{2} \right) \leq \frac{10.3 \text{ in.}}{2}$$
$$= 2.01 \text{ in.}$$

From AISC *Seismic Manual* Table 7-11B, Point E equations, the plastic neutral axis location, h_E, and corresponding angle, θ_2 is calculated as follows:

$$h_E = \frac{h_n}{2} + \frac{h}{4}$$
$$= \frac{2.01 \text{ in.}}{2} + \frac{10.3 \text{ in.}}{4}$$
$$= 3.58 \text{ in.}$$

$$\theta_2 = \pi - 2\arcsin \left(\frac{2h_E}{h} \right)$$
$$= \pi - 2\arcsin \left[\frac{2(3.58 \text{ in.})}{10.3 \text{ in.}} \right]$$
$$= 1.60 \text{ rad}$$

Determine P_E using the current equation (Equation 4):

$$P_E = P_A - \frac{1}{4}\left[F_y\left(d^2 - h^2\right) + \frac{0.95 f_c'}{2} h^2\right]\left(\theta_2 - \sin\theta_2\right)$$

$$= 750 \text{ kips} - \frac{1}{4}\left\{46 \text{ ksi}\left[\left(10.750 \text{ in.}\right)^2 - \left(10.3 \text{ in.}\right)^2\right] + \left[\frac{0.95\left(5 \text{ ksi}\right)}{2}\right]\left(10.3 \text{ in.}\right)^2\right\} \times \left[1.60 \text{ rad} - \sin\left(1.60 \text{ rad}\right)\right]$$

$$= 647 \text{ kips}$$

Determine P_E using the proposed equation (Equation 11):

$$P_E = P_A - \frac{1}{4}F_y\left(d^2 - h^2\right)\theta_2 - \frac{1}{8}0.95 f_c'h^2\left(\theta_2 - \sin\theta_2\right)$$

$$= 750 \text{ kips} - \frac{1}{4}\left(46 \text{ ksi}\right)\left[\left(10.75 \text{ in.}\right)^2 - \left(10.3 \text{ in.}\right)^2\right]\left(1.60 \text{ rad}\right) - \frac{1}{8}0.95\left(5 \text{ ksi}\right)\left(10.3 \text{ in.}\right)^2\left[1.60 \text{ rad} - \sin\left(1.60 \text{ rad}\right)\right]$$

$$= 537 \text{ kips}$$

From AISC *Seismic Manual* Table 7-11B, Point E equations, M_E is calculated as follows:

$$Z_{cE} = \frac{h^3}{6}\sin^3\left(\frac{\theta_2}{2}\right)$$

$$= \frac{\left(10.3 \text{ in.}\right)^3}{6}\sin^3\left(\frac{1.60 \text{ rad}}{2}\right)$$

$$= 67.2 \text{ in.}^3$$

$$Z_{sE} = \frac{d^3 - h^3}{6}\sin\left(\frac{\theta_2}{2}\right)$$

$$= \frac{\left(10.75 \text{ in.}\right)^3 - \left(10.3 \text{ in.}\right)^3}{6}\sin\left(\frac{1.60 \text{ rad}}{2}\right)$$

$$= 17.9 \text{ in.}^3$$

$$M_E = F_y Z_{sE} + \frac{0.95 f_c' Z_{cE}}{2}$$

$$= \left(46 \text{ ksi}\right)\left(17.9 \text{ in.}^3\right) + \frac{0.95\left(5 \text{ ksi}\right)\left(67.2 \text{ in.}^3\right)}{2}$$

$$= 983 \text{ kip-in.}$$

The results for the remaining points are presented in Table 1 along with an evaluation of Point E using the exact geometry (Geschwindner, 2010). These values are presented graphically in Figure 4 along with the results of the interaction diagram evaluated by dividing the cross section into many individual fibers, assuming a plastic neural axis location, assigning a plastic stress to each fiber according to the assigned material (i.e., steel or concrete) and whether is it in tension or compression, and numerically integrating the stresses to determine pairs of axial compression and bending moment. As can be observed from Figure 4, the current formula for P_E overestimates the axial compression at Point E, which results in an unconservative approximation of the interaction diagram. Additionally, the proposed equation not only results in a close approximation of the exact results, but, because of the consistent approximations in the formulas, the pair (M_E, P_E) lies nearly on the numerically determined interaction diagram, thus introducing nearly no error to the evaluation of the interaction diagram.

To confirm the observations from Figure 4, the error between the proposed Point E and the exact interaction diagram was computed over a range of cross sections. The error in this case can be computed as a function of two nondimensional parameters: the

Table 1. Example Interaction Diagram Anchor Points		
Point	**Bending Moment (kip-in.)**	**Axial Compression (kips)**
Point A	0	750
Point E—current	983	647
Point E—proposed	983	538
Point E—exact	1,030	525
Point C	1,400	396
Point D	1,620	198
Point B	1,400	0

tube slenderness ratio, D/t, and the material strength ratio, F_y/f'_c. Given the range of compact steel tubes and material strength limitations in the AISC *Specification* (2010), the calculations were performed for tube slenderness ratios ranging from $D/t = 10$ to $D/t = 125$ and material strength ratios ranging from $F_y/f'_c = 3.5$ to $F_y/f'_c = 25$. The maximum error was 0.1%, which occurred for $D/t = 10$ and $F_y/f'_c = 3.5$, confirming that the geometric approximation in the proposed formulas introduces nearly no error to the computation of the interaction diagram.
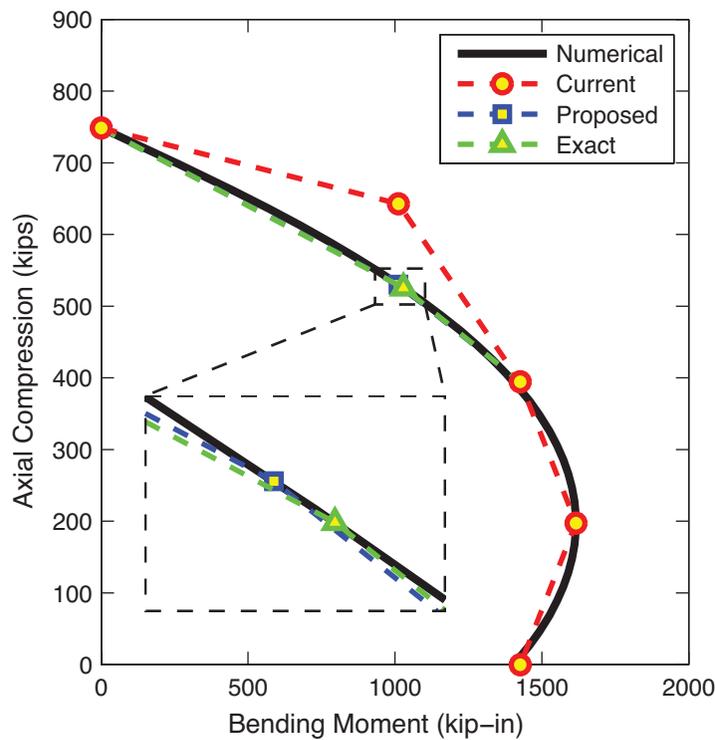

Fig. 4. Example interaction diagram.

CONCLUSIONS

The derivation of the plastic cross-section axial strength of circular, concrete-filled steel tubes at Point E was reviewed, and the formula currently provided in the AISC *Seismic Design Manual* (AISC, 2012) and the AISC *Design Examples* (AISC, 2011) was found to be unconservative. A new formula that utilizes assumptions about the geometry that are consistent with the corresponding formula for the flexural strength at Point E was derived. This formula was verified against a numerical evaluation of the interaction surface and found to be accurate in all cases. It is recommended that this paper's Equation be adopted for use in place of the current formula in all AISC publications.

ACKNOWLEDGMENTS

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