Strength of Singly Symmetric I-Shaped Beam-Columns

THEODORE V. GALAMBOS

ABSTRACT

The AISC-LRFD Specification provides a conservative prediction of the strength of singly symmetric I-shaped beam-columns bent about the axis of symmetry if the compression flange is larger than the tension flange. This paper presents a more economic method that bases the *in-plane capacity* on the attainment of the full plastic capacity of the cross-section, and the *lateral-torsional strength* on an inelastic modification of the elastic buckling solution. The proposed derivations are based on essentially the same philosophy as the ones for the doubly symmetric wide-flange design rules in the AISC Specification. However, for singly symmetric shapes it is not possible to arrive at simple approximate empirical interaction equations. The proposed methods are of necessity spreadsheet oriented, and an appendix to this paper provides a sample calculation scheme using the MATHCAD software as the vehicle of calculation.

INTRODUCTION

The Load and Resistance Factor Design Specification of the American Institute of Steel Construction (AISC, 1999) has an accurate way of treating the determination of the strength of doubly symmetric wide-flange beam-columns under compressive axial load. By using the interaction equations in Chapter H in conjunction with the provisions in Chapter E (columns), and Chapter F (beams) an efficient design procedure is available to the structural engineer. An extension of these design criteria to the singly symmetric (another term used frequently in the literature is *mono-symmetric*) I-shaped beam-column results in quite conservative designs when the compression flange is larger than the tension flange. This is especially so for the lateral-torsional buckling limit state. The predictions of the axial compression capacity and the in-plane and the lateral-torsional bending capacity in Chapters E and F of the AISC Specification, respectively, are reasonably accurate for these singly symmetric members. However, the beam-column interaction equations connecting the pivotal points of only axial force and only bending moment in the axial force-bending moment interaction space are too conservative for these shapes. This paper will present an alternate method that is more appropriate to the actual conditions existing in singly symmetric beam-columns. The method can deal rationally with the case of uniform bending about the axis of symmetry (the x-axis of the cross section). The cases of non-uniform bending and bi-axial bending are not discussed.

The paper will first consider the fully plastic in-plane capacity of a zero-length member subject to an axial force applied through the geometric centroid of the cross section and a bending moment applied in the plane of symmetry. The cross section is assumed to be fully yielded under the applied axial force and bending moment. The cross section capacity is then modified to account for the fact that under compression the strength of a real member of a given length is reduced from the fully yielded case. This reduction is a proportion of the in-plane column strength formulas in Chapter E of the AISC Specification. The length effect is approximately accounted for by rotating the zero length interaction curve about the pivot $P = 0$ and $M = M_p$ until it ends at the point $P = P_{cr}$ and $M = 0$.

The next portion of the paper considers the lateral-torsional buckling strength of a singly symmetric beam-column subjected to axial force through the geometric centroid of the cross section, and equal applied end-bending moments. The elastic lateral buckling behavior is defined by a quadratic equation that expresses the relationship between the applied axial force and the end-bending moment (Galambos, 1968 and 1998). For a given moment this equation can be solved for the elastic axial capacity. This capacity is then reduced to approximate the inelastic strength by using the procedure given in Appendix E3 of the AISC Specification (AISC, 1999).

The applied bending moments are moments that are amplified to account for second-order bending of the member and the story (B_1 and B_2, respectively, in Chapter C of the AISC Specification, or by explicit second-order analysis). The proposed methods are of necessity spreadsheet oriented, and an appendix provides a sample calculation scheme using the MATHCAD software as the vehicle of calculation. The example can be used to set up computational schemes by other spreadsheet programs, such as Excel or Quattro Pro.

IN-PLANE BEHAVIOR

The cross section of the singly symmetric shape is made up of three plates welded together into a wide-flange shape, as shown in Figure 1. The following derivation will develop the equations relating the axial load and the plastic bending moment when the steel is perfectly plastic, that is, the stress

Theodore V. Galambos is emeritus professor of structural engineering, University of Minnesota, Minneapolis, MN.

is equal to the yield stress F_y everywhere in the cross section. All three plates have the same yield stress.

The fully yielded cross section can have its plastic neutral axis in the top flange (Figure 2), in the web (Figure 3) or in the bottom flange (Figure 4). When the neutral axis is in the top flange, equilibrium of forces requires that:

$$P = F_y\left[-A + 2b_{f1}y_p\right] \tag{1a}$$

This equation can now be solved for y_p, the distance of the neutral axis from the top of the top flange:

$$y_p = \frac{A}{2b_{f1}}\left[p+1\right] \tag{1b}$$

where

$$p = \frac{P}{AF_y}$$

The limits of applicability of this equation are: $y_p = 0$ when $P = -AF_y$ (or $p = -1$, i.e. the whole cross section is yielded in tension); and $y_p = t_{f1}$ when:

$$p = \frac{2t_{f1}b_{f1}}{A} - 1 = \frac{2A_{f1}}{A} - 1$$

The plastic moment can be obtained by taking moments about the centroid of the cross section, where the axial force P is assumed to act:

$$M_{pc} = F_y\left[\begin{array}{l} b_{f1}y_p\left(y - \dfrac{y_p}{2}\right) - b_{f1}\left(t_{f1} - y_p\right)\left(y - \dfrac{t_{f2}+y_p}{2}\right) + \\ A_w\left(t_{f1} + \dfrac{h}{2} - y\right) + A_{f2}\left(d - y - \dfrac{t_{f2}}{2}\right) \end{array}\right] \tag{2}$$

The equations of the location of the plastic neutral axis and the plastic moment can be similarly derived for the other cases. The equations are given below.

Plastic neutral axis in the web:

$$y_p = \frac{A}{2t_w}\left(p+1\right) - \frac{A_{f1}}{t_w} + t_{f1} \quad \text{for} \quad \frac{2A_{f1}}{A} - 1 \le p \le \frac{2\left(A_{f1}+A_w\right)}{A} - 1 \tag{3}$$

$$M_{pc} = F_y\left\{\begin{array}{c} A_{f1}\left(y - \dfrac{t_1}{2}\right) + t_w\left\{\begin{array}{l} \left(y_p - t_{f1}\right)\left(y - \dfrac{t_{f1}}{2} - \dfrac{y_p}{2}\right) + \\ \left(t_{f1} + h - y_p\right)\left(\dfrac{t_{f1}+h+y_p}{2} - y\right) \end{array}\right\} \\ + A_{f2}\left(d - \dfrac{t_{f2}}{2} - y\right) \end{array}\right\} \tag{4}$$

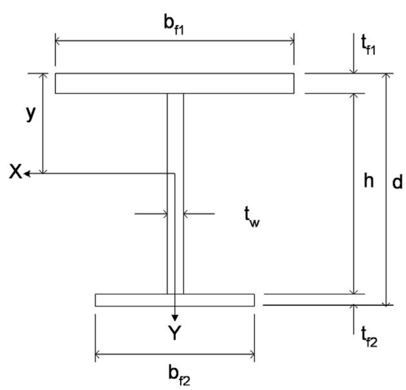


Fig.1. Cross section of the singly symmetric shape.

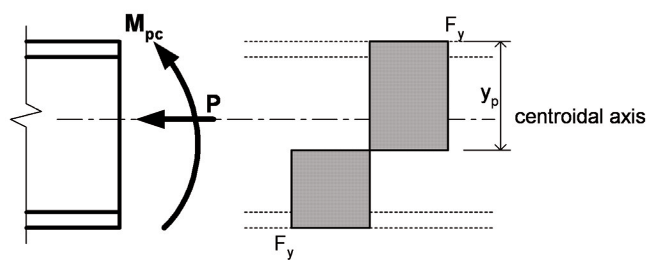


Fig. 3. Plastic neutral axis in the web.

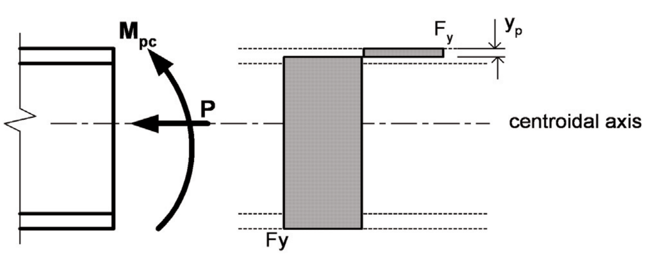


Fig. 2. Plastic neutral axis in the top flange.

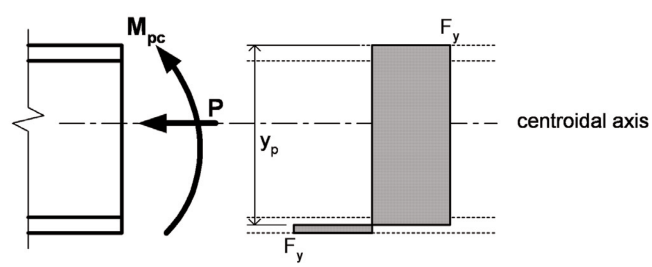


Fig. 4. Neutral axis in the bottom flange.

Plastic neutral axis in the bottom flange:

$$y_p = \frac{A(p-1)}{2b_{f2}} + d \quad \text{for} \quad \frac{2(A_{f1}+A_w)}{A} - 1 \le p \le 1 \tag{5}$$

$$M_{pc} = F_y \left[\begin{array}{l} A_{f1}\left(y - \dfrac{t_{f1}}{2}\right) - A_w\left(t_{f1} + \dfrac{h}{2} - y\right) \\ -b_{f2}\left\{ \begin{array}{l} \left(y_p - t_{f1} - h\right)\left(\dfrac{y_p + t_{f1} + h}{2} - y\right) \\ -\left(d - y_p\right)\left(\dfrac{d + y_p}{2} - y\right) \end{array} \right\} \end{array} \right] \tag{6}$$

The interaction curves for the full plastic capacity of a zero-length tee-shape is shown in Figure 5. The plastic limit envelope is the heavy solid line. The axial force P is applied through the geometric centroid of the cross section. The directions of the applied forces are shown in Figure 6 for the four quadrants of the interaction space. Also shown in Figure 5 are the curves representing the relationship between axial force and bending moment when the stress at the extreme fiber is equal to the yield stress, and the interaction curve according to the specification of the American Institute of Steel Construction (AISC, 1999). The axial force axis shows the non-dimensional ratio P/P_y, where $P_y = AF_y$ is the axial force when the whole cross section is yielded in compression or tension. The bending moment axis shows the non-dimensional ratio M/M_p, where the plastic moment M_p is defined by the formulas given in Appendix 1.

The formulas for the first yield interaction relationship for the first quadrant (P is compressive and M causes compression on the top of the top fiber) are as follows:

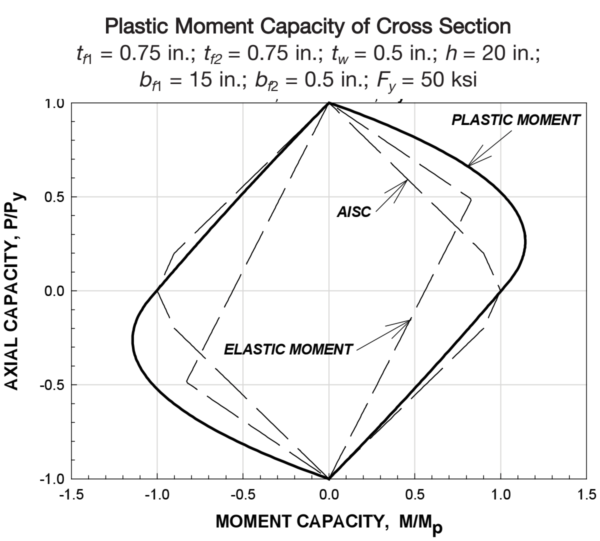


Fig. 5. Interaction diagram for a zero-length tee-shape.

For compressive yielding at the top of the top flange

$$\frac{P}{A} + \frac{My}{I_x} = F_y \tag{7}$$

For tensile yield at the bottom of the bottom flange

$$\frac{P}{A} - \frac{M(d-y)}{I_x} = -F_y \tag{8}$$

Similar expressions hold for the other three quadrants shown in Figure 6. The AISC-type interaction equations for the first quadrant of the zero-length member are:

$$\frac{P}{P_y} + \frac{8M}{9M_p} = 1.0 \quad \text{for} \quad \frac{P}{P_y} > 0.2 \tag{9}$$

$$\frac{P}{P_y} + \frac{8M}{9M_p} = 1.0 \quad \text{for} \quad \frac{P}{P_y} > 0.2 \tag{10}$$

In viewing Figure 5 it is evident that for a zero-length member the AISC interaction equations are conservative in the first and third quadrants, while in the second and fourth quadrant they are slightly unconservative for this particular cross section. Equations 9 and 10 are not strictly valid for tee-shapes according to the AISC Specification Section F1.2c. There the nominal moments are limited to $1.5M_y$ and M_y, respectively, depending on whether the flange or the stem are in compression (M_y is the yield moment). Because the Equations 9 and 10 are not exactly in conformance they are named "AISC-type formulas" herein.

The previous derivations, equations, and plots are applicable for the in-plane strength of the cross section, that is, for a zero-length member. In the AISC Specification (AISC, 1999) the formulas for the cross section are generalized by redefining the terms in the interaction equations: the applied moment, M, is defined as the amplified moment obtained from a second-order analysis of the frame. Alternately, M may be determined by the approximate procedure given in

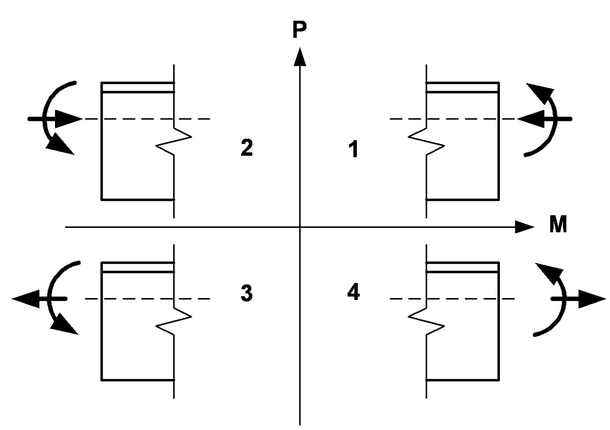


Fig. 6 Direction of axial force and bending moment on the cross section.

Chapter C, that is, $M = B_1 M_{nt} + B_2 M_{lt}$, where B_1 and B_2 are amplification factors, and M_{nt} is the moment due to applied forces causing no lateral translation, and M_{lt} is due to forces causing lateral translation of the story under consideration. M_{nt} and M_{lt} are obtained in this case by performing a first-order analysis of the frame. The maximum value of the axial force P is taken in the AISC procedure as the yield strength of the cross section, $P_y = AF_y$, when the axial force is in tension, and $P_{max} = P_{cr}$, the in-plane critical load computed by the column formulas in Section E2 (AISC, 1999). In effect, then, the zero-length interaction curve is rotated so that its end at $M = 0$ is anchored to the point $P = P_{cr}$. This is shown in Figure 7 for a singly symmetric member.

The geometric rotation of the interaction curve is accomplished by assuming that the compressive stress σ varies from the yield stress F_y at $P = 0$ to the critical stress F_{cr} when $P = P_{cr}$. For any value of P between $P = 0$ and $P = P_{cr}$ linear reduction is assumed, that is

$$\sigma = F_y - \left(F_y - F_{cr}\right)\frac{P}{P_{cr}} \tag{11}$$

The equations for the plastic capacity (Equations 1 through 6) are consequently modified as follows:

Plastic neutral axis in the top flange:

$$y_p = \frac{A}{b_{f1}}\left[\frac{p+1}{\frac{\sigma}{F_y}+1}\right] \quad \text{for } 0 \le \frac{p+1}{\frac{\sigma}{F_y}+1} \le \frac{A_{f1}}{A} \tag{12}$$

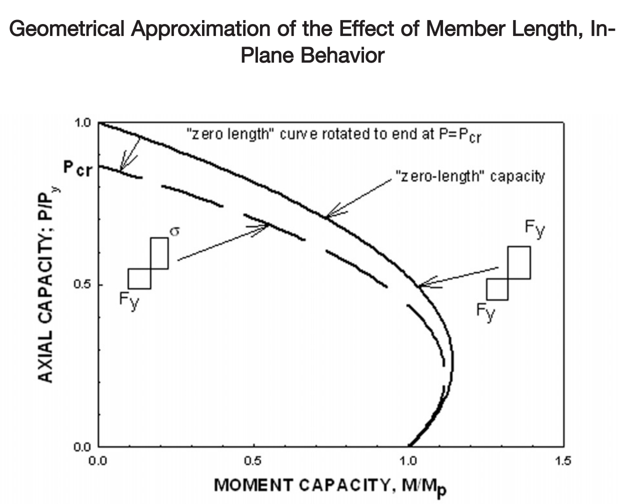


Fig. 7. Rotation of interaction curve.

$$\begin{aligned} M_{pc} &= \sigma\left[b_{f1} y_p \left(y - \frac{y_p}{2}\right)\right] \\ &+ F_y \left[\begin{aligned} &-b_{f1}\left(t_{f1} - y_p\right)\left(y - \frac{t_{f1}+y_p}{2}\right) \\ &+ A_w\left(t_{f1} + \frac{h}{2} - y\right) + A_{f2}\left(d - y - \frac{t_{f2}}{2}\right)\end{aligned}\right]\end{aligned} \tag{13}$$

Plastic neutral axis in the web:

$$y_p = \frac{A}{t_w}\left[\frac{p+1}{\frac{\sigma}{F_y}+1}\right] - \frac{A_{f1}}{t_w} + t_{f1} \quad \text{for } \frac{A_{f1}}{A} \le \left[\frac{p+1}{\frac{\sigma}{F_y}+1}\right] \le \frac{A_w + A_{f1}}{A} \tag{14}$$

$$\begin{aligned} M_{pc} &= A_{f1}\left(y - \frac{t_{f1}}{2}\right)\sigma \\ &+ t_w \left\{\begin{aligned} &\left(y_p t_{f1}\right)\left(y - \frac{t_{f1}}{2} - \frac{y_p}{2}\right)\sigma \\ &+ \left(t_{f1} + h - y_p\right)\left(\frac{t_{f1} + h + y_p}{2} - y\right)F_y \end{aligned}\right\} \\ &+ A_{f2}\left(d - \frac{t_{f2}}{2} - y\right)f_y \end{aligned} \tag{15}$$

Plastic neutral axis in the bottom flange:

$$y_p = \frac{A}{b_{f2}}\left[\frac{p+1}{\frac{\sigma}{F_y}+1}\right] - \frac{A_{f1} + A_w}{b_{f2}} + t_{f1} + h$$

$$\text{for } \frac{A_{f1} + A_w}{A} \le \left[\frac{p+1}{\frac{\sigma}{F_y}+1}\right] \le 1 \tag{16}$$

$$\begin{aligned} M_{pc} &= A_{f1}\left(y - \frac{t_{f1}}{2}\right)\sigma - A_w\left(t_{f1} + \frac{h}{2} - y\right)\sigma \\ &- b_{f2}\left\{\begin{aligned} &\left(y_p - t_{f1} - h\right)\left(\frac{y_p + t_{f1} + h}{2} - y\right)\sigma \\ &- \left(d - y_p\right)\left[\frac{d + y_p}{2} - y\right]F_y \end{aligned}\right\}\end{aligned} \tag{17}$$

The axial capacity P_{cr} in these equations is the critical load in the plane of symmetry (the y-y axis in Figure 1, i.e. buckling is about the x-x axis). The value of P_{cr} is determined by the column formulas in Section E2 of the AISC Specification:

$$\lambda_x = \frac{L}{\pi r_x}\sqrt{\frac{F_y}{E}} \tag{18}$$

$$P_{crx} = AF_{crx} \tag{19}$$

$$F_{crx} = \begin{bmatrix} 0.658^{\lambda_x^{\,2}} F_y \;\; \text{for } \lambda_x \le 1.5 \\ \dfrac{0.877 F_y}{\lambda_x^{\,2}} \;\; \text{for } \lambda_x > 1.5 \end{bmatrix} \tag{20}$$

Typical interaction diagrams are presented in Figures 8 and 9. The curves in Figure 8 are for built-up shapes: a doubly symmetric wide-flange shape, a cross section for which the bottom flange is half as wide as the top flange, and a tee shape. The solid lines represent in-plane strength, while the dashed lines are for lateral-torsional buckling behavior (to be discussed in the next section of this paper). The heavy lines are depicting the curves of the fully plastic capacity defined by Equations 12 through 17, and the thin lines represent the AISC-type interaction equations. The information in Figure 9 is for rolled shapes: a 2L8×4×¾ double angle, and a WT18×67.5 tee section. From these typical curves it can be observed that:

- The AISC-type interaction equations are an excellent approximation for a doubly symmetric wide-flange shape;
- In the second and fourth quadrants (see Figure 6 for definition of the directions of the cross-sectional forces), assuming that the top flange is larger than the bottom flange, the AISC-type curves are close to the fully plastic curves; and
- In the first and third quadrant the AISC–type approach can be quite conservative, especially for tee-shapes.

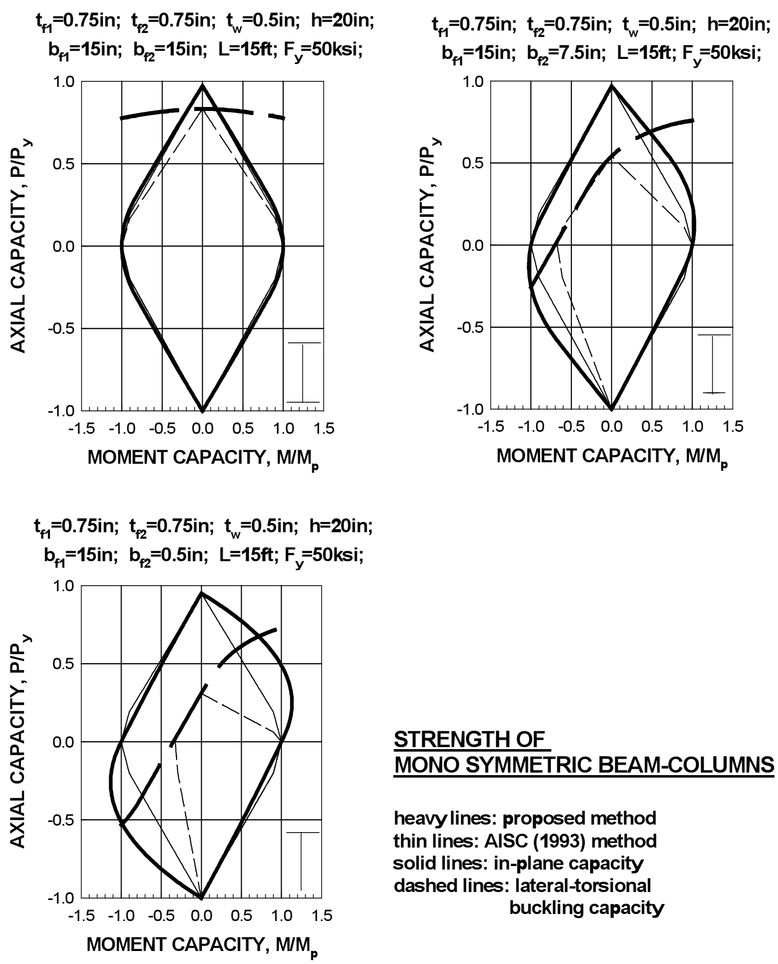


Fig. 8. Interaction curves for built-up singly symmetric shapes.

LATERAL-TORSIONAL BUCKLING

The elastic lateral-torsional interaction relationship between an axial force P and equal end bending moments M_o about the x-axis, causing single curvature deflection along the length of the beam-column, is given by the following quadratic equation (Galambos, 1968 and 1998):

$$\left(P_{ey}-P_e\right)\left(r_o^2 P_z - r_o^2 P_e + \beta_x M_o\right) = \left(M_o + P_e y_o\right)^2 \qquad (21)$$

where

$$P_{ey} = \frac{\pi^2 E I_y}{L^2} \qquad (22)$$

$$P_z = \frac{1}{r_o^2}\left[\frac{\pi^2 E C_w}{L^2} + GJ\right] \qquad (23)$$

The cross-sectional properties r_o, β_x, I_y, C_w, and J are defined in Appendix 1. The quadratic equation (Equation 21) is solved for P_e for a given M_o, and then an equivalent slenderness parameter λ_{ce} can be determined according to Appendix E3 of the AISC Specification, as follows:

$$\lambda_{ce} = \sqrt{\frac{A F_y}{P_e}} \qquad (24)$$

The critical lateral-torsional buckling load is then calculated using the AISC column formula:

$$P_{ltb} = A F_{ltb} \qquad (25)$$

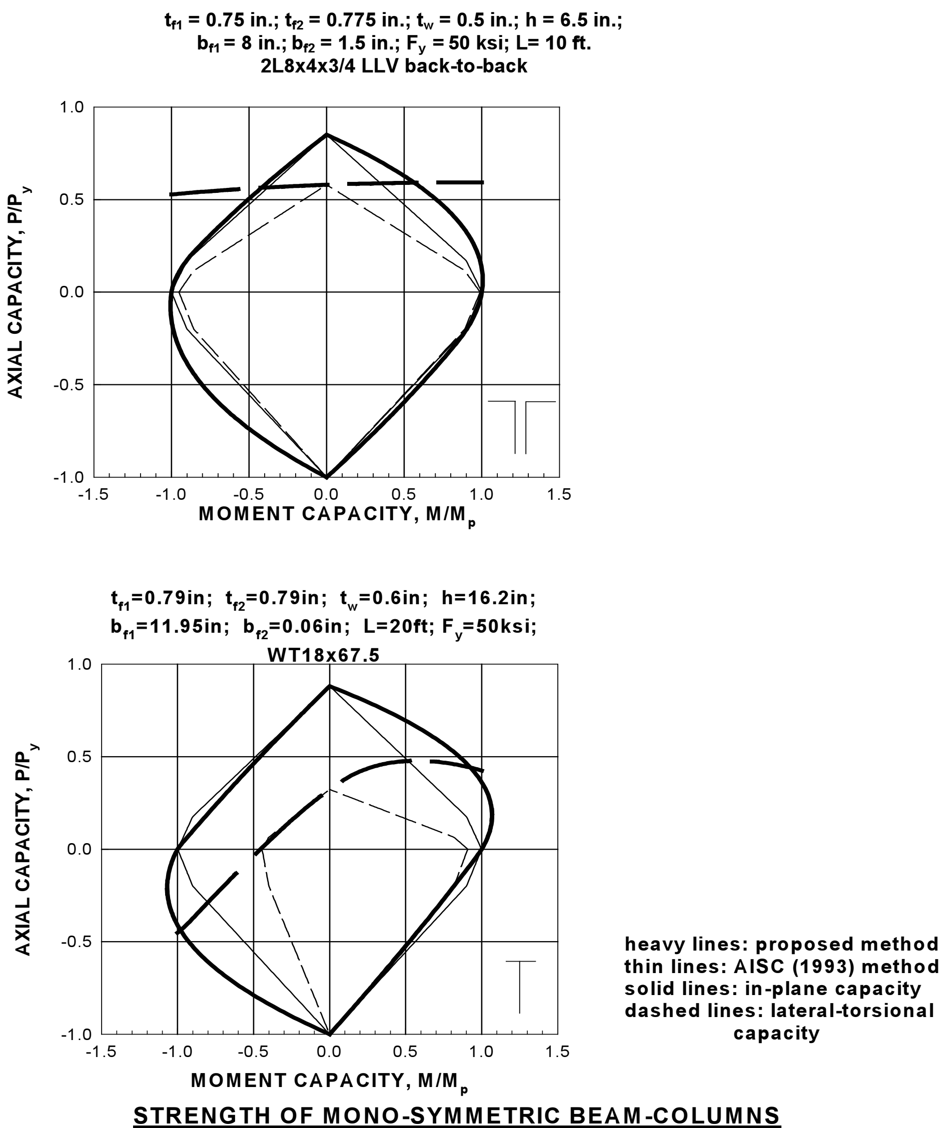


Fig.9. Interaction curves for rolled shapes.

$$F_{ltb} = \begin{bmatrix} 0.658^{\lambda_{ce}^2} F_y & \text{for } \lambda_{ce} \le 1.5 \\ \dfrac{0.877F_y}{\lambda_{ce}^2} & \text{for } \lambda_{ce} > 1.5 \end{bmatrix} \tag{26}$$

The resulting curves for the various cross sections are shown in Figures 8 and 9 by the heavy dashed lines. The AISC-type interaction curves are shown as the thin dashed lines. The in-plane strength is shown as solid lines, and the lateral-torsional strength is depicted as dashed lines. For a doubly symmetric wide-flange shape (curves in the top left corner of Figure 8) the in-plane strength computed by the method of this paper and the AISC-type interaction equations give an essentially identical result. For lateral-torsional buckling it is seen that this mode of failure governs only over a region of small moment and high axial force when the method of this paper is used, but that the AISC-type interaction equation method requires a reduction of the moment over all moment values. The differences between the AISC-type interaction equation predictions and the proposed method become more pronounced for sections that have a different flange top and bottom, or that are tee shapes. The AISC-type approach compares closely with the proposed method for in-plane strength prediction in the second and fourth quadrants, but it is conservative in the first and third quadrants. The lateral-torsional buckling strength is closely predicted by the AISC-type equations in the second quadrant (compression in the smaller flange), but they are quite conservative in the first quadrant (compression in the larger flange). In this case lateral-torsional buckling is not critical over significant regions of the domain of forces. The double angle case (top diagram in Figure 9) is a special case that shows a smaller effect of lateral-torsional buckling. The curves in Figures 8 and 9 illustrate the phenomena that are at work for these singly symmetric sections. The AISC-type method is a reasonably good approach when the compressed flange is the smaller one. When the compressed flange is the larger one, both the in-plane and the lateral-torsional buckling predictions of the AISC-type method can be significantly conservative.

The presentations in this paper are not amenable to easy manual calculation. For this reason spread sheets can be very advantageous. The enclosed Appendices 2 and 3 give programs for the MATHCAD 8 PROFESSIONAL software. Appendix 2 presents the calculation scheme for the proposed in-plane and lateral-torsional buckling methods. Appendix 3 gives the AISC Specification method for the lateral-torsional buckling checking. This program is presented to demonstrate that the proposed method and the AISC method require about the same amount of computational effort.

SUMMARY AND CONCLUSIONS WITH EXAMPLE

This paper has demonstrated that the interaction equations of the AISC give quite conservative designs for singly symmetric wide-flange shapes when the larger of the flanges is under compression. This conclusion can be seen in the curves in Figures 8 and 9 for several example beam-columns. In order to obtain a further feeling for the differences between the proposed and the AISC methods, a numerical example will be discussed below.

Problem statement: Compare the required strengths determined by the method of this paper and the AISC Specification for a simply supported tee-shaped beam-column subjected to a factored compressive axial load $P_u = 400$ kips through the centroid of the tee section, and to equal end moments $M_u = 2{,}000$ kip-in. These moments cause single curvature bending about the x-axis of the member such that the flange of the tee is in compression. The length of the member is 20 ft. The section is built up using a WT18×67.5 tee section and a bottom flange plate, and the yield stress of the steel is $F_y = 50$ ksi. The details of the strength check are given in the MATHCAD8 Professional programs reproduced in Appendices 2 and 3. Appendix 2 pertains to the method proposed in this paper, and Appendix 3 presents the steps of the AISC Specification.

In-plane strength: (member is laterally braced)

Proposed method: (Appendix 2)

For $P_u = 400$ kips the factored flexural strength is $\phi_b M_{pc} =$ 3,580 kip-in.

The margin is: $\phi_b M_{pc} - M_u = 1{,}580$ kip-in.

Ratio: $M_u / \phi_b M_{pc} = 2{,}000/3{,}580 = 0.56$

The beam-column can actually support 3,580 kip-in. of moment in the presence of a 400 kip axial force.

AISC-type Method:

From Appendix 2, $\dfrac{P_{nx}}{P_y} = 0.878$

From Appendix 2, $P_y = AF_y = 982$ kips

$\phi_c P_{nx} = 0.85 \times 0.878 \times 989 = 733$ kips

$\dfrac{P_u}{\phi_c P_{nx}} = 400/733 = 0.54 > 0.2$

From Appendix 2 (or from Appendix 3), the plastic moment $M_p = 4{,}505$ kip-in.

$\phi_b M_n = 0.9 M_p = 0.9 \times 4{,}505 = 4{,}055$ kip-in.

AISC Interaction Equation:

$$\frac{P_u}{\phi_c P_{nx}} + \frac{8}{9} \times \frac{M_u}{\phi_b M_n} = 0.54 + \frac{8 \times 2{,}000}{9 \times 4{,}055} = 0.98$$

The member could actually support a moment of 2,098 kip-in.

Ratio: proposed method/AISC interaction equation method = 3,580/2,098 = 1.71

The AISC LRFD specification also provides an alternate approach in Section H3: the maximum elastic stress in the cross section, $f_{max} \le \phi_b F_y = 0.9 \times 50 = 45$ ksi.

The compressive top flange elastic stress is

$$f_{tf} = \frac{P_u}{A} + \frac{M_u y}{I_x} = 45 \text{ ksi}$$

The maximum required moment can then be determined from the relationship

$$M_u = \left[45 - \frac{P_u}{A}\right]\frac{I_x}{y}$$

With $P_u = 400$ kips, $A = 19.634$ in.^{2}, $y = 5.011$ in., and $I_x = 633.075$ in.^{4}, the value of M_u is computed to be 3,111 kip-in.

$$\begin{aligned} f_{bf} &= \frac{P_u}{A} - \frac{M_u(d-y)}{I_x} \\ &= \frac{400}{19.634} - \frac{3{,}111 \times (17.78 - 5.011)}{633.075} \\ &= |-42.38| \text{ ksi } \le |-45| \text{ ksi} \end{aligned}$$

The stress in the bottom flange is

The required tensile stress in the bottom flange is less than 45 ksi, and so the maximum permitted bending moment is 3,111 kip-in. This is higher than the 2,098 kip-in. obtained by using the interaction equation method (3,111/2,098 = 1.48).

Ratio: Proposed method / AISC elastic limit method = 3,580/3,111 = 1.15.

Lateral-torsional buckling strength

Proposed method:

From Appendix 2 and for $M_u = 2{,}000$ kip-in, we get $\phi_c P_n = 399$ kips $\approx$ 400 kips. The member is just OK.

AISC method:

From Appendix 3 with the top flange in compression, the AISC interaction sum equals 1.94, indicating that the member is under strength. The beam-column can actually only support $\phi_c P_n = 144$ kips.

Ratio: proposed method / AISC method = 399/144 = 2.77

Another type of comparison of the proposed method with the AISC Procedure can be made by considering the computational effort required in determining the lateral-torsional buckling strengths in Appendices 2 and 3. The AISC method requires a solution of a complicated non-linear equation to obtain the value of L_r (see Appendix 3). The required computational effort is essentially the same for both approaches. For about the same amount of effort, then, one can use a theory that is less conservative than that of the AISC Specification.

REFERENCES

AISC (1999), *Load and Resistance Factor Design Specification for Structural Steel Buildings*, American Institute of Steel Construction, Chicago, IL

Galambos, T. V. (1968), *Structural Members and Frames*, Prentice-Hall, Englewood Springs, NJ

Galambos, T. V. (1998), *Guide to Stability Design Criteria for Metal Structures*, Wiley Interscience, New York, NY

APPENDIX 1.
Cross-sectional Properties (rev. 6/5/00)

The cross section consists of three rectangular plates that are welded together to form a singly symmetric I shape (Fig. A1-1).

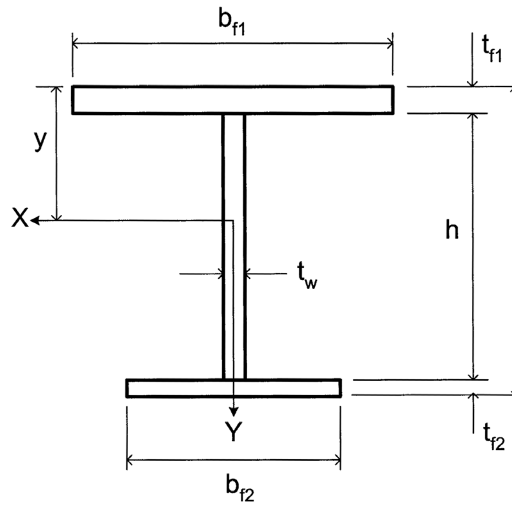


Fig. A1-1 Cross section

Areas:

$$A_{f1} := b_{f1} \cdot t_{f1} \qquad A_{f2} := b_{f2} \cdot t_{f2} \qquad A_w := h \cdot t_w \qquad A := A_{f1} + A_{f2} + A_w$$

Centroid and moment of inertia about x-axis:

$$y := \frac{1}{A} \cdot \left[\frac{A_{f1} \cdot t_{f1}}{2} + A_w \cdot \left(t_{f1} + \frac{h}{2} \right) + A_{f2} \cdot \left(d - \frac{t_{f2}}{2} \right) \right]$$

$$I_x := \frac{b_{f1} \cdot t_{f1}^3 + h^3 \cdot t_w + b_{f2} \cdot t_{f2}^3}{12} + A_{f1} \cdot \left(y - \frac{t_{f1}}{2} \right)^2 + A_w \cdot \left(t_{f1} + \frac{h}{2} - y \right)^2 + A_{f2} \cdot \left(d - \frac{t_{f2}}{2} - y \right)^2$$

Lateral-torsional buckling section properties:

$$y_{bar} := y - \frac{t_{f1}}{2}$$

$$d' := d - \frac{t_{f1} + t_{f2}}{2}$$

$$\alpha := \frac{1}{1 + \left(\frac{b_{f1}}{b_{f2}} \right)^3 \cdot \frac{t_{f1}}{t_{f2}}}$$

$$I_y := \frac{b_{f1}^3 \cdot t_{f1} + b_{f2}^3 \cdot t_{f2} + h \cdot t_w^3}{12}$$

$$y_o := -y_{bar} + \alpha \cdot d'$$

$$C_w := \frac{d'^2 \cdot b_{f1}^3 \cdot t_{f1} \cdot \alpha}{12}$$

$$r_o := \sqrt{y_o^2 + \frac{I_x + I_y}{A}}$$

$$H := 1 - \left(\frac{y_o}{r_o} \right)^2$$

$$J := \frac{b_{f1} \cdot t_{f1}^3 + b_{f2} \cdot t_{f2}^3 + d' \cdot t_w^3}{3}$$

$$\beta_x := \frac{1}{I_x} \cdot \left[(d' - y_{bar}) \cdot \left[\frac{b_{f2}^3 \cdot t_{f2}}{12} + b_{f2} \cdot t_{f2} \cdot (d' - y_{bar})^2 + \frac{t_w \cdot (d' - y_{bar})^3}{4} \right] \cdots + (-y_{bar}) \cdot \left(\frac{b_{f1}^3 \cdot t_{f1}}{12} + b_{f1} \cdot t_{f1} \cdot y_{bar}^2 + \frac{t_w \cdot y_{bar}^3}{4} \right) \right] - 2 \cdot y_o$$

Derived structural quantities:

Plastic moment about x-axis when axial force is zero

Location of plastic neutral axis measured from the top of the top flange

$$y_{p1} := \frac{A}{2 \cdot b_{f1}}$$

$$y_{p2} := \frac{A - 2 \cdot A_{f1}}{2 \cdot t_w} + t_{f1}$$

$$y_{p3} := d - \frac{A}{2 \cdot b_{f2}}$$

Plastic moment M_p

Neutral axis in the top flange:

$$M_{p1} := F_y \cdot \left[\frac{b_{f1}}{2} \cdot \left[y_{p1}^2 + (t_{f1} - y_{p1})^2 \right] + A_w \cdot \left(\frac{h}{2} + t_{f1} - y_{p1} \right) + A_{f2} \cdot \left(t_{f1} - y_{p1} + h + \frac{t_{f2}}{2} \right) \right]$$

Neutral axis in the web:

$$M_{p2} := F_y \cdot \left[A_{f1} \cdot \left(y_{p2} - \frac{t_{f1}}{2} \right) + \left[\frac{t_w}{2} \cdot \left[(y_{p2} - t_{f1})^2 + (h + t_{f1} - y_{p2})^2 \right] + A_{f2} \cdot \left(d - y_{p2} - \frac{t_{f2}}{2} \right) \right] \right]$$

Neutral axis in the bottom flange:

$$M_{p3} := F_y \cdot \left[A_{f1} \cdot \left(y_{p3} - \frac{t_{f1}}{2} \right) + A_w \cdot \left(y_{p3} - t_{f1} - \frac{h}{2} \right) + \frac{b_{f2}}{2} \cdot \left[(y_{p3} - t_{f1} - h)^2 + (d - y_{p3})^2 \right] \right]$$

$$M_p := \left| \begin{array}{ll} M_{p1} & \text{if } A \le 2 \cdot A_{f1} \\ M_{p2} & \text{if } 2\,A_{f1} \le A \le 2 \cdot (A_w + A_{f1}) \\ M_{p3} & \text{otherwise} \end{array} \right.$$

Elastic lateral-torsional buckling load under axial load only, P_{cre}:

$$P_{ey} := \frac{\pi^2 \cdot E \cdot I_y}{L^2}$$

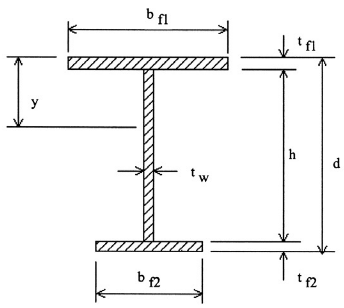
$$P_z := \frac{1}{r_o^2} \cdot \left(\frac{\pi^2 \cdot E \cdot C_w}{L^2} + G \cdot J \right)$$

$$P_{cre} := \frac{P_{ey} + P_z}{2 \cdot H} \cdot \left[1 - \sqrt{1 - \frac{4 \cdot P_{ey} \cdot P_z \cdot H}{(P_{ey} + P_z)^2}} \right]$$

APPENDIX 2.
In-plane and LTB Strength of Singly-symmetric I-shaped Beam-columns

Notes: All plates have the same yield stress
Axial force is positive in compression and negative in tension
Bending moment is always positive
Top flange is the flange that would be in compression under bending moment alone

Input Data: WT18x67.5



$ksi := 1000 \cdot psi$

$kip := 1000 \cdot lbf$

$b_{f1} := 11.95 \cdot in$

$b_{f2} := 0.6 \cdot in$

$h := 16.2 \cdot in$

$t_{f1} := 0.79 \cdot in$

$t_{f2} := 0.79 \cdot in$

$F_y := 50 \cdot ksi$ $\quad L := 20 \cdot ft$ $\quad d := t_{f1} + h + t_{f2}$

$E := 29000 \cdot ksi$ $\quad G := 0.385 \cdot E$ $\quad t_w := 0.6 \cdot in$

$P_u := 400 \cdot kip$ $\quad M_u := 2000 \cdot kip \cdot in$ $\quad \phi_b := 0.9$

$$\phi_c := \begin{cases} 0.85 & \text{if } P_u \ge 0 \\ 0.9 & \text{otherwise} \end{cases}$$

Areas:

$A_{f1} := b_{f1} \cdot t_{f1}$ $\quad A_{f2} := b_{f2} \cdot t_{f2}$ $\quad A_w := h \cdot t_w$

$A := A_{f1} + A_{f2} + A_w$ $\quad A = 19.634 \cdot in^2$

Centroid:

$$y := \frac{1}{A} \cdot \left[\frac{A_{f1} \cdot t_{f1}}{2} + A_w \cdot \left(t_{f1} + \frac{h}{2} \right) + A_{f2} \cdot \left(d - \frac{t_{f2}}{2} \right) \right] \qquad y = 5.011 \cdot in$$

Moment of inertia about x-axis

$$I_x := \frac{b_{f1} \cdot t_{f1}^3 + h^3 \cdot t_w + b_{f2} \cdot t_{f2}^3}{12} + A_{f1} \cdot \left(y - \frac{t_{f1}}{2} \right)^2 + A_w \cdot \left(t_{f1} + \frac{h}{2} - y \right)^2 + A_{f2} \cdot \left(d - \frac{t_{f2}}{2} - y \right)^2$$

$I_x = 633.075 \cdot in^4$

Plastic Moment when P=0

$$y_p := \begin{cases} \dfrac{A}{2 \cdot b_{f1}} & \text{if } A \le 2 \cdot A_{f1} \\ \dfrac{A - 2 \cdot A_{f1}}{2 \cdot t_w} + t_{f1} & \text{if } 2 \cdot A_{f1} \le A \le 2 \cdot (A_w + A_{f1}) \\ d - \dfrac{A}{2 \cdot b_{f2}} & \text{otherwise} \end{cases} \qquad y_p = 1.418 \cdot in$$

$$M_{p1} := F_y \cdot \left[\frac{b_{f1}}{2} \cdot \left[y_p^2 + (t_{f1} - y_p)^2 \right] + A_w \cdot \left(\frac{h}{2} + t_{f1} - y_p \right) + A_{f2} \cdot \left(t_{f1} - y_p + h + \frac{t_{f2}}{2} \right) \right]$$

$$M_{p2} := F_y \cdot \left[A_{f1} \cdot \left(y_p - \frac{t_{f1}}{2} \right) + \left[\frac{t_w}{2} \cdot \left[(y_p - t_{f1})^2 + (h + t_{f1} - y_p)^2 \right] + A_{f2} \cdot \left(d - y_p - \frac{t_{f2}}{2} \right) \right] \right]$$

$$M_{p3} := F_y \cdot \left[A_{f1} \cdot \left(y_p - \frac{t_{f1}}{2} \right) + A_w \cdot \left(y_p - t_{f1} - \frac{h}{2} \right) + \frac{b_{f2}}{2} \cdot \left[(y_p - t_{f1} - h)^2 + (d - y_p)^2 \right] \right]$$

$$M_p := \begin{cases} M_{p1} & \text{if } A \le 2 \cdot A_{f1} \\ M_{p2} & \text{if } 2\ A_{f1} \le A \le 2 \cdot (A_w + A_{f1}) \\ M_{p3} & \text{otherwise} \end{cases}$$

$M_p = 4.505 \cdot 10^3 \cdot kip \cdot in$

Lateral-torsional Buckling cross section properties:

$$y_{bar} := y - \frac{t_{f1}}{2} \qquad d' := d - \frac{t_{f1} + t_{f2}}{2}$$

$$\alpha := \frac{1}{1 + \left(\dfrac{b_{f1}}{b_{f2}} \right)^3 \cdot \dfrac{t_{f1}}{t_{f2}}} \qquad I_y := \frac{b_{f1}^3 \cdot t_{f1} + b_{f2}^3 \cdot t_{f2} + h \cdot t_w^3}{12} \qquad I_y = 112.65 \cdot in^4$$

$$y_o := -y_{bar} + \alpha \cdot d' \qquad y_o = -4.613 \cdot in \qquad C_w := \frac{d'^2 \cdot b_{f1}^3 \cdot t_{f1} \cdot \alpha}{12} \qquad C_w = 4.104 \cdot in^6$$

$$r_o := \sqrt{y_o^2 + \frac{I_x + I_y}{A}} \qquad J := \frac{b_{f1} \cdot t_{f1}^3 + b_{f2} \cdot t_{f2}^3 + d' \cdot t_w^3}{3}$$

$$\beta_x := \frac{1}{I_x} \cdot \left[\begin{array}{l} (d' - y_{bar}) \cdot \left[\dfrac{b_{f2}^3 \cdot t_{f2}}{12} + b_{f2} \cdot t_{f2} \cdot (d' - y_{bar})^2 + \dfrac{t_w \cdot (d' - y_{bar})^3}{4} \right] \ldots \\ + (-y_{bar}) \cdot \left(\dfrac{b_{f1}^3 \cdot t_{f1}}{12} + b_{f1} \cdot t_{f1} \cdot y_{bar}^2 + \dfrac{t_w \cdot y_{bar}^3}{4} \right) \end{array} \right] - 2 \cdot y_o$$

$J = 3.286 \cdot in^4$ $\quad r_o = 7.698 \cdot in$ $\quad \beta_x = 13.809 \cdot in$

$$P_{ey} := \frac{\pi^2 \cdot E \cdot I_y}{L^2} \qquad P_z := \frac{1}{r_o^2} \cdot \left(\frac{\pi^2 \cdot E \cdot C_w}{L^2} + G \cdot J \right)$$

$P_{ey} = 559.764 \cdot kip$ $\quad P_z = 619.374 \cdot kip$

$P_y := A \cdot F_y$ $\quad P_y = 981.725 \cdot kip$

Calculation of beam-column capacity

In-plane axial capacity, Sec. E1, AISC Specification

$$r_x := \sqrt{\frac{I_x}{A}} \qquad r_x = 5.678 \cdot in$$

$$\lambda_x := \frac{L}{\pi \cdot r_x} \cdot \sqrt{\frac{F_y}{E}} \qquad \lambda_x = 0.559$$

$$p_{xcrc} := \begin{cases} 0.658^{\lambda_x^2} & \text{if } \lambda_x \le 1.5 \\ \dfrac{0.877}{\lambda_x^2} & \text{otherwise} \end{cases} \qquad p_{xcrc} = 0.878$$

$$p_{xcr} := \begin{cases} p_{xcrc} & \text{if } P_u \ge 0 \\ 1 & \text{otherwise} \end{cases}$$

$$P := \frac{P_u}{\phi_c}$$

$$p := \frac{P}{P_y} \qquad p = 0.479$$

$$\sigma := F_y \cdot \left[(p_{xcr} - 1) \cdot \left(\frac{p}{p_{xcr}} \right) + 1 \right]$$

Plastic moment when axial force is present

$$y_{p1} := \frac{A}{b_{f1}} \cdot \left[\frac{1+p}{\left(\frac{\sigma}{F_y}+1\right)} \right]$$

$$M_{pc1} := \sigma \left[b_{f1} \cdot y_{p1} \cdot \left(y - \frac{y_{p1}}{2} \right) \right] + F_y \cdot \left[-b_{f1} \cdot (t_{f1} - y_{p1}) \cdot \left(y - \frac{y_{p1}}{2} - \frac{t_{f1}}{2} \right) \ldots \atop + A_w \cdot \left(\frac{h}{2} + t_{f1} - y \right) + A_{f2} \cdot \left(d - y - \frac{t_{f2}}{2} \right) \right]$$

$$y_{p2} := \left[\frac{A}{t_w} \cdot \frac{(p+1)}{\left(\frac{\sigma}{F_y}+1\right)} - \frac{A_{f1}}{t_w} + t_{f1} \right]$$

$$M_{pc2} := \sigma \cdot \left[A_{f1} \cdot \left(y - \frac{t_{f1}}{2} \right) + t_w \cdot (y_{p2} - t_{f1}) \cdot \left(y - \frac{t_{f1}}{2} - \frac{y_{p2}}{2} \right) \right] \ldots$$
$$+ F_y \cdot \left[t_w \cdot (h + t_{f1} - y_{p2}) \cdot \left(\frac{h + y_{p2} + t_{f2}}{2} - y \right) + A_{f2} \cdot \left(d - \frac{t_{f2}}{2} - y \right) \right]$$

$$y_{p3} := \frac{A}{b_{f2}} \cdot \frac{1+p}{\left(\frac{\sigma}{F_y}+1\right)} - \frac{A_{f1} + A_w}{b_{f2}} + t_{f1} + h$$

$$M_{pc3} := \sigma \cdot \left[A_{f1} \cdot \left(y - \frac{t_{f1}}{2} \right) - A_w \cdot \left(t_{f1} + \frac{h}{2} - y \right) - b_{f2} \cdot (y_{p3} - t_{f1} - h) \cdot \left(\frac{y_{p3} + t_{f1} + h}{2} - y \right) \right] \ldots$$
$$+ F_y \cdot \left[b_{f2} \cdot (d - y_{p3}) \cdot \left(\frac{d + y_{p3}}{2} - y \right) \right]$$

$$M_{pc} := \begin{cases} M_{pc1} & \text{if } 0 \le \dfrac{p+1}{\dfrac{\sigma}{F_y}+1} \le \dfrac{A_{f1}}{A} \\ M_{pc2} & \text{if } \dfrac{A_{f1}}{A} \le \dfrac{p+1}{\dfrac{\sigma}{F_y}+1} \le \dfrac{A_w + A_{f1}}{A} \\ M_{pc3} & \text{otherwise} \end{cases}$$

$M_{pc} = 3.977 \cdot 10^3 \text{ kip} \cdot \text{in}$

<u>Calculation of the inelastic critical lateral-torsional load by the equivalent slenderness method of Appendix E3</u>

$$M := \frac{M_u}{\phi_b}$$

$$a := r_o^2 - y_o^2$$

$$b := -\left[r_o^2 \cdot (P_{ey} + P_z) + (\beta_x + 2 \cdot y_o) \cdot M \right]$$

$$c := r_o^2 \cdot P_{ey} \cdot P_z + \beta_x \cdot P_{ey} \cdot M - (M)^2$$

$$P_{ltb} := \frac{-b - \sqrt{(b)^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

$$\lambda_{eq} := \sqrt{\frac{P_y}{|P_{ltb}|}}$$

$$P_{ltb} := \begin{cases} 0.658^{(\lambda_{eq})^2} & \text{if } \lambda_{eq} \le 1.5 \\ \dfrac{0.877}{(\lambda_{eq})^2} & \text{otherwise} \end{cases}$$

$$P_{ltb} := \begin{cases} P_{ltb} \cdot P_y & \text{if } P_{ltb} \ge 0 \\ -P_{ltb} \cdot P_y & \text{otherwise} \end{cases} \qquad P_{ltb} = 469.048 \text{ kip}$$

<u>Results:</u>

$F_y = 50 \text{ ksi}$ $L = 20 \text{ ft}$

$b_{f1} = 11.95 \text{ in}$ $t_{f1} = 0.79 \text{ in}$ $h = 16.2 \text{ in}$ $t_w = 0.6 \text{ in}$

$b_{f2} = 0.6 \text{ in}$ $t_{f2} = 0.79 \text{ in}$

$P_u = 400 \text{ kip}$ $M_u = 2 \cdot 10^3 \text{ kip} \cdot \text{in}$

<u>in-plane capacity</u>

$\phi_b \cdot M_{pc} = 3.58 \cdot 10^3 \text{ kip} \cdot \text{in}$ $\phi_b \cdot M_{pc} - M_u = 1.58 \cdot 10^3 \text{ kip} \cdot \text{in}$

$$\text{note1} := \begin{cases} \text{"in-plane capacity is OK"} & \text{if } \phi_b \cdot M_{pc} - M_u > 0 \\ \text{"design not OK"} & \text{otherwise} \end{cases}$$

note1 = "in-plane capacity is OK"

<u>lateral-torsional buckling capacity</u>

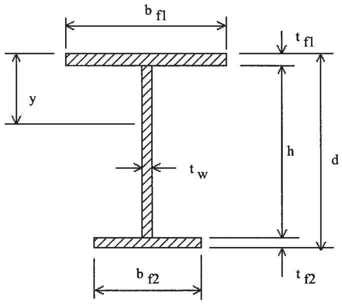
$\phi_c \cdot P_{ltb} = 398.691 \text{ kip}$ $\phi_c \cdot P_{ltb} - P_u = -1.309 \text{ kip}$

$$\text{note2} := \begin{cases} \text{"LTB capacity is OK"} & \text{if } \phi_c \cdot P_{ltb} - P_u > 0 \\ \text{"design not OK"} & \text{otherwise} \end{cases}$$

note2 = "design not OK"

APPENDIX 3.
AISC Interaction Calculations for LTB

Axial force P acts through centroid
P is a compressive force
WT 18x67.5 Section



$ksi := 1000 \cdot psi$

$kip := 1000 \cdot lbf$

$b_{fl} := 11.95 \cdot in$

$b_{f2} := 0.6 \cdot in$

$h := 16.2 \cdot in$

$t_{fl} := 0.79 \cdot in$

$t_{f2} := 0.79 \cdot in$

$d := t_{fl} + h + t_{f2}$

$t_w := 0.6 \cdot in$

$F_y := 50 \cdot ksi$ $\quad L := 20 \cdot ft$

$E := 29000 \cdot ksi$ $\quad G := 0.385 \cdot E$

$P_u := 400 \cdot kip$ $\quad M_u := 2000 \cdot kip \cdot in$

$\phi_c := 0.85$ $\quad \phi_b := 0.9$

Areas:

$A_{fl} := b_{fl} \cdot t_{fl}$ $\quad A_{f2} := b_{f2} \cdot t_{f2}$ $\quad A_w := h \cdot t_w$

$A := A_{fl} + A_{f2} + A_w$ $\quad A = 19.634 \cdot in^2$

Centroid:

$$y := \frac{1}{A} \cdot \left[\frac{A_{fl} \cdot t_{fl}}{2} + A_w \cdot \left(t_{fl} + \frac{h}{2} \right) + A_{f2} \cdot \left(d - \frac{t_{f2}}{2} \right) \right] \qquad y = 5.011 \cdot in$$

Moment of inertia:

$$I_x := \frac{b_{fl} \cdot t_{fl}^3 + h^3 \cdot t_w + b_{f2} \cdot t_{f2}^3}{12} + A_{fl} \cdot \left(y - \frac{t_{fl}}{2} \right)^2 + A_w \cdot \left(t_{fl} + \frac{h}{2} - y \right)^2 + A_{f2} \cdot \left(d - \frac{t_{f2}}{2} - y \right)^2$$

$I_x = 633.075 \cdot in^4$

Plastic Moment when P=0

$$y_{p1} := \frac{A}{2 \cdot b_{fl}}$$

$$y_{p2} := \frac{A - 2 \cdot A_{fl}}{2 \cdot t_w} + t_{fl}$$

$$y_{p3} := d - \frac{A}{2 \cdot b_{f2}}$$

$$M_{p1} := F_y \cdot \left[\frac{b_{fl}}{2} \cdot \left[y_{p1}^2 + \left(t_{fl} - y_{p1} \right)^2 \right] + A_w \cdot \left(\frac{h}{2} + t_{fl} - y_{p1} \right) + A_{f2} \cdot \left(t_{fl} - y_{p1} + h + \frac{t_{f2}}{2} \right) \right]$$

$$M_{p2} := F_y \cdot \left[A_{fl} \cdot \left(y_{p2} - \frac{t_{fl}}{2} \right) + \left[\frac{t_w}{2} \cdot \left[\left(y_{p2} - t_{fl} \right)^2 + \left(h + t_{fl} - y_{p2} \right)^2 \right] + A_{f2} \cdot \left(d - y_{p2} - \frac{t_{f2}}{2} \right) \right] \right]$$

$$M_{p3} := F_y \cdot \left[A_{fl} \cdot \left(y_{p3} - \frac{t_{fl}}{2} \right) + A_w \cdot \left(y_{p3} - t_{fl} - \frac{h}{2} \right) + \frac{b_{f2}}{2} \cdot \left[\left(y_{p3} - t_{fl} - h \right)^2 + \left(d - y_{p3} \right)^2 \right] \right]$$

$$M_p := \left| \begin{array}{ll} M_{p1} & \text{if } A \le 2 \cdot A_{fl} \\ M_{p2} & \text{if } 2\ A_{fl} \le A \le 2 \cdot \left(A_w + A_{fl} \right) \\ M_{p3} & \text{otherwise} \end{array} \right.$$

$M_p = 4.505 \cdot 10^3 \cdot kip \cdot in$

Lateral-torsional Buckling

$$y_{bar} := y - \frac{t_{fl}}{2} \qquad d' := d - \frac{t_{fl} + t_{f2}}{2}$$

$$\alpha := \frac{1}{1 + \left(\frac{b_{fl}}{b_{f2}} \right)^3 \cdot \frac{t_{fl}}{t_{f2}}} \qquad I_y := \frac{b_{fl}^3 \cdot t_{fl} + b_{f2}^3 \cdot t_{f2} + h \cdot t_w^3}{12} \qquad I_y = 112.65 \cdot in^4$$

$$y_o := -y_{bar} + \alpha \cdot d' \quad y_o = -4.613 \cdot in \qquad C_w := \frac{d'^2 \cdot b_{fl}^3 \cdot t_{fl} \cdot \alpha}{12} \qquad C_w = 4.104 \cdot in^6$$

$$r_o := \sqrt{y_o^2 + \frac{I_x + I_y}{A}} \qquad J := \frac{b_{fl} \cdot t_{fl}^3 + b_{f2} \cdot t_{f2}^3 + d' \cdot t_w^3}{3}$$

$r_o = 7.698 \cdot in$ $\qquad J = 3.286 \cdot in^4$

$$P_{ey} := \frac{\pi^2 \cdot E \cdot I_y}{L^2} \qquad P_z := \frac{1}{r_o^2} \cdot \left(\frac{\pi^2 \cdot E \cdot C_w}{L^2} + G \cdot J \right)$$

$P_{ey} = 559.765 \cdot kip$ $\qquad P_z = 619.374 \cdot kip$

$$H := 1 - \frac{y_o^2}{r_o^2} \qquad P_y := A \cdot F_y \qquad P_y = 981.725 \cdot kip$$

Torsional flexural capacity of the column:

$$P_{cre} := \frac{P_{ey} + P_z}{2 \cdot H} \left[1 - \sqrt{1 - \frac{4 \cdot P_{ey} \cdot P_z \cdot H}{\left(P_{ey} + P_z \right)^2}} \right] \qquad P_{cre} = 367.391 \cdot kip$$

$$\lambda_{ce} := \sqrt{\frac{P_y}{P_{cre}}} \qquad \lambda_{ce} = 1.635$$

$$P_{cru} := \left| \begin{array}{ll} 0.658^{\lambda_{ce}^2} & \text{if } \lambda_{ce} \le 1.5 \\ \dfrac{0.877}{\lambda_{ce}^2} & \text{otherwise} \end{array} \right. \qquad P_{cru} = 0.328 \qquad P_n := P_y \cdot P_{cru}$$

$P_n = 322.202 \cdot kip$

Flexural buckling moment:
Appendix F

note: Shapes may be "rolled" or "welded"; value of residual stress will be different for each shape.

shape := "rolled"

$$F_r := \left| \begin{array}{ll} 10 \cdot ksi & \text{if shape = "rolled"} \\ (16.5 \cdot ksi) & \text{otherwise} \end{array} \right. \qquad F_r = 10 \cdot ksi$$

$$F_L := F_y - F_r \qquad F_L = 40 \cdot ksi$$

Flexural buckling moment when top flange is in compression:

$$S_{xct} := \frac{I_x}{y} \qquad S_{xct} = 126.348 \cdot in^3$$

$$S_{xtt} := \frac{I_x}{d - y} \qquad S_{xtt} = 49.577 \cdot in^3$$

$$M_{rt} := \min\left(\begin{bmatrix} S_{xct} \cdot F_L \\ S_{xtt} \cdot F_y \end{bmatrix} \right) \qquad M_{rt} = 2.479 \cdot 10^3 \cdot kip \cdot in$$

$M_p = 4.505 \cdot 10^3 \cdot kip \cdot in$

$$I_{yct} := \frac{b_{fl}^3 \cdot t_{fl}}{12} \qquad I_{yct} = 112.344 \cdot in^4 \qquad I_y = 112.65 \cdot in^4$$

$$r_{yct} := \sqrt{\frac{I_{yct}}{A_{f1}}} \qquad r_{yct} = 3.45 \cdot in \qquad r_y := \sqrt{\frac{I_y}{A}}$$

$$L_{pt} := \frac{300 \cdot r_{yct}}{\sqrt{\frac{F_y}{ksi}}} \qquad L_{pt} = 146.357 \cdot in \qquad r_y = 2.395 \cdot in$$

$$B_{1t} := 2.25 \cdot \left(2 \cdot \frac{I_{yct}}{I_y} - 1\right) \cdot \frac{h}{L} \cdot \sqrt{\frac{I_y}{J}}$$

$$B_{2t} := 25 \cdot \left(1 - \frac{I_{yct}}{I_y}\right) \cdot \frac{I_{yct}}{J} \cdot \left(\frac{h}{L}\right)^2$$

initial guess for L_r to start iteration going: $L_{rrt} := L_{pt}$

$$L_{rt} := \mathrm{root}\left[\frac{\pi \cdot \sqrt{E \cdot G \cdot I_y \cdot J}}{L_{rrt}} \cdot \left[\left[2.25 \cdot \left(2 \cdot \frac{I_{yct}}{I_y} - 1\right) \cdot \frac{h}{L_{rrt}} \cdot \sqrt{\frac{I_y}{J}}\right] \ldots + \sqrt{1 + \left[2.25 \cdot \left(2 \cdot \frac{I_{yct}}{I_y} - 1\right) \cdot \frac{h}{L_{rrt}} \cdot \sqrt{\frac{I_y}{J}}\right]^2 \ldots + 25 \cdot \left(1 - \frac{I_{yct}}{I_y}\right) \cdot \frac{I_{yct}}{J} \cdot \left(\frac{h}{L_{rrt}}\right)^2}\right] - M_{rt}, L_{rrt}\right]$$

$L_{rt} = 615.682 \cdot in$

$L = 240 \cdot in$

$$M_{cr1t} := \frac{\pi \cdot \sqrt{E \cdot G \cdot I_y \cdot J}}{L} \cdot \left(B_{1t} + \sqrt{1 + B_{2t} + B_{1t}^{\,2}}\right)$$

$$M_{crt} := \min\left(\begin{bmatrix} M_{cr1t} \\ M_p \end{bmatrix}\right) \qquad M_{crt} = 4.505 \cdot 10^3 \cdot kip \cdot in \qquad M_p = 4.505 \cdot 10^3 \cdot kip \cdot in$$

$$M_{nt} := \begin{cases} M_p & \text{if } L \le L_{pt} \\ \left[M_p - (M_p - M_{rt}) \cdot \frac{L - L_{pt}}{L_{rt} - L_{pt}}\right] & \text{if } L_{pt} < L \le L_{rt} \\ M_{crt} & \text{otherwise} \end{cases} \qquad M_{nt} = 4.1 \cdot 10^3 \cdot in \cdot kip$$

I = sum of interaction equation; must be equal to or less than unity

$$I_{top} := \begin{cases} \left(\frac{P_u}{\phi_c \cdot P_n} + \frac{8}{9} \cdot \frac{M_u}{\phi_b \cdot M_{nt}}\right) & \text{if } \frac{P_u}{\phi_c \cdot P_n} > 0.2 \\ \frac{1}{2} \cdot \frac{P_u}{\phi_c \cdot P_n} + \frac{M_u}{\phi_b \cdot M_{nt}} & \text{otherwise} \end{cases}$$

$I_{top} = 1.942$

Flexural buckling moment when bottom flange is in compression:

$$S_{xcb} := \frac{I_x}{d - y} \qquad S_{xcb} = 49.577 \cdot in^3$$

$$S_{xtb} := \frac{I_x}{y} \qquad S_{xtb} = 126.348 \cdot in^3$$

$$M_{rb} := \min\left(\begin{bmatrix} S_{xcb} \cdot F_L \\ S_{xtb} \cdot F_y \end{bmatrix}\right) \qquad M_{rb} = 1.983 \cdot 10^3 \cdot kip \cdot in$$

$M_p = 4.505 \cdot 10^3 \cdot kip \cdot in$

$$I_{ycb} := \frac{b_{f2}^{\,3} \cdot t_{f2}}{12} \qquad I_{ycb} = 0.014 \cdot in^4 \qquad I_y = 112.65 \cdot in^4$$

$$r_{ycb} := \sqrt{\frac{I_{ycb}}{A_{f2}}} \qquad r_{ycb} = 0.173 \cdot in \qquad r_y := \sqrt{\frac{I_y}{A}}$$

$$L_{pb} := \frac{300 \cdot r_{ycb}}{\sqrt{\frac{F_y}{ksi}}} \qquad L_{pb} = 7.348 \cdot in$$

$$B_{1b} := 2.25 \cdot \left(2 \cdot \frac{I_{ycb}}{I_y} - 1\right) \cdot \frac{h}{L} \cdot \sqrt{\frac{I_y}{J}} \qquad B_{2b} := 25 \cdot \left(1 - \frac{I_{ycb}}{I_y}\right) \cdot \frac{I_{ycb}}{J} \cdot \left(\frac{h}{L}\right)^2$$

initial guess for L_r to start iteration going: $L_{rr} := L_{pb}$

$$L_{rb} := \mathrm{root}\left[\frac{\pi \cdot \sqrt{E \cdot G \cdot I_y \cdot J}}{L_{rr}} \cdot \left[\left[2.25 \cdot \left(2 \cdot \frac{I_{ycb}}{I_y} - 1\right) \cdot \frac{h}{L_{rr}} \cdot \sqrt{\frac{I_y}{J}}\right] \ldots + \sqrt{1 + \left[2.25 \cdot \left(2 \cdot \frac{I_{ycb}}{I_y} - 1\right) \cdot \frac{h}{L_{rr}} \cdot \sqrt{\frac{I_y}{J}}\right]^2 \ldots + 25 \cdot \left(1 - \frac{I_{ycb}}{I_y}\right) \cdot \frac{I_{ycb}}{J} \cdot \left(\frac{h}{L_{rr}}\right)^2}\right] - M_{rb}, L_{rr}\right]$$

$L_{rb} = 258.588 \cdot in \qquad L = 240 \cdot in$

$$M_{cr1b} := \frac{\pi \cdot \sqrt{E \cdot G \cdot I_y \cdot J}}{L} \cdot \left(B_{1b} + \sqrt{1 + B_{2b} + B_{1b}^{\,2}}\right)$$

$$M_{crb} := \min\left(\begin{bmatrix} M_{cr1b} \\ M_p \end{bmatrix}\right) \qquad M_{crb} = 2.036 \cdot 10^3 \cdot kip \cdot in \qquad M_p = 4.505 \cdot 10^3 \cdot kip \cdot in$$

$$M_{nb} := \begin{cases} M_p & \text{if } L \le L_{pb} \\ \left[M_p - (M_p - M_{rb}) \cdot \frac{L - L_{pb}}{L_{rb} - L_{pb}}\right] & \text{if } L_{pb} < L \le L_{rb} \\ M_{crb} & \text{otherwise} \end{cases} \qquad M_{nb} = 2.17 \cdot 10^3 \cdot in \cdot kip$$

$$I_{bottom} := \begin{cases} \left(\frac{P_u}{\phi_c \cdot P_n} + \frac{8}{9} \cdot \frac{M_u}{\phi_b \cdot M_{nb}}\right) & \text{if } \frac{P_u}{\phi_c \cdot P_n} > 0.2 \\ \frac{1}{2} \cdot \frac{P_u}{\phi_c \cdot P_n} + \frac{M_u}{\phi_b \cdot M_{nb}} & \text{otherwise} \end{cases}$$

$I_{bottom} = 2.371$

Calculation when P is in tension:

Moment causes compression in top flange:

Note: interaction sum = J

$$J_{top} := \begin{cases} \left(\frac{P_u}{\phi_b \cdot P_y} + \frac{8}{9} \cdot \frac{M_u}{\phi_b \cdot M_{nt}}\right) & \text{if } \frac{P_u}{\phi_b \cdot P_y} > 0.2 \\ \frac{1}{2} \cdot \frac{P_u}{\phi_b \cdot P_y} + \frac{M_u}{\phi_b \cdot M_{nt}} & \text{otherwise} \end{cases}$$

$J_{top} = 0.934$

Moment causes compression in bottom flange:

$$J_{bottom} := \begin{cases} \left(\frac{P_u}{\phi_b \cdot P_y} + \frac{8}{9} \cdot \frac{M_u}{\phi_b \cdot M_{nb}}\right) & \text{if } \frac{P_u}{\phi_b \cdot P_y} > 0.2 \\ \frac{1}{2} \cdot \frac{P_u}{\phi_b \cdot P_y} + \frac{M_u}{\phi_b \cdot M_{nb}} & \text{otherwise} \end{cases}$$

$J_{bottom} = 1.363$