A Modified Equation for Expected Maximum Shear Strength of the Special Segment for Design of Special Truss Moment Frames

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Seismic behavior of the special truss moment frame (STMF) system has been studied both analytically and experimentally by Goel and Itani (1994) and Basha and Goel (1995) at the University of Michigan during the past 15 years and has been incorporated into the AISC *Seismic Provisions for Structural Steel Buildings* (AISC, 2005), hereafter referred to as the AISC Seismic Provisions. This system consists of a truss girder with a special segment designed to behave inelastically under severe earthquakes while the other members, including girder-to-column connections, outside the special segment remain essentially elastic. The special segment can be made with X-diagonal or Vierendeel web members, as shown in Figure 1. When an STMF is subjected to lateral seismic forces, the induced shear force in the middle of the joist girder is resisted primarily by the chord members and the web diagonals of the special segment. After yielding and buckling of the diagonal members, plastic hinges will form at the ends of the chord members as shown in Figure 2. The yield mechanism of the frame is a combination of yielding of all special segments in the frame plus the plastic hinges at the column bases (Figure 2). The plastic hinges at the base may be in the columns or in the base connection, whichever is practically feasible. Design of STMFs starts with designing the special segment. Based on a capacity design approach, other elements are then designed to remain elastic under the shear forces in the middle of the truss girder generated by fully yielded and strain hardened special segments (in other words, the expected vertical shear strength, V_{ne}), along with other external forces.

The expected maximum shear strength of the special segment, V_{ne}, is given in the current AISC Seismic Provisions [Equation 12-1 in AISC (2005)] as:

$$V_{ne} = \frac{3.75 R_y M_{nc}}{L_s} + 0.075 E_s I \frac{\left(L - L_s\right)}{L_s^3} + R_y\left(P_{nt} + 0.3 P_{nc}\right)\sin\alpha \tag{1}$$

where

R_y	=	yield stress modification factor
M_{nc}	=	nominal flexural strength of the chord members of the special segment
$E_s I$	=	flexural elastic stiffness of the chord members of the special segment
L	=	span length of the truss
L_s	=	length of the special segment, center-to-center of supports
P_{nt}	=	nominal axial tension strength of diagonal members of the special segment
P_{nc}	=	nominal axial compression strength of diagonal members of the special segment
α	=	angle of diagonal members with the horizontal

The first two terms of Equation 1 were derived based on a study of a Vierendeel special segment (without X-braces) (Basha and Goel, 1994). One of the assumptions made in the derivation was that the elastic moment at the ends of

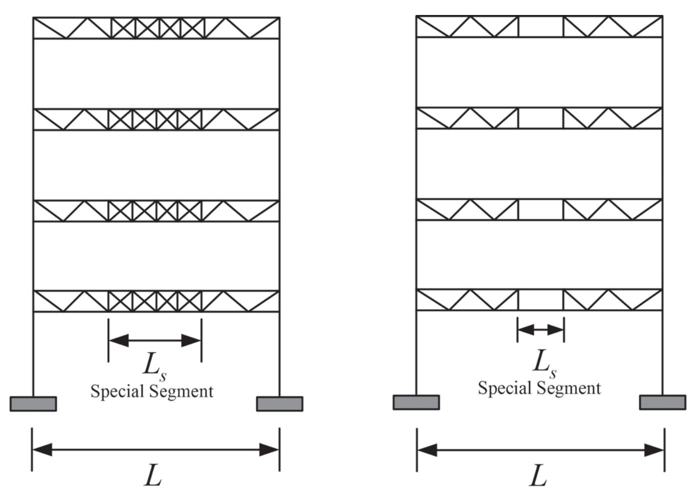

Fig. 1. STMF with different configurations of special segment.

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chord members of the special segment results from vertical translation only, in other words, the effect of end rotation is neglected as shown in Figure 3a. This assumption leads to overestimation of the elastic stiffness of the chord members, which in turn results in a higher coefficient, 0.075, in the second term of Equation 1. Nevertheless, this overestimation has a small influence on V_{ne} if the moment of inertia of the chord member is small. However, for heavier chord members the overestimation can be quite large because of their large moment of inertia and the second term of Equation 1 is directly related to the member's moment of inertia. Because the members outside the special segment, such as vertical members, diagonal members, connections and columns are designed based on V_{ne}, any overestimation would result in an overly conservative design of those members.

PROPOSED MODIFICATION

A remedy to the aforementioned problem is to use a more realistic flexural elastic stiffness of the chord members. This is done by the approach that follows.

If the chord member has no end rotation (fixed end condition), the elastic moment at the chord end can be given by:

$$M = \frac{6E_s I\theta}{L_s} \quad (2)$$

where

θ = relative vertical displacement at chord ends divided by the length of special segment

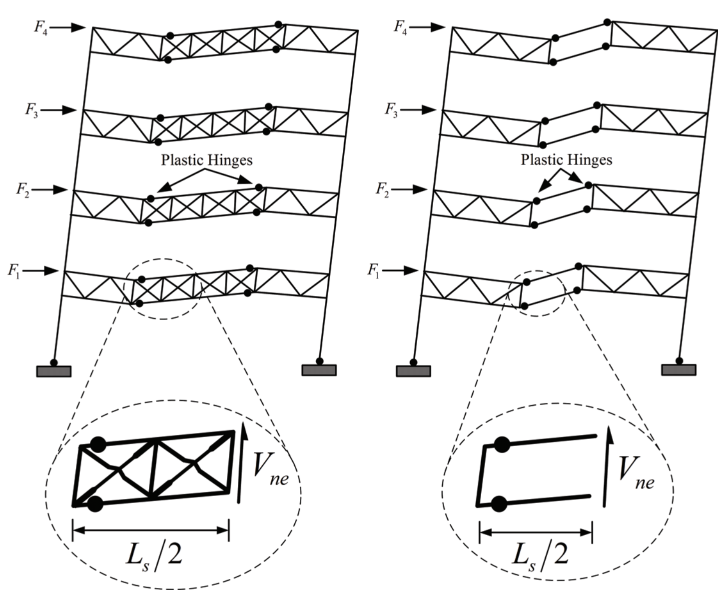

Fig. 2. Mechanism of STMF with different special segments.

Hence, the elastic stiffness is:

$$\frac{M}{\theta} = \frac{6E_s I}{L_s} = k \quad (3)$$

This elastic stiffness will decrease by allowing the end rotation to occur. For the extreme case, in other words, when the chord member has pinned ends as shown in Figure 3b, the elastic stiffness is equal to zero. True elastic stiffness is somewhere between these two extreme cases. It is assumed here that the true elastic stiffness can be approximated by:

$$k = \frac{3E_s I}{L_s} \quad (4)$$

By using this formulation, the maximum chord end moment is determined as follows:

Referring to Figure 4, the chord moment-rotation relation can be modeled by a bilinear curve, in which the inelastic stiffness is ηk. From Equation 4, the maximum elastic rotation is:

$$\theta_e = \frac{M_p L_s}{3E_s I} \quad (5)$$

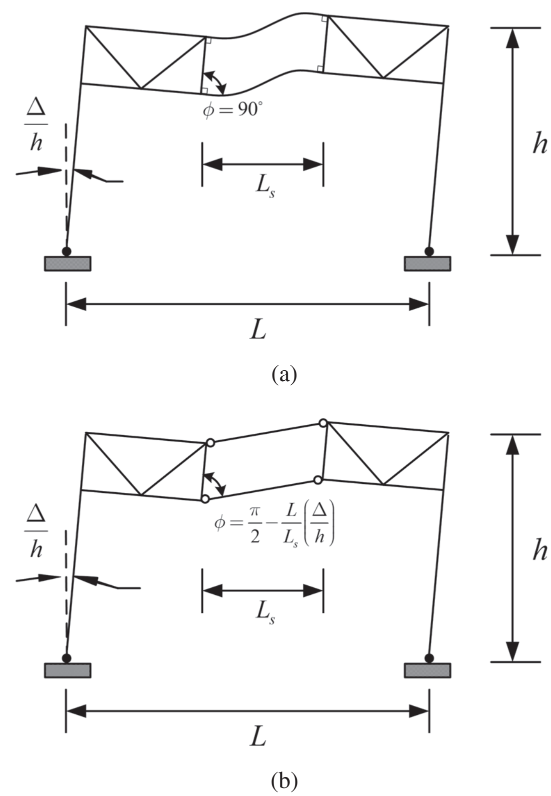

Fig. 3. Deformation of chord of the Vierendeel special segment: (a) no chord end rotation; (b) free chord end rotation.

The maximum rotation of the chord member can be obtained by using the geometric relation. Thus,

$$\theta_u = \frac{L}{L_s}\left(\frac{\Delta}{h}\right) \tag{6}$$

where

(Δ/h) = story drift

Hence the plastic rotation is:

$$\theta_p = \theta_u - \theta_e = \frac{L}{L_s}\left(\frac{\Delta}{h}\right) - \frac{M_p L_s}{3E_s I} \tag{7}$$

The expression for maximum moment can be written as (see Figure 4):

$$\begin{aligned} M_{max} &= M_p + \eta k \theta_p \\ &= (1-\eta) R_y M_{nc} + 3E_s I \eta \left(\frac{L}{L_s^2}\right)\left(\frac{\Delta}{h}\right) \end{aligned} \tag{8}$$

where

η = ratio of the post-yield slope to the elastic slope of the assumed bilinear moment-rotation model of the chord member

The expected maximum shear strength of the special segment, V_{ne}, is then calculated as:

$$V_{ne} = \frac{4M_{max}}{L_s} \tag{9}$$

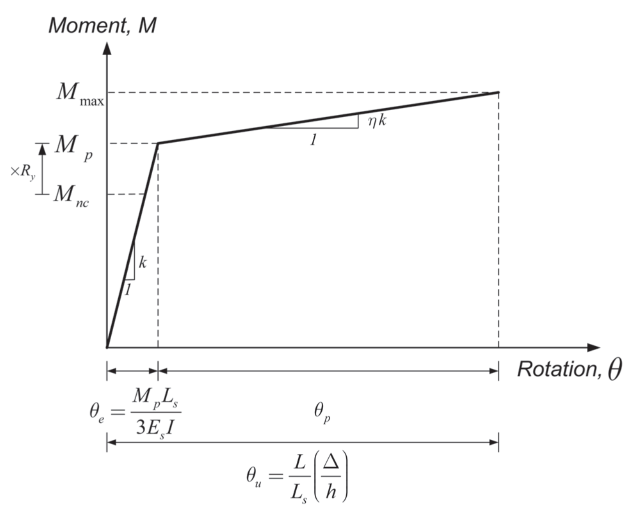

Fig. 4. Moment-rotation relationship of chord member.

It is noted that the slope, η, only accounts for strain hardening; that is, the expected plastic moment, M_p, was used instead of the nominal strength, M_{nc}, as shown in Figure 4. Based on tests on STMF subassemblages in which the special segment was composed of double-angle sections, Basha and Goel (1994) found that a value of η equal to 10% was adequate for the post-yield slope (see Figure 5). This value was also found adequate for double-channel sections based on tests conducted by Parra-Montesinos, Goel and Kim (2006). Figure 6 shows the backbone curve of hysteretic loops for a typical double-channel element, as well as the corresponding bilinear model. By using η = 10%, and Δ/h = 0.03 in Equation 8, Equation 9 becomes:

$$V_{ne} = \frac{3.6 R_y M_{nc}}{L_s} + 0.036 E_s I \frac{L}{L_s^3} \tag{10}$$

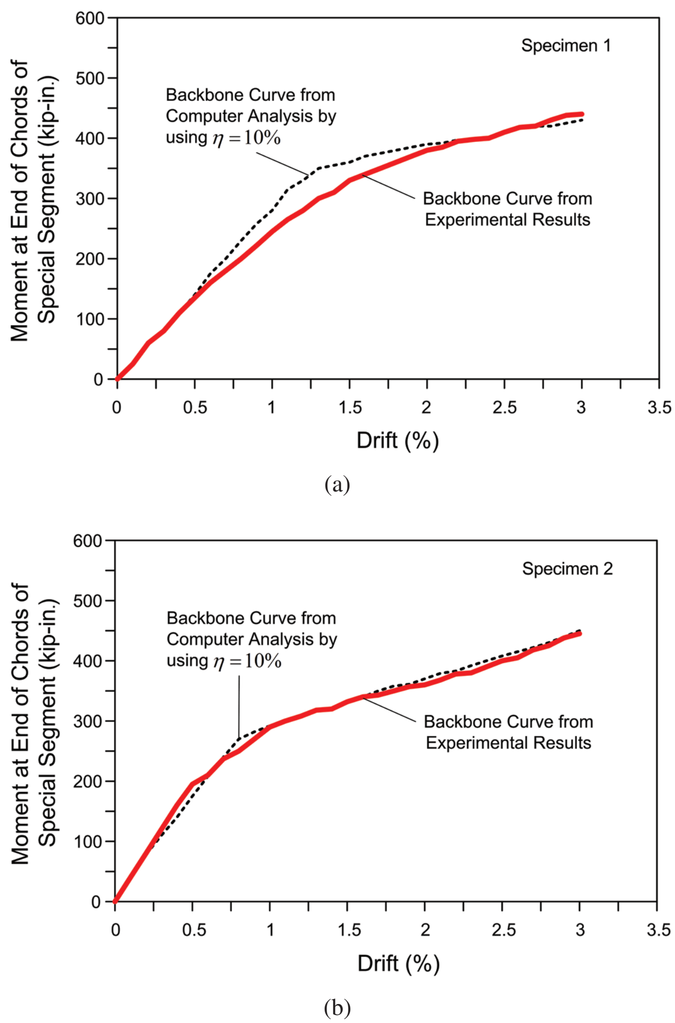

Fig. 5. Moment versus drift plots of subassemblage tested, chord member was a built-up section composed of double-angles and plate (Basha and Goel, 1994).

The value of $\Delta/h = 0.03$ was chosen because current design practice is generally based on a limiting drift ratio of around 0.02. If the maximum rotation capacity, θ_p , of a chord member is available, Equation 9 can also be expressed by:

$$V_{ne} = \frac{4R_y M_{nc}}{L_s} + 1.2E_s \frac{I}{L_s^2}\theta_p \qquad (11)$$

The second term in Equation 11 accounts for the contribution of strain-hardening in the chord members. It is noted that Dusicka, Itani and Sahai (2002) derived a similar equation for V_{ne}. Their expression, however, did not account for strain-hardening effects.

VERIFICATION OF THE MODIFIED EQUATION

The proposed modification was verified by previous test results as well as nonlinear static and dynamic analyses as described in the following.

Previous Test Results

Equation 10 was first verified by the test results (Parra-Montesinos et al., 2006) on double-channel (2C10×25, I = 182.2 in.^{4}, Z = 46.2 in.^{3}, F_y = 50 ksi) elements used for chord members of STMF, in which the maximum shear capacity was measured at 150 kips when the corresponding story drift reached 3%. The value of V_{ne} calculated from Equation 10, by using R_y = 1.1, gives the same exact value, in other words, 150 kips.

Equation 10 was further verified using the test results of a subassemblage STMF (Basha and Goel, 1994), in which the chord members of the Vierendeel special segment were made of a built-up section composed of double angles and a plate (2L3×3×½ and PL1×2¼). The built-up section had a rather small moment of inertia (I = 5.87 in.^{4}). The maximum end moment at the story drift of 3% was 440 kip-in., which corresponds to V_{ne} = 26.2 kips. The estimated value from Equation 10 is 24.4 kips, which is about 7.3% lower than the experimental value. On the other hand, the AISC equation (Equation 1) gives a value of 28.4 kips, about 8.5% higher than the testing data.

Nonlinear Static Analysis

Equation 10 was then verified by investigating the maximum developed shears in the Vierendeel special segments of a seven-story STMF (Chao and Goel, 2006). The chord member at the fifth floor of this frame was made of 2C10×25. Modeling of the moment-rotation relation for the chord members was based on that shown in Figure 6. Pushover analysis was performed until the interstory drift at the fifth level reached 3%, as shown in Figure 7. It was found that the average V_{ne} was about 143 kips, which is close to the predicted value, 150 kips. While the proposed equation gives good agreement with the test and analysis results, the V_{ne} as calculated by using the AISC expression (Equation 1) is 177.3 kips, about 20% higher than that obtained from Equation 10. It should be noted that both the AISC

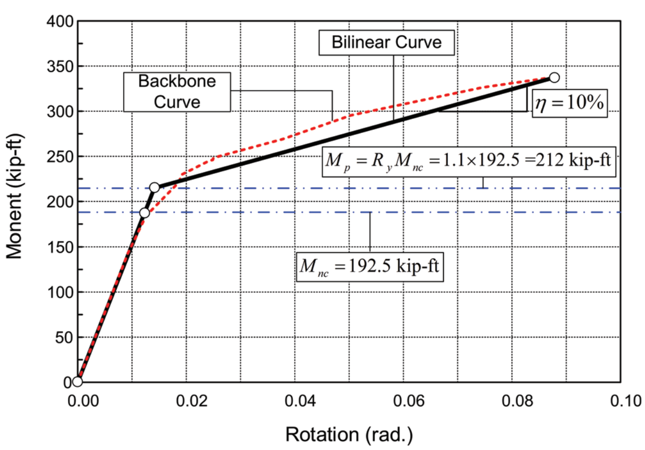

Fig. 6. Modeling of the backbone curve of double-channel element using bilinear curve.

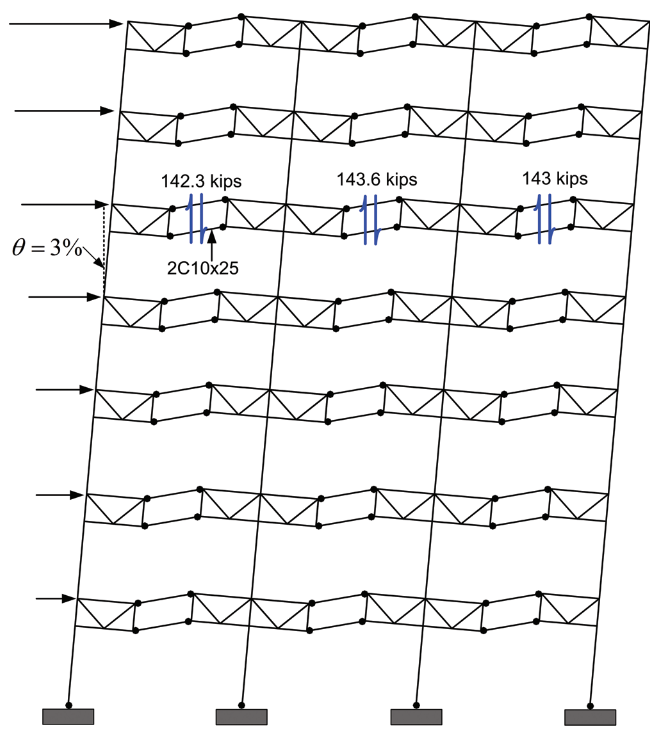

Fig. 7. Maximum developed shears in the fifth-floor special segments when the study STMF was statically pushed until the fifth floor reached 3% story drift.

expression and Equation 10 are based on a story drift equal to 3%. If the expected maximum story drift is well below 3%, the AISC expression would be even more conservative.

It should also be noted that the L_s used in Equation 10 is taken as center-to-center distance of the vertical members at the ends of the special segment. In reality, the distance between the plastic hinges, L_p, will be smaller than L_s. This length in the University of Michigan double-channel tests was formed to be about $0.82L_s$ and the V_{ne} in the seven-story STMF analysis was calculated based on $0.85L_s$. Both of them have almost the same V_{ne} as that obtained from Equation 10. This suggests that Equation 10 is still slightly conservative even when L_s is used instead of L_p in the denominator of Equation 10.

Nonlinear Dynamic Analysis

Two nine-story STMFs with Vierendeel type special segments (Figure 8), representing the class of essential facilities (in other words, hospital buildings) as well as ordinary office/residential occupancy type, were designed for evaluating the proposed modification (Chao and Goel, 2006). Double-channel sections were used for chord members. For ordinary building types, the design target drifts of 2% and 3% for 10% in 50 years and 2% in 50 years design hazard levels, respectively, were chosen. The corresponding numbers for essential facilities were 1.5% and 2.25%. Design spectral values were based on NEHRP Provisions (FEMA, 2001) for the San Francisco site. After the final design work was completed, nonlinear dynamic analyses were conducted to study the responses. Nine, 10% in 50 years, and five, 2% in 50 years, SAC Los Angeles region ground motions representing the two design hazard levels were used for the nonlinear response history analyses.

The results showed that yielding was limited to the special segments only, while the other elements remained elastic. This suggests that the proposed expression for V_{ne} was quite adequate to ensure elastic performance of the elements outside the special segments. Figures 9 and 10 show the maximum developed shears in the special segments for the ordinary and essential STMFs, respectively. It can be seen that the AISC equation significantly overestimates the expected shear strength, which would lead to undue over-design of elements outside the special segments. On the other hand,

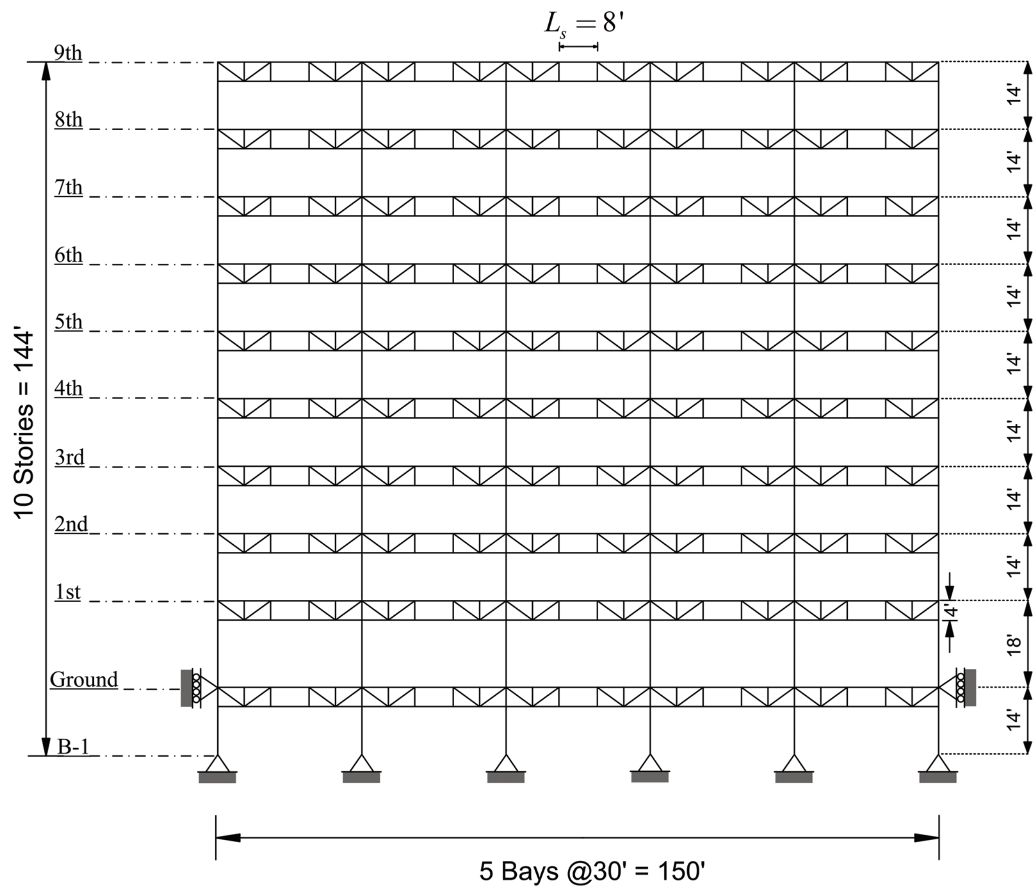

Fig. 8. Elevation of study nine-story STMFs.

the proposed expression (Equation 10) is closer to the actual developed shears while maintaining some safety margin.

Therefore, based on the preceding discussion, it is suggested that the current AISC expression (with the web diagonals also included in the special segment) can be modified as:

$$V_{ne} = \frac{3.6R_y M_{nc}}{L_s} + 0.036E_s I \frac{L}{L_s^3} + R_y\left(P_{nt} + 0.3P_{nc}\right)\sin\alpha \quad (12\text{-}1)$$

or

$$V_{ne} = \frac{4R_y M_{nc}}{L_s} + 1.2E_s \frac{I}{L_s^2}\theta_p + R_y\left(P_{nt} + 0.3P_{nc}\right)\sin\alpha \quad (12\text{-}2)$$

VIERENDEEL SPECIAL SEGMENTS WITH INTERMEDIATE VERTICAL MEMBERS

A special segment can contain multiple Vierendeel panels by adding intermediate vertical members, as shown in Figure 11. One benefit of using multiple Vierendeel panels is that the redundancy of the seismic energy dissipation mechanism increases. It also has the advantages of allowing more flexibility in mechanical and architectural layout, as well as reducing the rotational ductility demands on special segment chords (Valley and Hooper, 2002). It is very likely that during minor to moderate earthquake events, inelastic deformation would only occur in the intermediate vertical members, which could be replaced relatively easily. In addition, the size of chord members can be reduced because of additional

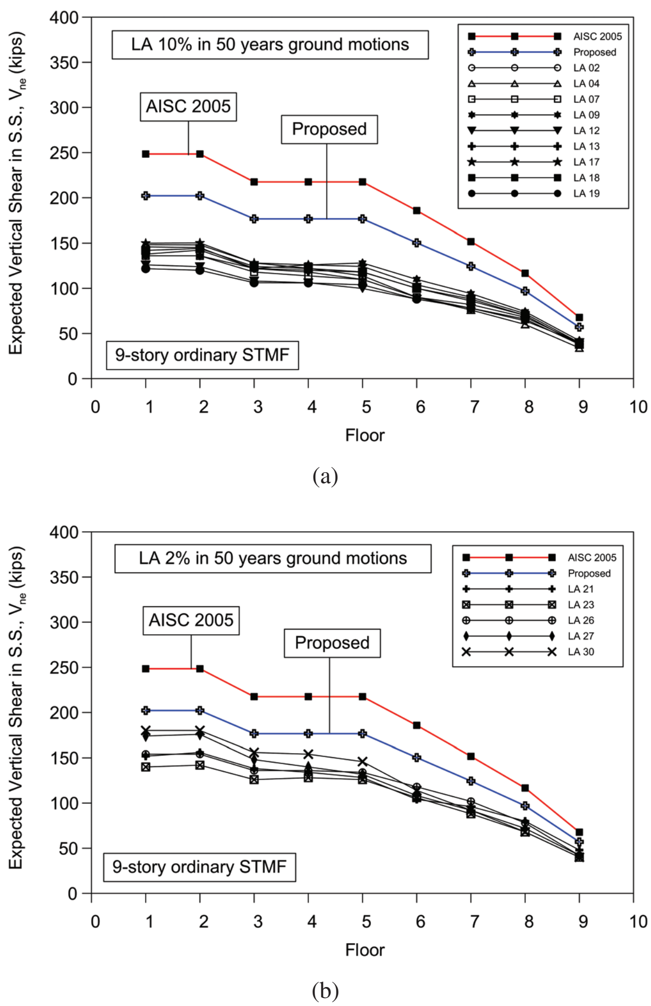

Fig. 9. Maximum developed shears in special segments in the nine-story ordinary STMF subjected to 10%/50 and 2%/50 ground motions.

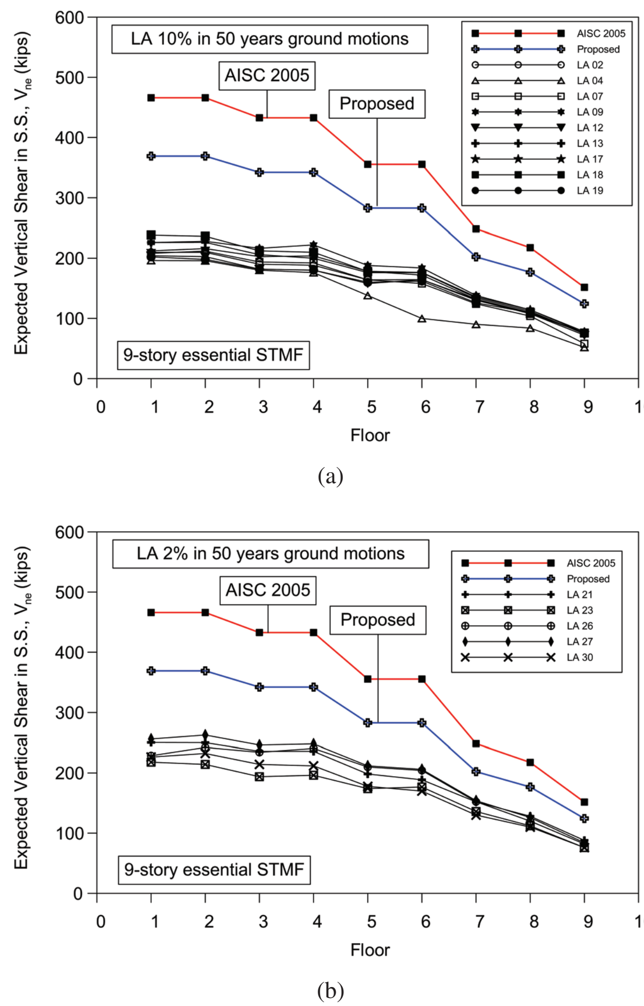

Fig. 10. Maximum developed shears in special segments in the nine-story essential STMF subjected to 10%/50 and 2%/50 ground motions.

strength due to those intermediate vertical members. In the case where multiple Vierendeel panels are present, the calculation of V_{ne} should include the contribution from intermediate vertical members. It can be shown that both the chord member and the intermediate vertical member have the same plastic rotation when the yield mechanism is reached (Figure 11). Thus the maximum moment developed in the chord members and intermediate vertical members can be obtained from Equation 8, as follows:

$$\left(M_c\right)_{max} = \left(1-\eta\right) R_y M_{nc} + 3E_s I_c \eta \left(\frac{L}{L_s^2}\right)\left(\frac{\Delta}{h}\right) \tag{13}$$

$$\left(M_v\right)_{max} = \left(1-\eta\right) R_y M_{nv} + 3E_s I_v \eta \left(\frac{L}{L_s^2}\right)\left(\frac{\Delta}{h}\right) \tag{14}$$

where

$(M_c)_{max}$, $(M_v)_{max}$ = maximum expected developed moments in the chord member and intermediate vertical member, respectively

I_c = moment of inertia of the chord member

I_v = moment of inertia of the intermediate vertical member

It should be noted that the strain-hardening ratio tends to increase when the member length decreases (Engelhardt and Popov, 1989). For a typical chord member, a 10% strain-hardening ratio may be reasonable but the strain-hardening ratio for the vertical member might be actually higher due to its much shorter length. In this study, the chord and intermediate vertical members were assumed to have the same strain-hardening ratio.

The expected maximum shear strength of the special segment with one intermediate vertical member is then calculated as (Figure 12):

$$V_{ne} = \frac{4\left(M_c\right)_{max}}{L_s} + \frac{2\left(M_v\right)_{max}}{L_s} \tag{15}$$

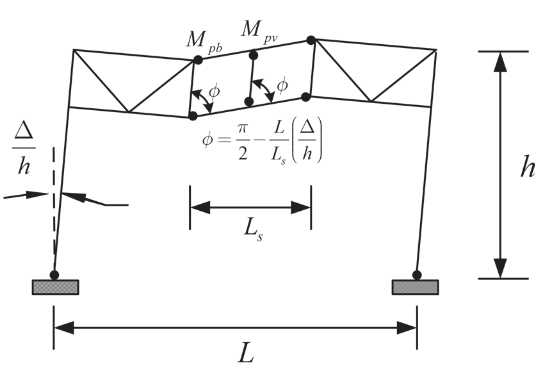

Fig. 11. Yield mechanism of STMF with multiple Vierendeel panels.

For a special segment with two intermediate vertical members (Figure 13):

$$V_{ne} = \frac{4\left(M_c\right)_{max}}{L_s} + \frac{4\left(M_v\right)_{max}}{L_s} \tag{16}$$

In general, the expected maximum shear strength of a special segment with intermediate vertical members can be expressed as:

$$V_{ne} = \frac{4\left(M_c\right)_{max}}{L_s} + \left(\frac{m}{2}\right)\frac{4\left(M_v\right)_{max}}{L_s} \tag{17}$$

where

m = number of intermediate vertical members

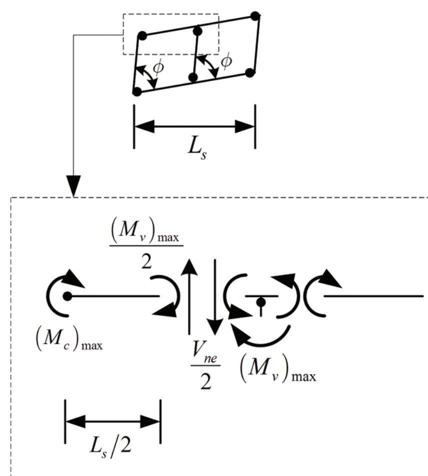

Fig. 12. Calculation of V_{ne} *for two Vierendeel panels.*

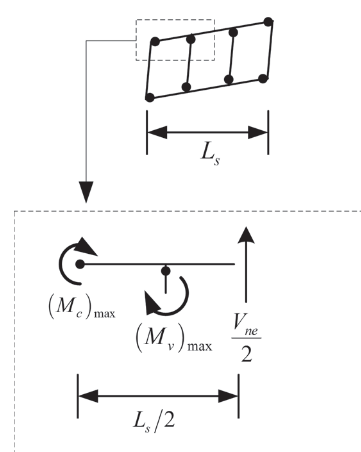

Fig. 13. Calculation of V_{ne} *for three Vierendeel panels.*

By using $\eta = 0.1$, and $\Delta/h = 0.03$, V_{ne} will be given by:

$$V_{ne} = \left(\frac{3.6 R_y M_{nc}}{L_s} + 0.036 E_s I_c \frac{L}{L_s^3} \right) + \frac{m}{2} \left(\frac{3.6 R_y M_{nv}}{L_s} + 0.036 E_s I_v \frac{L}{L_s^3} \right) \tag{18-1}$$

or

$$V_{ne} = \left(\frac{4 R_y M_{nc}}{L_s} + 1.2 E_s \frac{I_c}{L_s^2} \theta_{pc} \right) + \frac{m}{2} \left(\frac{4 R_y M_{nv}}{L_s} + 1.2 E_s \frac{I_v}{L_s^2} \theta_{pv} \right) \tag{18-2}$$

where

θ_{pc} and θ_{pv} = maximum rotation capacity of the chord and intermediate vertical members, respectively

A NOTE ON THE DESIGN OF MULTIPLE VIERENDEEL PANELS

It should be noted that, as illustrated in Figure 11, while the vertical members generally have smaller length than the chord members, the vertical members experience the same plastic rotation with the chord members when the yield mechanism forms. As pointed out by Engelhardt and Popov (1989), as member length decreases, flexural yielding tends to be confined to a smaller region at the ends of the member leading to larger curvature and bending strain demands for the same plastic rotation. This higher demand on bending strains in turn results in a higher possibility of fracture at welded connections at the member ends. In addition, the reduced length of the plastic region can cause problems of flange buckling and lateral-torsional buckling in flexural yielding members. As a consequence, until further experimental results are available, it is suggested that the intermediate vertical members should be treated as secondary members to prevent possible instability of the frame in case premature failure occurs in the vertical members. The term "secondary members" is used here in the sense that most of the truss strength and energy dissipation capacity should be provided by the chord members. In addition, plastic hinges must be avoided in the chord members except at chord ends; therefore, the moment capacity of vertical members has to be limited so that the moment in the chord members at sections adjacent to the vertical members is less than the moment capacity of chord members after the vertical members yield.

Therefore, to design intermediate vertical members as secondary members, it is suggested at this time that at least 70% of the input energy be dissipated by the chord members and the remainder by intermediate vertical members, unless further research can show that yielding of intermediate vertical members is not detrimental to the overall performance of an STMF. Therefore, the following equation can be applied at a given floor level (Chao and Goel, 2006):

$$\frac{30\%}{70\%} = \frac{\left(2 m_i M_{pvi}\right)\left(\pi/2 - \phi\right)}{\left(4 M_{pbi}\right)\left(\pi/2 - \phi\right)} \tag{19}$$

$$M_{pvi} \approx \frac{M_{pbi}}{m_i} \tag{20}$$

where

M_{pvi} = required plastic moment capacity of the intermediate vertical members at the ith level

M_{pbi} = required plastic moment capacity of the chord members at the ith level

m_i = number of intermediate vertical members at the ith level

The term $\pi/2 - \phi$ represents the rotation of the chord and intermediate vertical members (see Figure 11). It should also be mentioned that the design of chord and intermediate vertical members can be easily implemented by using the plastic design method (Chao and Goel, 2006). In that case, the internal work stored in a special segment can be expressed by:

$$\left(4 M_{pbi} + 2 m_i M_{pvi}\right)\theta = \left(4 M_{pbi} + 2 m_i \frac{M_{pbi}}{m_i}\right)\theta = 6 M_{pbi}\theta = \text{External Work} \tag{21}$$

If the design of chord members is performed based on Equation 21, then the design of intermediate vertical members should follow Equation 20.

SUMMARY AND CONCLUSION

A revised equation for maximum expected shear strength, V_{ne}, was derived by using a more realistic assumption and validated by experimental results as well as nonlinear static and dynamic analyses. Based on extensive nonlinear dynamic analyses, it was found that the current AISC equation for V_{ne} in the special segments can significantly overestimate the expected shear strength, which in turn leads to undue overdesign of members outside the special segments, such as vertical members, diagonal members, connections and columns. The values given by the proposed equation were closer to the actual developed shears while maintaining some safety margin. A design equation of V_{ne} for STMF using multiple Vierendeel panels in the special segments was also proposed.

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