Evaluating Single-Angle Compression Struts Using an Effective Slenderness Approach

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Single angles are often used as web members of a truss or as bracing members for building frames or equipment support structures. In almost all of these situations, the angles are attached at each end by one of the legs to a gusset plate or to a projecting leg or stem of the chord member of a truss. Since these angles are loaded eccentrically about a nonprincipal axis and restrained in differing amounts about the *x*- and *y*-axes, evaluation of their compressive load capacity is difficult. This document presents a simple procedure for evaluating the compressive load capacity of single angles loaded at their ends through one leg by evaluation of an equivalent slenderness ratio so that the angle can be designed as a pinned-end axially loaded compression member.

BACKGROUND

Currently the general procedure used by a designer for evaluating the axial capacity of a single angle supported by one leg at each end would be to treat the angle as a beam-column. To do this, the designer needs to know the eccentricity of the load relative to the gusset plate and the degree of restraint about the *x*- and *y*-axes.

The actual eccentricity in the angle is less than the distance from centerline of gusset if there is any restraint about the *x*-axis (see Figure 1). This was illustrated by Lutz (1998) in his examination of an angle when the end of the gusset plate is fixed. Woolcock and Kitipornchai (1986) in their single-angle design procedure recommended reduction of the eccentricity to $\bar{y} - t/2$, where *t* is the angle thickness, as long as the angle web members were all placed to one side of the chord stem or gusset plate. (See Figure 1.)

Consideration of the restraint of the single angle about the *y*-axis (axis perpendicular to the plane of the gusset plate) is also very important in obtaining a good value of the compressive capacity. The angle is rotationally well restrained about the *y*-axis, which means that the stress along the connected leg will be approximately uniform. In order to achieve a uniform stress along the connected leg, the axial load should be applied along a line such that the ratio of the eccentricities is $e_z/e_w = I_z/I_w$, as explained by Lutz (1996). This line is shown in Figure 1.

Therefore, to achieve a value analytically for the capacity that is not extremely conservative, the designer must consider the restraint provided about both geometric axes, as well as the reduced eccentricity produced by the *x*-axis restraint. Since the effective length factors can be readily estimated or evaluated only about the geometric axes, it is difficult to determine the effective slenderness of the angle. Lutz (1992) suggested a means of obtaining a critical slenderness based on the effective length factors about the geometric axes. This expression is empirical, not a theoretically developed expression.

The various design procedures for single angles were examined by Lutz, Temple, and Sakla (1996). Use of the 1999 LRFD AISC provisions (AISC, 1999) for single angles with due consideration of the end restraint as discussed earlier can provide capacities that compare well with test results (Lutz, 1998). Other procedures (Woolcock and Kitipornchai, 1986; ASCE, 1997) were also examined by Lutz (1998).

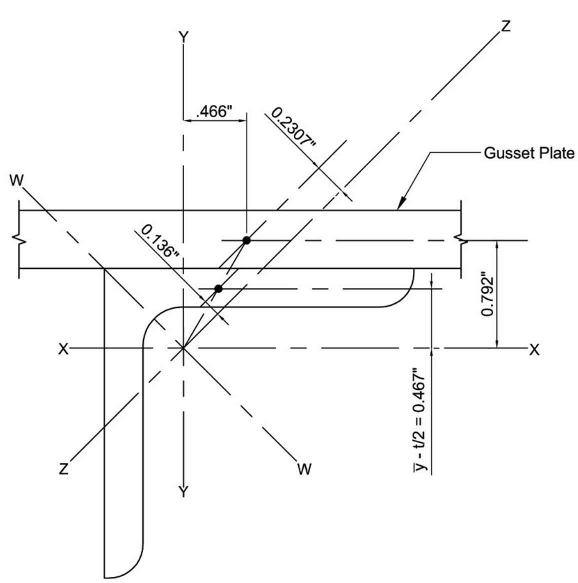

Fig. 1. L2×2×¼ showing load locations with $e_z/e_w = I_z/I_w$.

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NEW PROCEDURE

The single-angle design procedure employed by ASCE (1997) for latticed steel transmission towers forms the basis of the simplified design procedure included in the 2005 American Institute of Steel Construction *Specification for Structural Steel Buildings* (AISC, 2005), hereafter referred to as the 2005 AISC Specification. The procedure applies to members that (1) are loaded at the ends in compression through the same one leg, (2) are attached by welding or by minimum two-bolt connections, and (3) have no intermediate transverse loads. An equivalent slenderness is obtained such that the design of the single angle can be for axial compression only.

The equivalent slenderness ratio KL/r per ASCE (1997) for angles supported through one leg is:

For members with normal framing eccentricities at both ends

$$\frac{KL}{r} = 60 + 0.5\left(\frac{L}{r_z}\right) \text{ when } 0 \leq \frac{L}{r_z} \leq 120 \qquad \text{(1a)}$$

for members partially restrained against rotation at both ends

$$\frac{KL}{r} = 46.2 + 0.615\left(\frac{L}{r_z}\right) \text{ when } 120 \leq \frac{L}{r_z} \leq 250 \qquad \text{(1b)}$$

This expression is intended to apply to equal-leg angles used in space trusses. The radius of gyration is about the z-axis (see Figure 1).

The intent in the new AISC Specification was to include unequal-leg angles as well as equal-leg angles, so Equations 1a and 1b were rewritten in terms of L/r_x, as originally proposed by Mengelkoch and Yura (2002). Tests by Trahair, Usami, and Galambos (1969), Woolcock and Kitipornchai (1986), and Mengelkoch and Yura (2002) all indicated that the buckling was primarily about the x-axis due to the eccentricity of load about the x-axis coupled with the high degree of restraint about the y-axis. The 1969 and 2002 references included tests of unequal-leg angles.

Therefore, for equal-leg angles or unequal-leg angles connected through the longer leg that are web members of box or space trusses with adjacent web members attached to the same side of the gusset plate or chord:

$$\frac{KL}{r} = 60 + 0.8\left(\frac{L}{r_x}\right) \text{ when } 0 \leq \frac{L}{r_x} \leq 75 \qquad \text{(2a)}$$

$$\frac{KL}{r} = 45 + \left(\frac{L}{r_x}\right) \leq 200 \text{ when } \frac{L}{r_x} > 75 \qquad \text{(2b)}$$

Equation 2b limits L/r_x to a maximum of 155.

However, for the case where the unequal-leg angles are connected through the shorter leg, the test data (Usami and Galambos, 1971; Mengelkoch and Yura, 2002) and the theoretical capacities determined in the 1971 reference tend to be lower for comparable L/r_x values than found for equal-leg angles. Due to having the shorter leg stiffened rotationally by the chord, the stiffness about the y-axis is closer to the stiffness of the longer leg (about the x-axis) which tends to force the buckling axis to be closer to the z-axis of the angle and away from the x-axis. Due to the previously noted behavior and due to limited tests of such angles, the slenderness expressions are modified and a limiting slenderness is imposed.

The KL/r from Equations 2a and 2b must be increased by adding a slenderness of $6[(b_l/b_s)^2 - 1]$ where b_l = longer leg of angle and b_s = shorter leg of angle, but this increased slenderness shall not be less than $0.82L/r_z$. The $0.82L/r_z$, which represents the slenderness at $KL/r = 200$ in Equation 2b for equal-leg angles, provides a governing slenderness for the more slender unequal-leg angles. This slenderness limit does not need to be checked for equal-leg angles.

Equations 2a and 2b can only be used for angles with leg length ratios less than 1.7 when connected through the shorter leg due to lack of testing of angles with larger leg length ratios. This eliminates angles L6×3½, L7×4, L8×4, and L9×4 from consideration.

Data from tests by Trahair, Usami, and Galambos (1969) and by Mengelkoch and Yura (2002) of single angles with fixed ends (Figure 2b) are plotted in Figure 3 and compared with the nominal strength using Equations 2a and 2b modified as noted for unequal-leg angles. The criteria are intended to address the case where there is significant rotational restraint from the chord, but certainly less than the fixed-end condition. The criteria produce an average value 75.5% of the fixed-end values with a 0.188 coefficient of variation. The benefit of the larger end restraint of the Trahair et al. (1969) data is evident in Figure 3.

To address the case where a smaller amount of rotational restraint about the x-axis is anticipated, a second set of expressions is to be used. For equal-leg angles or unequal-leg angles connected through the longer leg that are individual angles or are web members of planar trusses with adjacent web members attached to the same side of the gusset plate or chord:

$$\frac{KL}{r} = 72 + 0.75\left(\frac{L}{r_x}\right) \text{ when } 0 \leq \frac{L}{r_x} \leq 80 \qquad \text{(3a)}$$

$$\frac{KL}{r} = 32 + 1.25\left(\frac{L}{r_x}\right) \leq 200 \text{ when } \frac{L}{r_x} > 80 \qquad \text{(3b)}$$

Equation 3b limits L/r_x to a maximum of 134.4.

For unequal-leg angles with leg length ratios less than 1.7, and connected through the shorter leg, the KL/r from Equations 3a and 3b shall be increased by adding $4[(b_l/b_s)^2 - 1]$, but this KL/r shall not be less than $0.95L/r_z$.

In this case the nominal strength is compared in Figure 4 with tests by Foehl (1948) and by Trahair et al. (1969) of single angles in compression with the pinned-end condition illustrated in Figure 2a. The criteria produce an average value 99.8% of the pinned values with a 0.109 coefficient of variation.

COMPARISON OF NEW PROCEDURE WITH CURRENT GENERAL PROCEDURE

The capacity of an L2×2×¼ with a 50 ksi (345 MPa) yield strength evaluated using the AISC single-angle specification (AISC, 2000) was compared to the equivalent slenderness approach included in the 2005 AISC Specification.

In the case for planar trusses, an $e_x = \bar{y} - t/2$ was employed (Woolcock and Kitipornchai, 1986) along with the e_y as illustrated in Figure 1. In evaluating the axial component a $k_{eff} = 0.866$ was used as would be computed with $k_x = 1$ and $k_y = 0.65$ from a procedure suggested by Lutz (1992). A plot comparing results is shown in Figure 5. The results are very close at smaller *KL/r* values. One would have to use a slightly smaller e_x at larger *KL/r* values to match the effective slenderness approach.

In the case for space trusses, an $e_x = 0.6\bar{y}$ was arbitrarily considered for the full range of *KL/r*. The e_y used was based on maintaining the e_z/e_w ratio as illustrated in Figure 1. To consider the effect of the restraint about the *x*-axis, k_x was assumed at 0.85. This k_x along with a $k_y = 0.65$ led to use of $k_{eff} = 0.766$. A plot comparing results of the equivalent slenderness approach with the general AISC single-angle design procedure is shown in Figure 6. The results are very close for large *KL/r* values. An $e_x = 0.8\bar{y}$ would produce a much better agreement for smaller *KL/r* values.

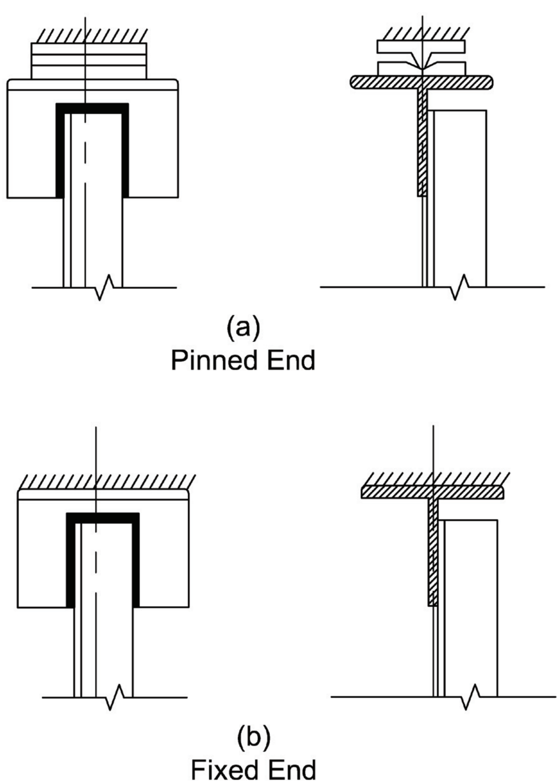

Fig. 2. Test end conditions for single-angle struts.

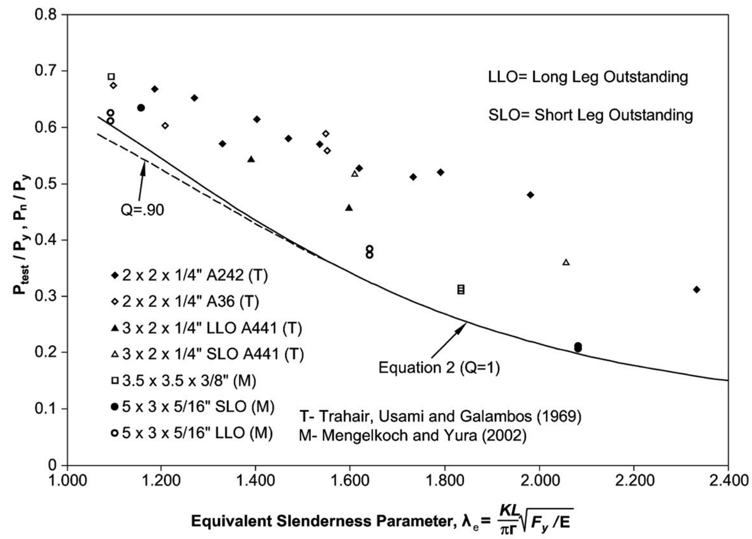

Fig. 3. Equation 2 compared with fixed-end test data.

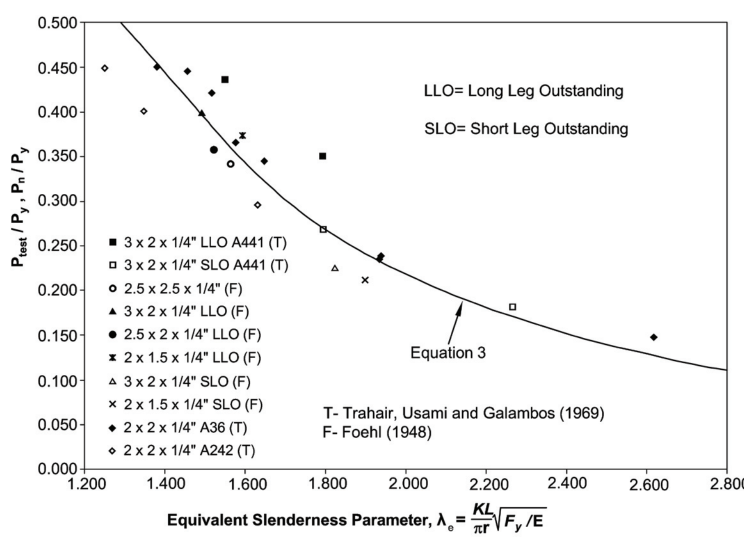

Fig. 4. Equation 3 compared with pinned-end test data.

COMPARISON OF NEW PROCEDURE WITH OTHER EQUIVALENT SLENDERNESS PROCEDURES

The Eurocode 3 and the British Standard BS5950 also employ equivalent slenderness procedures to permit the design of single-angle struts as axially loaded members with pinned ends. The Eurocode 3 expression is virtually identical to the proposed AISC approach for equal-leg angles in space trusses when $L/r_x > 60$ as illustrated in Figure 7. The BS5950 expression becomes quite close as well when the equation, $KL/r = 0.85L/r_z$, controls.

However, the European procedures produce much higher capacities for angles with smaller L/r_x values. The capacity per the proposed AISC approach for planar trusses falls below the European procedures for the entire range of slenderness.

APPLICATIONS OF NEW PROCEDURE

The following two application examples illustrate the use of the effective slenderness procedure for single angles.

Example 1

Determine the design compression strength of an L4×4×¼ used as a web member of a planar truss where all web members are welded to the same side of the truss chords. The angle is 67 in. long and is made of ASTM A 36 steel.

$$L/r_x = 67/1.25 = 53.6$$

Since L/r_x is less than 80, use Equation 3a to determine KL/r.

$$\frac{KL}{r} = 72 + 0.75(53.6) = 112$$

Since $b/t = 4/0.25 = 16$ is greater than $0.45\sqrt{29{,}000/36} = 12.8$, local buckling reduces the axial capacity.

$$Q_s = 1.34 - 0.76\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}} = 0.911 = Q$$

Since $KL/r < 4.71\sqrt{E/QF_y} = 140$,

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y$$

$$\text{with } F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2(29{,}000)}{(112)^2} = 22.8 \text{ ksi}$$

Therefore,

$$F_{cr} = 0.911\left[0.658^{\left(\frac{0.911(36)}{22.8}\right)}\right]36$$

$$= 18.0 \text{ ksi}$$

The design compression strength with $\phi = 0.90$ is

$$\phi F_{cr}A = 0.9(18.0)(1.94) = 31.4 \text{ kips}$$

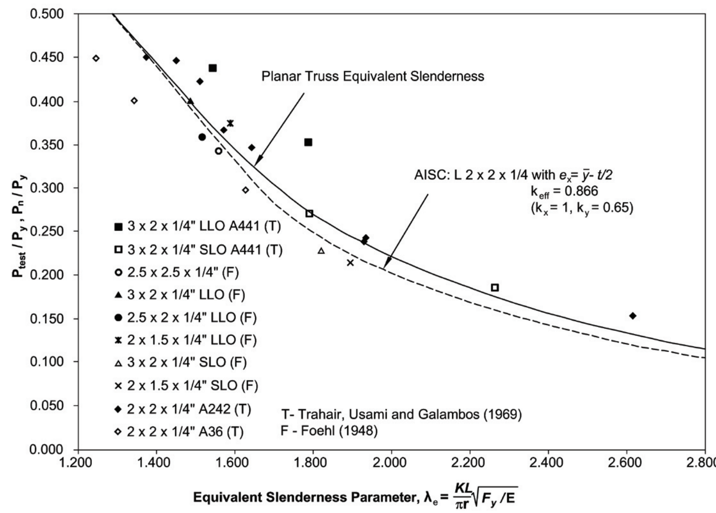

Fig. 5. Comparison of general AISC procedure with new approach—planar trusses.

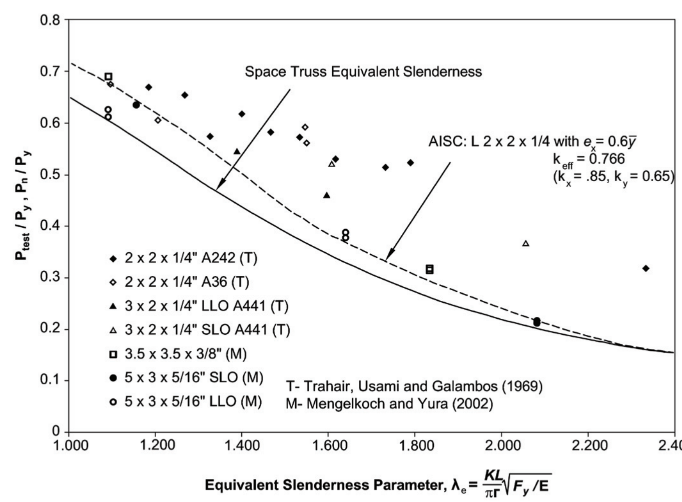

Fig. 6. Comparison of general AISC procedure with new approach—space trusses.

Example 2

Determine the design compression strength of an L4×3×5⁄16 used as a web member of a box truss. The 3 in. leg is welded to the chord. The angle is 120 in. long and is made of ASTM A 36 steel.

Since the shorter leg is connected to the chord, compute

$$6\left[\left(\frac{b_l}{b_s}\right)^2-1\right]=6\left[\left(\frac{4}{3}\right)^2-1\right]=4.7$$

with $r_x = 1.27$, $L/r_x = 120/1.27 = 94.5$.

Since $L/r_x > 75$, find KL/r by using Equation 2b and adding 4.7.

$$\frac{KL}{r}=4.7+45+94.5=144$$

Check: $0.82L/r_z = 0.82(120)/0.638 = 154 > 144$.

Use $KL/r = 154$, which is greater than

$$4.71\sqrt{\frac{E}{F_y}}=4.71\sqrt{\frac{29{,}000}{36}}=134$$

Therefore, $F_{cr}=0.877F_e=0.877\left(\frac{\pi^2(29{,}000)}{(154)^2}\right)$

$$=10.6 \text{ ksi}$$

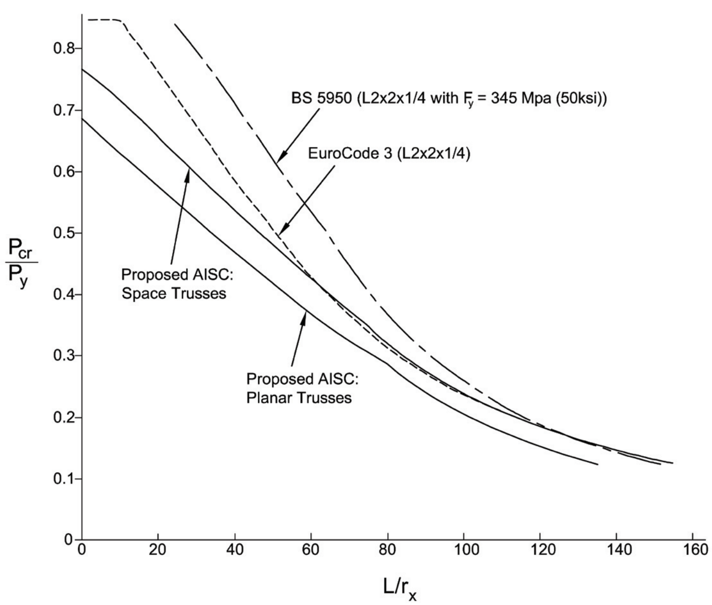

Fig. 7. Comparison of new AISC and other effective slenderness procedures.

The design compression strength with $\phi = 0.90$ is

$$\phi F_{cr}A = 0.9(10.6)(2.09) = 19.9 \text{ kips}$$

CONCLUSIONS

An effective slenderness procedure, which will appear in the 2005 AISC Specification (AISC, 2005), is presented to permit the design of single-angle compression struts supported by one leg at the ends as axially loaded pinned-end members. The procedure addresses the cases where little rotational restraint about the axis parallel to the connected leg is to be considered and also for the situations where there is a more significant rotational restraint as would occur in space trusses. The procedure permits the simple evaluation of a conservative capacity for these single-angle struts in lieu of a more detailed analysis.

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