The Behavior and Load-Carrying Capacity of Unstiffened Seated-Beam Connections

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ABSTRACT

Unstiffened seated-beam connections are often used to connect a beam to a column. For many years this connection was designed by using tabular methods in the AISC Design Manuals, including the most recent LRFD Manual. The manual tables are generated based on the *required* bearing length method developed in 1940s. This paper investigates the validity of this method by examining the formulations of the model and analyzing the connection behavior. The interactions between connection components are discussed and more rational and accurate models are developed for connections consisting of a flexible angle and a stiff beam. Through comparison of various results, the current LRFD procedures are assessed and practical implications of this research are summarized.

INTRODUCTION

An often used simple connection between a beam and column web is the unstiffened seated-beam connection, where the end reaction is supported by an unstiffened angle bolted or welded to the column as shown in Figure 1. The angle connecting the upper portion of the beam to the column, optionally located either on the top flange or the upper part of the web of the beam, is only for stability considerations. The entire load is assumed to be transmitted through the bottom or "seat" angle.

In both the 4th and 9th edition of the AISC Manual,[1-2] the procedure for the design of a seated connection is the so-called *required* bearing length method developed in 1940s. In this procedure, the location of the reaction is assumed to be at the center of the effective bearing length N required for beam web local yielding (Figure 2). The bending moment on the critical section of the angle, assumed at the base of the fillet on the outstanding leg (Figure 2), is determined by taking the beam reaction times the distance to the critical section. The thickness of the seat angle is then determined by limiting the flexural stress on this critical section. This procedure is also incorporated in the latest AISC LRFD Manual.[3]

Seated connection tests conducted 50 years ago[4-6] were probably the experimental foundation for the AISC Manual design procedures. Most of the experiments on seated connections reported after 1980s[7-11] studied the moment-rotation characteristics of the seated connections as semi-rigid connections other than their load-carrying capacity as simple connections. Recently reported experiments by Ellifritt et al.[42-44] addressed stiffened seated connections only, where the strength of column web or the welding between seat and column often control the behavior of connections.

Although extensive theoretical analysis on the behavior of seated-beam connection can be found in the literature, almost all of them focused on the influence of connection flexibility on the performance of columns or frames as a whole,[12-23] or on modeling connection moment-rotation relations,[24-27] or both topics.[28-34] Prior analytical research efforts on the load-carrying capacity of seated connections were very limited,[35-37] probably because this type of connections has had a very good performance record for many years. While the current AISC/LRFD procedures, translated from the previous ASD procedures,[38] have the merit of simple formulations and appear to be reasonably conservative, experimental research by Roeder and Dailey[39] raised a number of questions regarding the design procedure because it was found that the safety of seated connections was not provided by factors suggested in the design calculation.

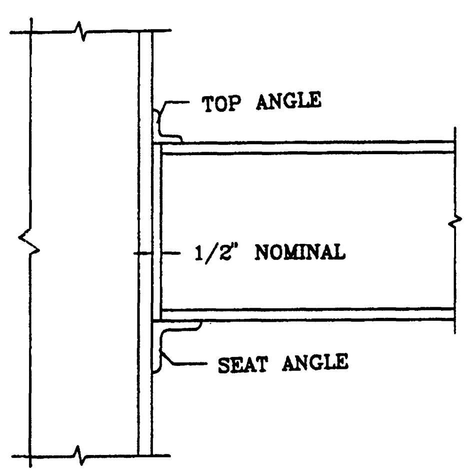

Fig. 1. Unstiffened seated-beam connection.

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This paper studies the behavior and load-carrying capacity of unstiffened seated beam connections consisting of a flexible angle and a strong beam, where the strength of the seat angle itself will control the structural behavior of the connection (the bolts' related strength are assumed to be adequate). Connections involving a thick angle and a weak beam will not be the main topic of the present study, because the limit states of beam web crippling or local yielding would be more critical in that case.

The first part of this paper briefly reviews the AISC LRFD procedures. Focusing on the effects of beam-bottom-flange to seat-angle attaching bolts, the second part studies the behavior of the connections by analyzing the interactions between the seat-angle and the supported-beam, from which simplified models for the calculation of the load-carrying capacity of the seated connections will be justified. Then, the load-carrying capacity of the seated connections, for both no-bolt and bolted cases, are determined from a failure mechanism analysis that uses a plastic hinge method.[40] Finally, through the comparison of various methods, the validity of the LRFD procedures are evaluated and assessed and the practical implications of this research emphasized.

REVIEW OF THE AISC/LRFD PROCEDURES

The design loads for the AISC/LRFD Manual[3] tables are based on two possible limit states: (1) excessive bending stress on the seat angle, and (2) local web yielding of the supported beam. When computing the load-carrying capacity of the connection, both limit states must be considered and the governing condition is the one which provides the most conservative results.

The basic assumptions used in the procedures are that: (1) the critical section is located at the toe of the fillet of the outstanding leg; (2) the reaction occurs at the center of the effective bearing length N required for the local web yielding limit state; (3) the angles fillet radius is ⅜-in. and the beam setback is ¾-in., which is ¼-in. more than the nominal setback to allow for possible mill underrun in beam length.

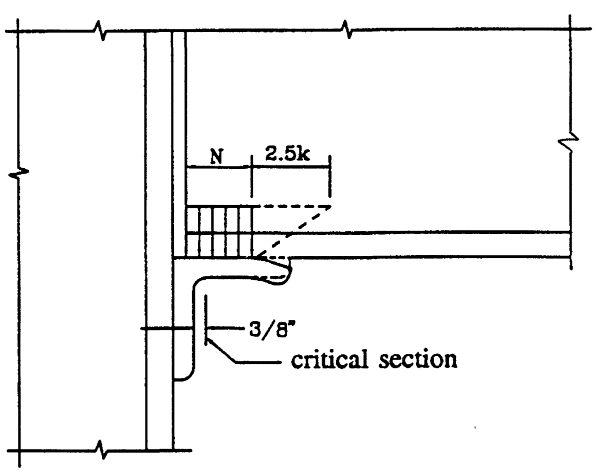


Fig. 2. AISC assumption for bearing stress.

The plastic moment of the angles outstanding leg is:

$$M_p = \frac{F_{y\text{-}angle} L t_a^2}{4} \tag{1}$$

The flexural design strength, based on a nominal resistance factor of 0.9, is:

$$\phi_b M_n = 0.9 M_p = 0.225 F_{y\text{-}angle} L t_a^2 \tag{2}$$

The design strength can be expressed in terms of the reaction:

$$\phi_b M_n = \phi_b (eR) \tag{3}$$

Thus, the equation for the limit state of flexure is:

$$\phi_b R = \frac{0.225 F_{y\text{-}angle} L t_a^2}{e} \tag{4}$$

The equation for the limit state of local web yielding is:

$$F_{y\text{-}beam} \geq \frac{\phi_b R}{t_w (N + 2.5k)} \tag{5}$$

where

$\phi_b R$ = magnitude of the reaction corresponding to the design flexural strength of the angle
e = distance from the angle fillet to the location of reaction
t_a = angle leg thickness
L = angle length
t_w = beam web thickness
$F_{y\text{-}angle}$ = yield stress of angle
$F_{y\text{-}beam}$ = yield stress of beam
k = distance from outer face of flange to the web toe of fillet
N = beam bearing length

Three cases are considered, depending on the relative values of N, k and the angle leg dimension.

Case I—Basic Case ($2.5k \leq N \leq 3.25$ inches)

The following equation relating the eccentricity of R and the bearing length is obtained by examining the geometry in Figure 3(a):

$$\frac{N}{2} + \frac{3}{4} = t_a + \frac{3}{8} + e \tag{6}$$

There are three equations (4, 5 and 6) and three unknowns ($\phi_b R$, N and e), which can be solved to provide the following expression for $\phi_b R$ and N by assuming the flexural strength and web yielding limit states are achieved simultaneously:

$$\frac{(\phi_b R)^2}{2t_w F_{y\text{-}beam}} + \phi_b R\left(\frac{3}{8} - t_a - \frac{5}{4}k\right) - 0.225 F_{y\text{-}angle} L t_a^2 = 0 \qquad (7)$$

$$N = \frac{\phi_b R}{t_w F_{y\text{-}beam}} - 2.5k \qquad (8)$$

Case II—N is less than $2.5k$

If the value of N resulting from the solution of Equation 8 is less than $2.5k$, the reaction is assumed to be located at one-fourth of the distance $(N + 2.5k)$ as shown in Figure 3(b). The geometrical equation is rewritten as:

$$\frac{N + 2.5k}{4} + \frac{3}{4} = t_a + \frac{3}{8} + e \qquad (9)$$

Resolving Equations 4, 5 and 9 gives the following quadratic in $\phi_b R$:

$$\frac{(\phi_b R)^2}{4t_w F_{y\text{-}beam}} + \phi_b R\left(\frac{3}{8} - t_a\right) - 0.225 F_{y\text{-}angle} L t_a^2 = 0 \qquad (10)$$

Case III—N is greater than 3.25 inches

If the value of N resulting from the solution of Equation 8 is greater than 3.25 inches, which is the maximum possible bearing length for a 4-in. angle, N is assumed to be 3.25 inches, as shown in Figure 3(c). In this case, the governing limit state is local web yielding and the resulting maximum load capacity is:

$$\phi_b R = (3.25 + 2.5k) t_w F_{y\text{-}beam} \qquad (11)$$

In summary, the LRFD procedures for computing the maximum load-carrying capacity of an unstiffened seat angle are as follows:

1. Assume Case I applies, and use Equation 7 and Equation 8 to compute, respectively, the values of $\phi_b R$ and N;
2. If N is greater than $2.5k$ and less than 3.25 inches, the value for $\phi_b R$ is valid and the solution process stops;
3. If N is less than $2.5k$, Case II applies and Equation 10 should be used to calculate a new value for $\phi_b R$; and
4. If N from Step (1) is greater than 3.25 inches, Case III applies and Equation 11 should be used to calculate a new value for $\phi_b R$.

This algorithm was used to generate Table 9-6 found on page 9-136 of the AISC LRFD Manual of Steel Construction.[3] In generating the tables, the following approximations for k are used for simplicity:

$$k = 2.5t_w \text{ for } t_w < 5/16 \qquad (12a)$$

$$k = 2.75t_w \text{ for } t_w \geq 5/16 \qquad (12b)$$

Example 1

Using the LRFD procedures to compute the design strength of an unstiffened seated connection.

Given

$t_w = 9/16$-in., $t_a = 1/2$-in., $L = 8$-in.

Assume

Beam set back = 3/4-in., angle outstanding leg length = 4 in.

Use ASTM A36 steel for both the seat angle and the supported beam.

Solution

From Equation 12b:

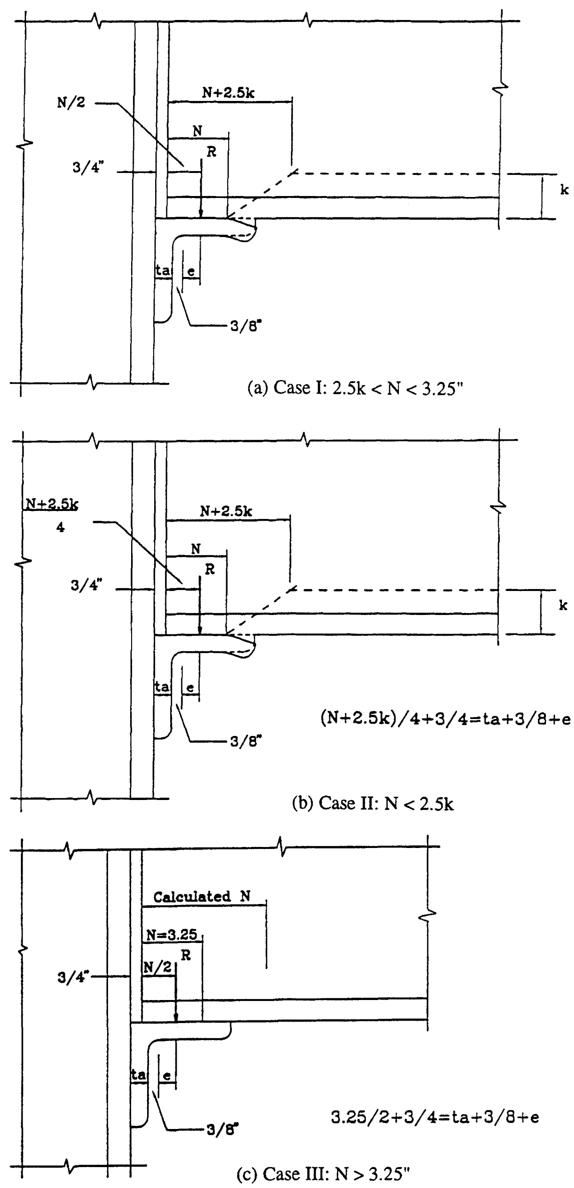


Fig. 3. LRFD procedures.

$$k = 2.75t_w = (2.75)\left(\frac{9}{16}\right) = 1.547 \text{ in.}$$

Substitute t_w = 9⁄16-in., t_a = ½-in., L = 8 in., k = 1.547 in. and $F_{y\text{-}beam} = F_{y\text{-}angle}$ into Equation 7:

$$\frac{(\phi_b R)^2}{40.5} - 2.06\phi_b R - 16.2 = 0$$

Solving the quadratic equation gives:

$$\phi_b R = 90.6 \text{ kips}$$

Using Equation 8:

$$N = \frac{\phi_b R}{t_w F_{y\text{-}beam}} - 2.5k = \frac{90.6}{(9/16)(36)} - (2.5)(1.547)$$

$$N = 0.608 \text{ in.} < 2.5k = (2.5)(1.547) = 3.867 \text{ in.}$$

Since N is less than $2.5k$, then Case II applies.

Substitute t_w = 9⁄16-in., t_a = ½-in., L = 8 in., and $F_{y\text{-}beam} = F_{y\text{-}angle}$ into Equation 10:

$$\frac{(\phi_b R)^2}{81} - 0.125\phi_b R - 16.2 = 0$$

Solving the quadratic equation yields the following:

$$\phi_b R = 41.6 \text{ kips}$$

INCONSISTENCIES IN THE LRFD PROCEDURES

The above LRFD procedures are essentially derived from prior ASD procedures[1,2] developed as early as the 1940s. The assumed distribution of forces within the connection appear to be highly idealized and somewhat unrealistic. Some inconsistencies in the formulations of the procedures are described in the forthcoming sections.

Unrealistic Values of Effective Bearing Length *N*

For a particular seat angle size, the load-carrying capacity of the connection should not exceed some limit value regardless of the supported beam size. However, the resultant design strength from the LRFD procedures could increase without bound, for a seat angle of a given thickness, should the beam web thickness keep increasing. At this point, the LRFD procedures are inconsistent when applied to connections consisting of a flexible-angle and thick-web-beam.

If the beam web were made thicker, the effective bearing length N required based on the local web yielding limit state would decrease. If the beam-web thickness t_w is large enough, the required bearing length N may become zero, or even a negative value. In these cases, the limit state of excessive bending stress on the seat angle should be the sole control condition, and the equation for the limit state of local web yield,

$$F_{y\text{-}beam} \geq \frac{\phi_b R}{t_w(N + 2.5k)} \qquad (5)$$

should be of no significance. However, if this equation is still enforced, as in the LRFD procedures, the resultant value of N will become unrealistic, i.e. a very small or even negative value.

Considering the geometric relations:

$$\frac{N}{2} + \frac{3}{4} = t_a + \frac{3}{8} + e \text{ for Case I} \qquad (6)$$

or

$$\frac{N + 2.5k}{4} + \frac{3}{4} = t_a + \frac{3}{8} + e \text{ for Case II} \qquad (9)$$

The unrealistic bearing length N would lead to a very small value of eccentricity e, which would in turn produce a very high design strength $\phi_b R$ when substituted into the equation for the limit state of flexure:

$$\phi_b R = \frac{0.225 F_{y\text{-}angle} L t_a^2}{e} \qquad (4)$$

The inconsistency is an unsafe factor and will be illustrated more clearly in the following example.

Example 2

Rework Example 1 to illustrate the inconsistency of an unrealistic effective bearing length N in the LRFD procedures.

Solution

Substitute $\phi_b R$ = 41.6 kips, $F_{y\text{-}beam}$ = 36 ksi, t_w = 9⁄16-in. and k = 1.547 in. into Equation 5:

$$36 \geq \frac{41.6}{(9/16)\,[N + (2.5)(1.547)]}$$

Solve for N:

$$N = \frac{41.6}{(9/16)(36)} - (2.5)(1.547) = -1.813 \text{ in.}$$

$N < 0$, **unrealistic!**

Because N is less than zero there is no true bearing between the angle and steel beam. Regardless this fact, if we substitute N = −1.813 in., k = 1.547 in., t_a = ½-in. into Equation 9 and solve for e:

$$\frac{-1.813 + (2.5)(1.547)}{4} + \frac{3}{4} = \frac{1}{2} + \frac{3}{8} + e$$

e = 0.3886 in., **very small!**

Underestimating the eccentricity would lead to a very large design strength, therefore the LRFD procedures are unsafe in this aspect. (It should be pointed out that the overall LRFD procedures usually produce satisfactory and safe designs since, as will be noted later, the procedures are overly conservative in other aspects.) An examination of the total 140 cases listed in AISC Table 9-6 on unstiffened seated connections reveals that more than one third (38 percent) have unrealistic values of effective bearing length N, i.e. half (50 percent) of those cases controlled by Case II ($N < 2.5k$) have negative or zero values of beam bearing length N.

Shear Force Effect on the Yielded Material

Since the values of eccentricity e (the distance from the toe of angle fillet to the location of reaction) are generally of the same order of magnitude when compared with the angle thickness, the shear force might have a significant reduction effect on the plastic moment capacity of the critical section. Ignoring the shear force effect, as in the LRFD procedures, may lead to an overestimation of the load-carrying capacity of the connection when the seat angle is thick. Drucker's yield criterion[41] for the combined plastic bending moment M_{ps} and shear force R_s at the critical section has the form:

$$\frac{M_{ps}}{M_0} + \left(\frac{R_s}{R_{os}}\right)^4 \le 1 \tag{13}$$

where

M_0 and R_{os}= pure plastic bending moment capacity and shear force capacity, without coupling of seat-angle, respectively.

Using the Tresca's yield criterion, we have:

$$M_0 = \frac{F_{y\text{-}angle} L t_a^2}{4} \tag{14a}$$

$$R_{os} = \frac{L t_a F_{y\text{-}angle}}{2} \tag{14b}$$

Substitute Equations 14a and 14b into Equation 13 and rearranging:

$$M_{ps} = \frac{F_{y\text{-}angle} L t_a^2}{4}\left[1 - \frac{16 R_s^4}{(L t_a F_{y\text{-}angle})^4}\right] \tag{1a}$$

This is the plastic moment capacity of the critical section considering the reduction effect due to shear force. Using Equation 1a instead of Equation 1, Equation 4 will have the following modified form:

$$\phi_b R_s = \frac{0.225 F_{y\text{-}angle} L t_a^2}{e}\left[1 - \frac{16(\phi_b R_s)^4}{(L t_a F_{y\text{-}angle})^4}\right] \tag{4a}$$

where

R_s= load-carrying capacity of seat angle including the effect of shear force.

Also, Equations 7 and 10 will be modified to the following forms, respectively,

$$\frac{3.6 F_{y\text{-}angle} L t_a^2}{(L t_a F_{y\text{-}angle})^4}(\phi_b R_s)^4 + \frac{(\phi_b R)^2}{2 F_{y\text{-}beam} t_w} + \left(\frac{3}{8} - t_a - 1.25k\right)(\phi_b R) - 0.225 F_{y\text{-}angle} L t_a^2 = 0 \tag{7a}$$

$$\frac{3.6 F_{y\text{-}angle} L t_a^2}{(L t_a F_{y\text{-}angle})^4}(\phi_b R_s)^4 + \frac{(\phi_b R)^2}{4 F_{y\text{-}beam} t_w} + (0.375 - t_a)(\phi_b R) - 0.225 F_{y\text{-}angle} L t_a^2 = 0 \tag{10a}$$

The value of $\phi_b R_s$ can be determined by an iteration procedure.

Example 3

Using the LRFD procedures with the modified equations (7a and 10a) to compute the design strength of an unstiffened seated connection made with a thick seat angle, and assessing the maximum possible reduction effect of shear force on the load-carrying capacity of seat angle.

Given

t_w = 9⁄16-in., t_a = 1.0 in., L = 6.0 in.

Assume

Beam set back = ¾-in., angle outstanding leg length = 4 in.

Use A36 steel for both seat angle and supported beam.

Solution:

Assume $2.5k \le N \le 3.25$ in., Case I—Basic Case

Substitute t_w = 9⁄16-in., t_a = 1.0 in., L = 6.0 in., k = 1.547 in. and $F_{y\text{-}beam} = F_{y\text{-}angle}$ = 36 ksi into Equation 7a;

$$\frac{(\phi_b R_s)^4}{2{,}799{,}360} + \frac{(\phi_b R)^2}{40.5} - 2.559(\phi_b R) - 48.6 = 0$$

$\phi_b R_s$ = 105.4 kips

Using Equation 8:

$$N = \frac{105.4}{(9/16)(36)} - (2.5)(1.547)$$

N = 1.337 in. < $2.5k$ = 3.867 in.

Therefore, Case II applies.

Substitute t_w = 9⁄16-in., t_a = 1 in., L = 6 in., k = 1.547 in. and $F_{y\text{-}beam} = F_{y\text{-}angle}$ = 36 ksi into Equation 10a:

$$\frac{(\phi_b R_s)^4}{2{,}799{,}360} + \frac{(\phi_b R_s)^2}{81} - 0.625(\phi R_s) - 48.6 = 0$$

$\phi_b R_s$ = 82.3 kips

From LRFD Table 9-6:

$\phi_b R = 93$ kips

This example indicates that the overestimation of the load-carrying capacity is 13 percent by neglecting the effect of shear force in the LRFD procedure.

THE BEHAVIOR OF SEATED BEAM CONNECTIONS

When designing a flexible angle, it is important to understand how the angle is being loaded, and how it reacts to this load. Figure 4(a) shows a simply supported beam placed on a seat angle. Due to loading on the beam, the beam deflects and its end rotates (Figure 4(b)). Consequently, the point of contact of the reaction R tends to move outward. This increase in moment arm increases the bending moment on the seat, causing the leg of the angle to deflect downward. As the deflected leg takes the same slope as the loaded beam, the point of contact moves back, Figure 4(c). Two key issues involved in determining the load-carrying capacity of seated connections are the location of the critical sections (type of failure modes) and the point of reaction R (bearing stress distribution).

The Location of the Critical Sections

If a seated connection is used without attachment to the beam, Figure 5(a), it seems plausible to take the critical section as the net section through the upper bolt line,[37] i.e. b-b section in Figure 5(a), since this section has the smallest net area and largest eccentricity. However, if the second-order effect is considered, it can be shown that the section at the toe of the fillet on the leg which is bolted to the column, i.e. a-a section in Figure 5(a), is more critical than the section through the upper bolt line.

Figure 5(b) shows the resultant reaction R acting on a deformed angle. Since the reaction force R has to be perpendicular to the deformed surface of seat angle, it creates a horizontal component $R\sin\theta$ and a vertical component $R\cos\theta$. From the figure, it can be seen that:

$$M_{a\text{-}a} = e_1 R\cos\theta - kR\sin\theta \qquad (15a)$$

$$M_{b\text{-}b} = e_2 R\cos\theta - lR\sin\theta \qquad (15b)$$

where

$M_{a\text{-}a}$ and $M_{b\text{-}b}$ = bending moment values acting on the seat angle at Sections a-a and b-b, respectively.

By subtracting Equation 15b from Equation 15a and rearranging:

$$M_{a\text{-}a} - M_{b\text{-}b} = R[(e_1 - e_2)\cos\theta + (l - k)\sin\theta] \qquad (16)$$

Considering the fact that $e_1 \approx e_2$ and $l >> k$, Equation 16 can be simplified as:

$$M_{a\text{-}a} - M_{b\text{-}b} \approx R(l - k)\sin\theta >> 0 \qquad (16a)$$

That is, the bending moment on Section a-a will be much higher than that on Section b-b as the rotation angle θ and reaction R increase. Therefore, the critical section should be taken at Section a-a. The corresponding failure mode is shown in Figure 5(c).

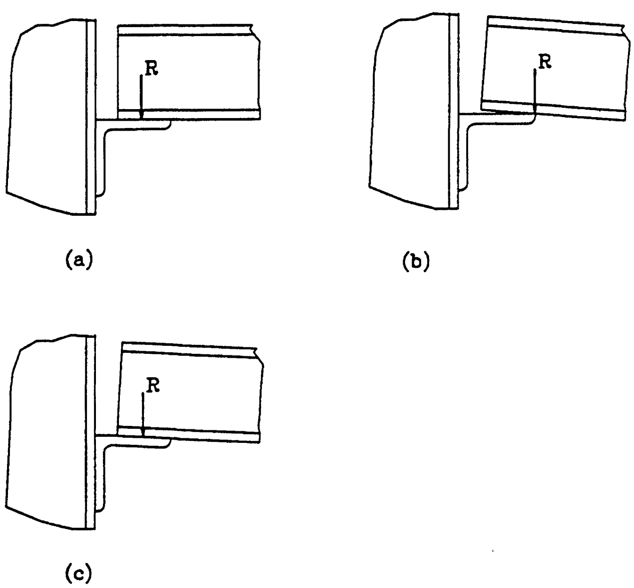


Fig. 4. Reaction location for beam on seat angle.

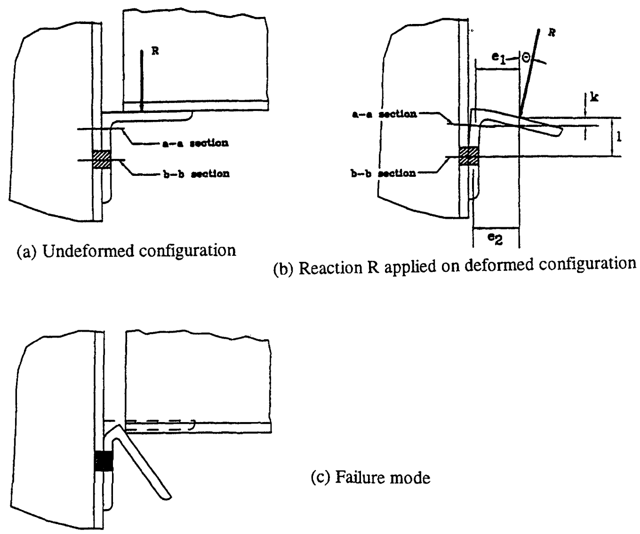


Fig. 5. Location of critical section for no-bolt case.

The beam should typically be attached to the seat as shown in Figure 6(a). Theoretically, the rotation of the beam at the end creates a horizontal force that tends to restrain the angle pull away from the column. The critical section for flexure is therefore taken at or near the base of the fillet on the outstanding leg, i.e. Section b-b in Figure 6(a). As a matter of fact, this is one of the basic assumptions used in the AISC LRFD procedures as reviewed in the preceding section. However, this assumption may not be valid when the entire length of the supported beam (not only one of the beam ends) is considered. As illustrated in Figure 7, the larger the beam end rotation (α), the shorter the net distance between the beam ends (because the beam would be deflected and curved) and the larger the gap (Δl) between the beam end and column face. Depending on the span and stiffness of the supported beam, the beam end rotation may not necessarily generate a force that always restrains the angle pull away from the column.

In order to estimate the maximum capacity of the seat angle, the beam was assumed to be very stiff and the effect of beam end rotation was neglected. As a result, the correct failure mode would be the one shown in Figure 6(b), where the beam end is assumed to be a rigid body moving downward.

The Point of Reaction *R*

Consider a simple beam of length L_b supported by cantilever beams of length l_a at both ends as shown in Figure 8. If we idealize the outstanding-seat-angle-legs as the cantilever beams, this simple model can be utilized to analyze the character of the shifting point of reaction R.

The assumptions embedded in the following derivations are: (1) the material is perfectly elastic; (2) the beams are initially straight and prismatic, with plane cross-sections remaining plane after deformation; and (3) the assumed point of contact for R is located where the two deformed surfaces are tangent to each other.

Based on mechanics of material, the slopes of deflected chords of the cantilever and simple beams due to a concentrated load P can be written in the following forms, respectively:

$$y_a' = \begin{cases} \dfrac{Px}{4E_aI_a}(2x_0 - x) & \text{for } 0 \le x \le x_0 \\ \dfrac{Px_0^2}{4E_aI_a} & \text{for } x_0 \le x < l_a \end{cases} \tag{17a}$$

$$y_b' = \begin{cases} \dfrac{P}{16E_bI_b}(L_b - 2x_0)^2 & \text{for } 0 \le x \le x_0 \\ \dfrac{P}{16E_bI_b}[(L_b - 2x_0)^2 - 4(x - x_0)^2] & \text{for } x_0 < x < L_b \end{cases} \tag{17b}$$

where

y_a' and y_b' = slopes of the deflected chords of the cantilever and simple beams, respectively

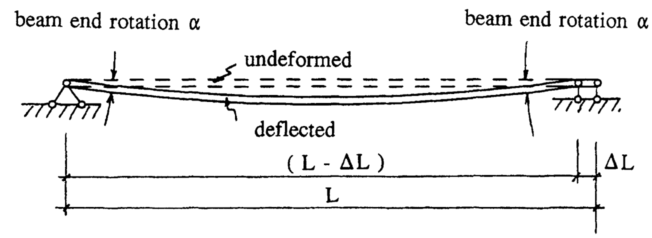
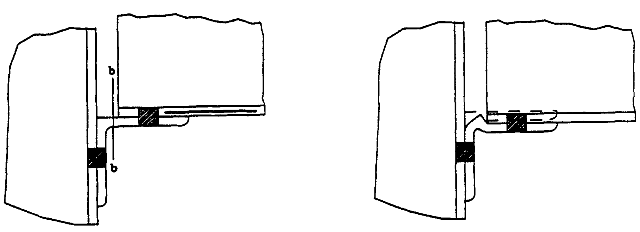


Fig. 7. Correlation between beam end rotation and net span.



(a) Undeformed configuration

(b) Failure mode

Fig. 6. Beam attached to seat angle.

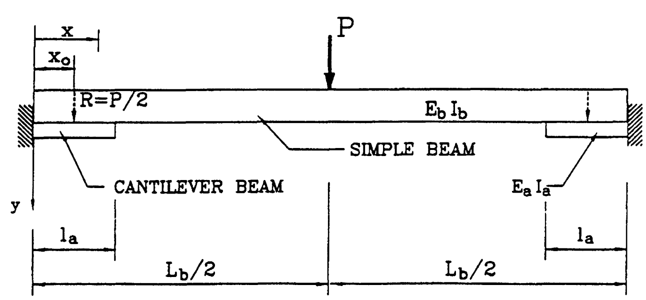


Fig. 8. Idealized model for evaluating shift of reaction point R.

E_a and E_b = elastic moduli of the cantilever and simple beam, respectively

I_a and I_b = moment of inertia (with respect to the bending axis) of the cantilever and simple beams, respectively

x, x_0, l_a, and L_b= dimensions as shown in Figure 8.

From Equation 17a and 17b, it is noted that the conditions for single contact point are satisfied automatically, i.e.

$$y_a' < y_b' \text{ for } x < x_0 \tag{18a}$$

$$y_a' = y_b' \text{ for } x = x_0 \tag{18b}$$

$$y_a' > y_b' \text{ for } x > x_0 \tag{18c}$$

By substituting Equations 17a and 17b into Equation 18b, we can write the following expression and solve for x_0:

$$\frac{Px_0^2}{4E_aI_a} = \frac{P}{16E_bI_b}(L_b - 2x_0)^2$$

$$0 \le x_0 = \frac{L_b/2}{1 + \sqrt{(E_bI_b)/(E_aI_a)}} \le l_a$$

Considering the fact that $I_b >> I_a$ and $E_a = E_b$ for practical seated connections made of steel, the above equation can be simplified as:

$$0 \le x_0 \approx \frac{1}{2}\sqrt{\frac{I_a}{I_b/L_b^2}} \le l_a \text{ for concentrated load} \tag{19a}$$

If the simple beam in Figure 8 is subjected to an evenly distributed load w throughout its span L_b instead of the concentrated load P, then the following expression for x_0 can be similarly derived:

$$0 \le x_0 \approx \sqrt{\frac{I_a}{6I_b/L_b^2}} \le l_a \text{ for distributed load} \tag{19b}$$

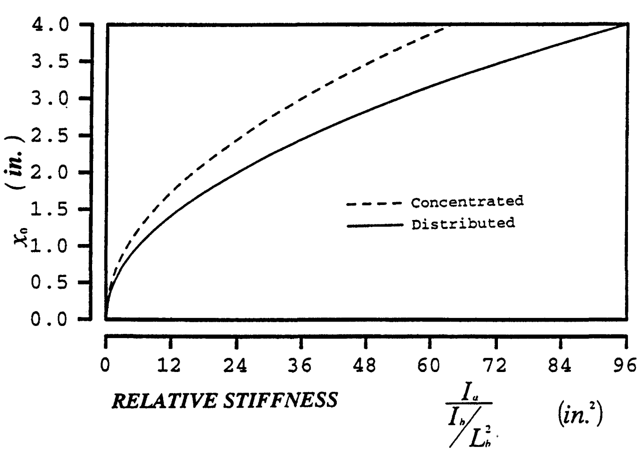


Fig. 9. Relationship between relative stiffness and location of reaction point R.

Equations 19a and 19b are plotted in Figure 9 for comparison. It can be seen that the point of reaction R (or bearing stress distribution pattern) depends on load conditions and relative stiffness of seat-angle-leg and supported-beam. When all other conditions are identical, the x_0 value for concentrated load is 22.5 percent more than for the distributed load case. When the dimensions of the supported beam (i.e., I_b and L_b) are given, then the bearing stress distribution will depend mainly on the thickness of the angle leg. If the leg of the angle is thin, it will deflect easily. Consequently, the point of contact will shift to the end of the beam (i.e. x_0 decreases), and the triangular distribution of Figure 10(a) is justified. If the leg of the angle is stiff, it will deflect less and the point of contact will extend farther out along the leg (i.e. x_0 increases), and thereby the reaction contact moves towards the outer edges as in Figure 10(b).

The Effect Of Beam-Bottom-Flange To Seat-Angle Attaching Bolts

Considering the beam-angle-column subassemblage as shown in Figure 11(a), the overall effect of bolting the beam to the seat angle is to add redundancy to the connection subassemblage. The structural subassemblage, which is statically determinate before bolting, Figure 11(b), will become statically indeterminate when the bolts are installed and properly tightened, Figure 11(c). For the no-bolt case shown in Figure 11(b), a single plastic hinge formed at any location in the angle will lead to a sudden collapse of the beam. The corresponding load-carrying capacity can be determined using simple mechanics. For the bolted case shown in Figure 11(c), the formation of one plastic hinge in the seat angle does

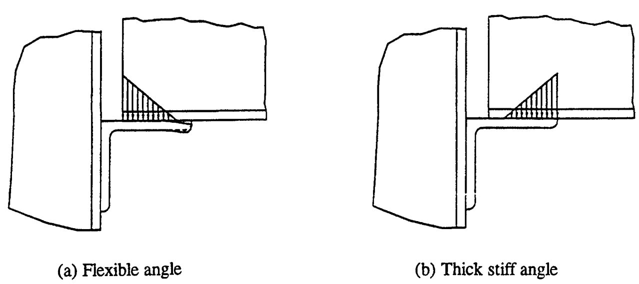


Fig. 10. Bearing stress distribution patterns.

not necessarily result in the failure of the connection. The load-capacity needs to be determined by plastic hinge analysis using assumed failure mechanisms.

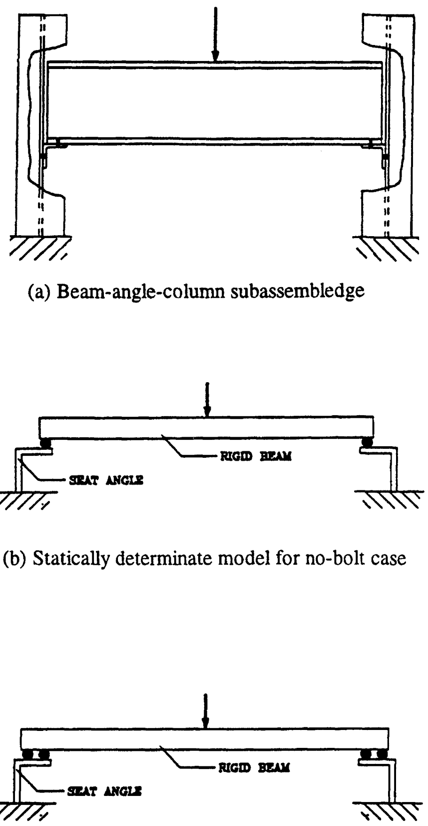
The local effects of the beam-bottom-flange to seat-angle bolts are different depending on the relative stiffness of the seat angle and the supported beam. Figure 12 illustrates the effect of the bolts for a connection consisting of a thick angle and a weak beam. Figure 12(c) indicates that tightening the bolts is equivalent to adding a clockwise moment M_b to the no-bolt case, Figure 12(b). This clockwise moment M_b is unfavorable to the performance of the connection since it increases the bending moment on the seat angle from M_{ub} to $M_{ub} + M_b$. Also, it would increase the bearing reaction concentration at the toe of the outstanding angle leg and might overstress the beam web in bearing. The limit state of beam web crippling or local yield is, therefore, more likely to control in this case. Figure 13 is for connections involving a flexible angle and a strong beam. Tightening the bolts is essentially the same as applying a counter clockwise moment M_b to the no-bolt case, Figure 13(b) and (c). This counter clockwise moment M_b would improve the performance of the connection by reducing the bending moment on the seat angle ($M_{bt} = M_{ub} - M_b$) and increasing the length of contact for proper support of the beam web. Attaching the beam to the seat using properly tightened bolts can significantly increase the load carrying capacity of seated connections consisting of a flexible angle and a strong beam, since the strength of the seat angle itself would control the structural behavior of this kind of connection and the failure mode of seat angle would shift from the one shown in Figure 6(c) to that in Figure 7(b) due to bolting.

THE LOAD-CARRYING CAPACITY OF SEATED-BEAM CONNECTIONS

The preceding section shows that the beam-bottom-flange to seat-angle attachment bolts have a significant effect on the behavior of seated connections. When the bolts are not installed and tightened during construction, the load-carrying capacity of the connection should be calculated using the model shown in Figure 11(b). When the bolts have been installed and tightened during construction, then the model shown in Figure 11(c) should be used to compute the design strength of the connection. The following sections will demonstrate that the load-carrying capacity is dramatically different for the no-bolt case and the bolted cases.

No-Bolt Case

If the beam is not attached to the seat angle, the load-carrying capacity of the connection is directly related to the value of eccentricity, which is in turn a function of the relative stiffness

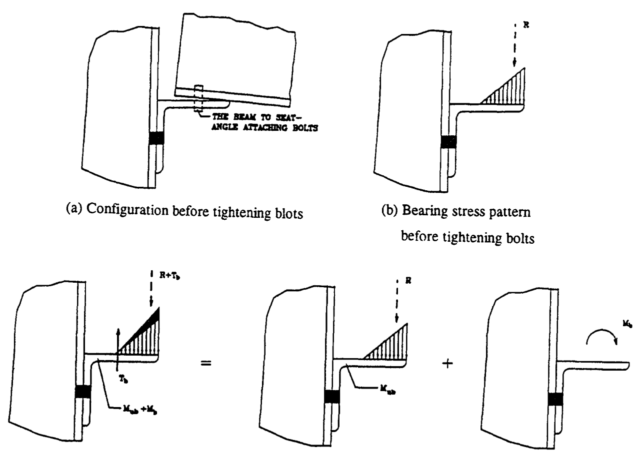


(a) Beam-angle-column subassembledge

(b) Statically determinate model for no-bolt case

(c) Statically indeterminate model for bolted case

Fig. 11. Overall effect of attaching beam to seat angles.



(a) Configuration before tightening blots

(b) Bearing stress pattern before tightening bolts

Fig. 12. Local effect of tightening the bolts (thick angle and flexible beam case).

parameter at the initial elastic stage as discussed earlier. The smaller the parameter

$$\left(\frac{I_a}{I_b/L_b^2}\right)$$

the smaller the eccentricity (see Equation 19 and Figure 9). For connections consisting of a flexible angle and a strong beam, both the triangular and parabolic bearing stress distributions shown in Figure 14 can be considered as rational models under service load conditions. Their eccentricity values can be estimated using the following equations:

$$e = \frac{N}{3} + b_s - \frac{t_a}{2} \text{ for triangular case} \quad (20a)$$

and

$$e = \frac{N}{4} + b_s - \frac{t_a}{2} \text{ for parabolic case} \quad (20b)$$

However, it should be noted that as the load increases, plastification would be developed and Equation 19 and Figure 9 no longer apply. As the outstanding leg of the angle deflects downward due to plastic and elastic deformations, the eccentricity would keep decreasing as the limit load is approached. At the same time, we note that the value of eccentricity cannot be physically less than that of beam setback minus one half of the angle thickness ($b_s - t_a/2$). Therefore, if we assume that $e = b_s - t_a/2$, then the maximum load-carrying capacity can be computed.

Since the eccentricity e has the same order of magnitude when compared with angle thickness, we must therefore consider the effect of axial load on the yielding of material due to bending. The nondimensionalized expression for the reduced plastic moment capacity of a rectangular section can be written as:[40]

$$\frac{M_{pc}}{M_o} = 1 - \left(\frac{R}{R_{0c}}\right)^2 \quad (21)$$

where

M_{pc} = reduced moment capacity due to the presence of the compressive force R that acts on the critical section

M_o and R_{oc} = pure plastic bending moment capacity and axial load capacity, without coupling of seat angle, respectively.

Substitute $M_{pc} = eR$ into Equation 21 and rearrange:

$$\left(\frac{1}{R_{0c}}\right)^2 R^2 + \left(\frac{e}{M_0}\right) R - 1 = 0 \quad (22)$$

where

$$R_{0c} = F_{y\text{-}angle} L t_a \quad (23)$$

$$M_0 = \frac{F_{y\text{-}angle} L t_a^2}{4} \quad (24)$$

Substituting Equations 23 and 24 into Equation 22, and using $e = b_s - t_a/2$, then:

$$\frac{1}{(F_{y\text{-}angle} L t_a)^2} R^2 + \frac{4b_s - 2t_a}{F_{y\text{-}angle} L t_a^2} R - 1 = 0 \quad (22a)$$

where

R = load-carrying capacity of seat angle including the effect of axial force.

Equation 22a can be solved for positive values of R, then the design strength can be obtained by multiplying R by a resis-

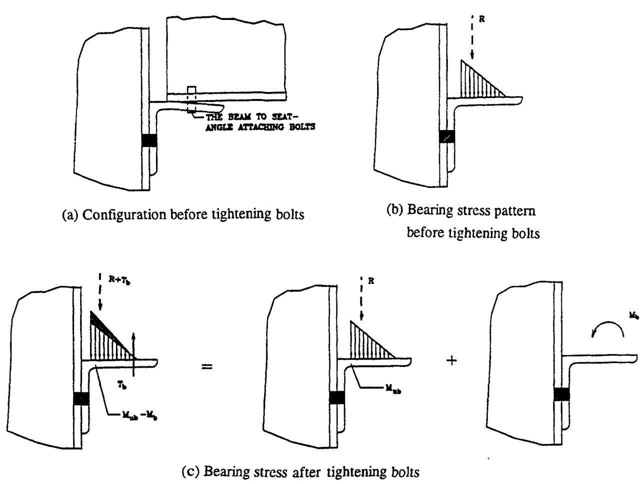


Fig. 13. Local effect of tightening bolts (flexible angle and strong beam case).

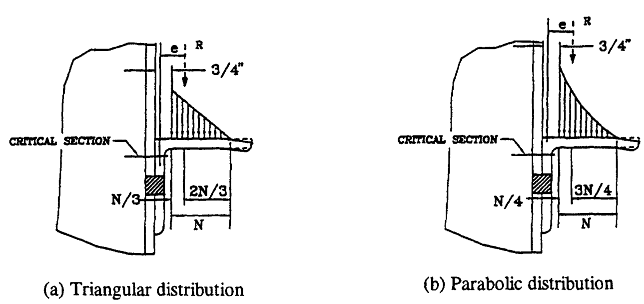


Fig. 14. Bearing stress pattern for flexible angle and stiff beam case.

Table 1.
Summary of Load-Carrying Capacity Calculated Using Various Theoretical Models*

Angle Length (in.)	Angle Thickness (in.)	Rational Models (kips)		LRFD** (kips)	
		No-bolt Case	Bolted Case	$F_y = 36$	$F_y = 50$
6	3/8	11.8	21.7	23.5	27.7
	1/2	22.9	37.0	36.8	44.7
	5/8	38.9	53.1	50.6	62.4
	3/4	60.4	69.4	64.6	80.4
	1	N/A	102.0	93.0	117.0
8	3/8	15.8	28.9	27.2	32.0
	1/2	30.6	49.3	41.6	50.3
	5/8	51.9	70.8	56.5	69.2
	3/4	80.5	92.5	71.6	88.5
	1	N/A	136.0	102.0	128.0

* Assume that beam web is thick enough so that the strength of seat angle controls; $F_{y\text{-}angle} = 36$ ksi;
** Values correspond to $t_W = 9/16$-in.

tance factor of 0.9. The results of selected examples from these procedures are listed in Table 1 for comparison purposes.

Bolt-Tightened Case

The collapse mechanism for a seated connection consisting of a flexible angle and a strong beam is shown in Figure 15 when the beam bottom flange is properly attached to the seat angle. Since the distance between the two plastic hinges is the same order of magnitude when compared with the angle thickness, then the strength reduction effects due to shear force and axial load on the plastic moment capacity of the seat angle should be considered using Equations 13 and 21, respectively.

The work equation for the mechanism shown in Figure 15 is given by:

$$M_{ps}\theta + M_{pc}\theta = R\left(b_s - \frac{t_a}{2}\right)\theta \tag{25}$$

From Equations 13 and 21, respectively, the moment values are given as:

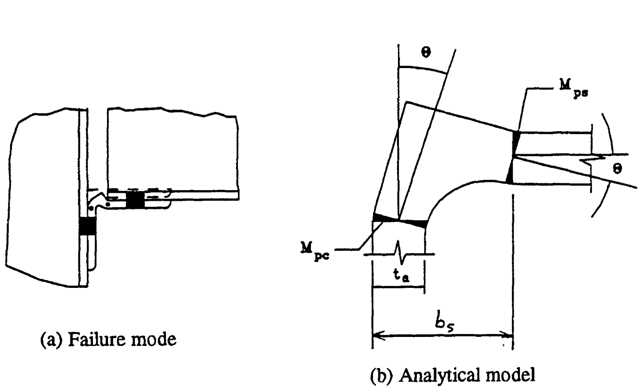
$$M_{ps} = M_0\left[1 - \left(\frac{R}{R_{0s}}\right)^4\right] \tag{26a}$$

$$M_{pc} = M_0\left[1 - \left(\frac{R}{R_{0c}}\right)^2\right] \tag{26b}$$

Substituting M_{ps} and M_{pc} from Equations 26a and 26b, and using Equations 14b, 23 and 24 for R_{os}, R_{oc}, and M_o, respectively, Equation 25 becomes:

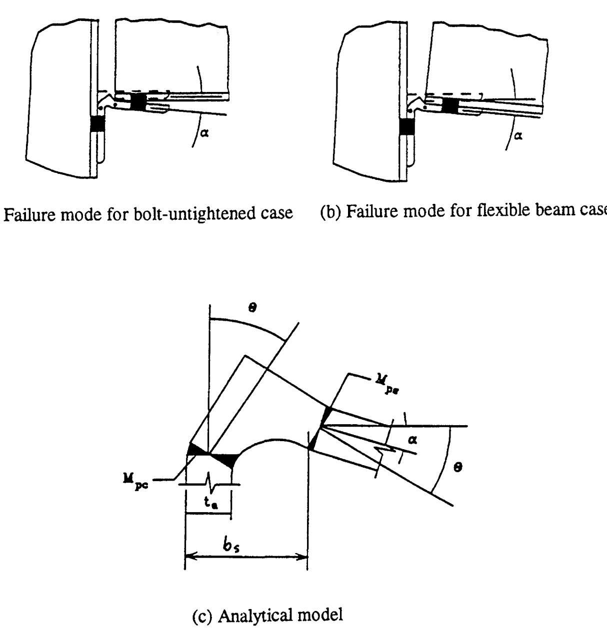
$$\frac{16}{(F_{y\text{-}angle}Lt_a)^4}R^4 + \frac{1}{(F_{y\text{-}angle}Lt_a)^2}R^2 + \frac{4b_s - 2t_a}{F_{y\text{-}angle}Lt_a^2}R - 2 = 0 \tag{27}$$

The value of R can be determined by an iteration procedure. Then, the design strength can be obtained by multiplying R by a resistance factor of 0.9. The results of selected examples from this method are listed in Table 1. It should be pointed out that these design strengths are determined solely from the limit state of plastic collapse of the seat angle. They represent



(a) Failure mode
(b) Analytical model

Fig. 15. Failure mode for bolt-tightened case.



(a) Failure mode for bolt-untightened case
(b) Failure mode for flexible beam case
(c) Analytical model

Fig. 16. Failure mechanism for bolt-untightened or flexible beam case.

an upper bound load carrying capacity of seated connection, since for the failure mode shown in Figure 15 the beam has to be rigid and the beam end rotation equals to zero. In reality, the beam can not be truly rigid and some finite beam end rotation will always exist. When the beam end rotation is significant and can not be neglected, then the model for the bolt-untightened case can be modified to consider the effect of beam end rotation on the load-carrying capacity of a seated-beam connection as discussed in the following section.

Bolt-Untightened Case

A situation between the no-bolt and the bolt-tightened cases could occur if the bolts connecting the beam bottom flange to seat angle are installed but not tightened. For example, if the nut is threaded onto the shank of the bolt, but not fully turned until contact is achieved between the bottom of the nut and the connecting surface, then the failure mode should be like the one shown in Figure 16(a). A similar failure mode shown in Figure 16(b) applies for the case when the bolts are tightened but the beam is flexible and beam end rotation equals a finite value of α. Analytically, both failure modes can share the same collapse mechanism for the seat angle as shown in Figure 16(c), except that for the bolt-untightened case the value of α depends on the position of nuts, while for the flexible beam case the beam end rotation α is a function of beam stiffness EI, beam span length L and load distribution patterns. Again, the effects of shear force and axial load on the plastic moment capacity of seat angle must be considered.

The work equation for the mechanism shown in Figure 16(c) is given by:

$$M_{ps}(\theta - \alpha) + M_{pc}\theta = R\left(k - \frac{t_a}{2}\right)\theta \qquad (28)$$

where

M_{ps} and M_{pc} are given by Equations 26a and 26b respectively.

In writing Equation 28, it is tactically assumed that the reaction R is close to the beam end so that the effect of rotation α on the work done by reaction R can be neglected.

Similar to the derivation from Equation 25 to Equation 27, Equation 28 can be simplified to the following fourth-order equation:

$$\frac{1-\alpha/\theta}{(F_{y\text{-}angle}Lt_a)^4}R^4 + \frac{1}{(F_{y\text{-}angle}Lt_a)^2}R^2 + \frac{4k-2t_a}{F_{y\text{-}angle}Lt_a^2}R - \left(2 - \frac{\alpha}{\theta}\right) = 0 \qquad (29)$$

For the bolt-untightened case shown in Figure 16(a), the rotation angle α is a constant as far as the position of nuts are known. Therefore, the load-carrying capacity R will be a function of only θ for given seat angle sizes. Given a specific

Table 2.
The Comparison of LRFD Design Strength Relative to the Results of Plastic Hinge Method*

Angle length (in.)	6				
Angle thickness (in.)	⅜	½	⅝	¾	1
F_y = 36 ksi	8.3%	–0.5%	–4.7%	–6.9%	–8.6%
F_y = 50 ksi	27.6%	20.8%	17.5%	15.9%	14.9%
Angle length (in.)	8				
Angle thickness (in.)	⅜	½	⅝	¾	1
F_y = 36 ksi	–5.9%	–15.6%	–20.2%	–22.6%	–24.8%
F_y = 50 ksi	10.7%	2.0%	–2.3%	–4.3%	–5.7%

* "+" = overestimate; "–" = underestimate.

value of θ, the value of R can be determined from Equation 29 by a simple iteration procedure. Thus, the R-θ relationship curve can be plotted using this method.

For the flexible beam case shown in Figure 16(b), the beam end rotation α is a linear function R, it can be expressed in the following general form:

$$\alpha = \Psi R \qquad (30)$$

where

$\Psi = \frac{l^2}{12EI}$ for uniformly distributed load on a simple beam

$\Psi = \frac{l^2}{8EI}$ for a concentrated load at the mid-span of a simple beam.

Substituting α from Equation 30 into Equation 29 and rearranging we have:

$$\frac{\Psi/\theta}{(F_{y\text{-}angle}Lt_a)^4}R^5 - \frac{1}{(F_{y\text{-}angle}Lt_a)^4}R^4 - \frac{1}{(F_{y\text{-}angle}Lt_a)^2}R^2 - \left(\frac{4k-2t_a}{F_{y\text{-}angle}Lt_a^2} + \frac{\Psi}{\theta}\right)R + 2 = 0 \qquad (31)$$

Again, the load-carrying capacity R is a function of θ. Given a specific value of θ, the corresponding R can be determined from Equation 31 using an iteration procedure. Thus, the R-θ relation curve can be plotted.

THE VALIDITY OF LRFD PROCEDURES AND PRACTICAL IMPLICATIONS

The load-carrying capacity of selected examples calculated using various analytical models described in the preceding sections are summarized and compared in Tables 1 and 2. All the values in the Table 1 include the appropriate resistance factors.

It can be observed from Table 2 that, based upon the strength given by the plastic hinge method, the LRFD method overestimates the load-carrying capacity by 15 percent to 28 percent when L = 6 in. and $F_{y\text{-}beam}$ = 50 ksi. Moreover, the LRFD method underestimates the strength by 6 percent to 25 percent when L = 8 in. and $F_{y\text{-}beam}$ = 36 ksi. Note, however, that the strength comparisons are more favorable (within ±10 percent) for the remaining cases listed in Table 2. Considering the inconsistencies and shortcomings of the LRFD procedures discussed previously, it is perhaps surprising that the design values generated from these procedures are not significantly off when compared with the results obtained using more rational and accurate models.

In the LRFD procedures, the unrealistic (very small or negative) value of effective bearing length N or its resultant small eccentricity $\acute{e}$, together with neglect of the shear force effect on the plastic moment capacity will always result in overestimating the load-carrying capacity and lead to an unsafe design. However, since the LRFD procedures completely ignore both of the local and overall beneficial effects of beam-bottom-flange to seat-angle attaching bolts and employ an unrealistic and overly conservative failure mechanism, they tend to underestimate the load-carrying capacity and lead to an extremely conservative design. The combined effect of these opposite factors make the LRFD procedures reasonably safe, although accidently in many cases. However, the safety is not provided by the factors suggested in the design calculations.

The comparisons made in Tables 1 and 2 are within the range of variables listed in the LRFD Manual tables (i.e. ⅜-in. $\leq t_a \leq$ 1 in. and $t_w \leq$ 9⁄16-in.). When the dimensions of the connection components exceed these ranges (although this may not happen very often in practice), the LRFD procedures will overestimate the load-carrying capacity more frequently. Therefore, it is reasonable to suggest that the design strength obtained using the plastic hinge analysis should be incorporated into the current LRFD procedures as an upper bound limiting strength. In other words, whenever the predictions of LRFD procedures for load-carrying capacity exceed those obtained using a simple plastic hinge analysis, the later should be used as design strength.

Practical Implications

It is noted from Table 1 that the seated connections of unbolted cases generally have load-carrying capacities much lower than the design strengths of bolted connections. Moreover, the failure of an unbolted seated connection, Figure 5(c), would develop suddenly and without warning, which is generally not acceptable in engineering practice. Therefore, it is very important to install and tighten the beam-bottom-flange to seat-angle attaching bolts as soon as possible after the beam is placed on the seat-angle during construction. A seated connection should not carry any additional load from other structural members until the bolts are installed and securely tightened.

From the comparison made in Table 2, it is obvious that when L = 6 in. and $F_{y\text{-}beam}$ = 50 ksi the LRFD results are not acceptable for safety consideration. This is because seat angles with a short length have less effective area to resist the load. In these cases the plastic moment capacity of seat angles become more sensitive to the reduction effect of axial load and shear force which are not accounted for in the LRFD formulations. Since the effects of axial load and shear force are, respectively, second and fourth order effects [see Equations 22a and 4a], their reduction effects would decrease rapidly as the angle length increases. This is why for L = 8 in. angle length the LRFD results are much more reasonable. According to the analysis made above, it is advisable to use seat angles with lengths greater than 8 in. whenever possible in practice. If a seat angle length less than 6 in. must be used, then care should be exercised in sizing the seat angle thickness in order to ensure a safe design.

CONCLUSION

It is concluded that the current LRFD procedures for unstiffened seated connections are somewhat irrational, although they often produce acceptable results. The LRFD procedures employ a highly idealized and somewhat unrealistic distribution of forces within the connection, and they assume incorrect failure modes which ignore some critical behavioral factors. Since some of factors ignored in the LRFD procedures are unsafe while others are overly conservative, the combined effect of opposite factors may explain why seated connections have resulted in satisfactory designs for many years without significant problems. However, the current LRFD procedures are valid only within a particular limited range of connection parameters.

Improvements in the design procedures of seated connections may be desirable. It seems more reasonable to check the strength of the seat angle and beam web local yielding and crippling separately, rather than to account for them together as in the current LRFD procedures. Moreover, the level of safety can be more accurately controlled if the model that is used to predict the strength is rational and behaviorally correct.

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DISCUSSION

The Behavior and Load-Carrying Capacity of Unstiffened Seated Beam Connections

Paper by W. H. YANG, W. F. CHEN and M. D. BOWMAN
(3rd Quarter, 1997)

Discussion by C. J. Carter, W. A. Thornton and T. M. Murray

The authors raise a valid criticism in their assessment of the AISC procedure for the design of unstiffened seated connections. Specifically, the anomaly in the formulation of the design strength that occasionally results in a negative bearing length needs to be corrected. An additional shortcoming of the AISC procedure that was not noted in the subject paper is that, while the local web yielding limit state is an integral part of the formulation of the seat angle design strength, the web crippling limit state is not although it is checked after a seat angle has been selected. A revised procedure that addresses these concerns is detailed at the end of this discussion.

Additionally, a number of other concerns raised need clarification or comment:

1. The authors describe the AISC procedure for the design of unstiffened seated connections as irrational, primarily because it does not model the exact behavior of the seat angle. Instead, the AISC procedure is based upon a simplified cantilever bending model for bending of the outstanding angle leg. Despite the author's characterizations, the AISC procedure is entirely a rational method—one in which a simplified approach is utilized to determine the answer to a problem that is more complex. Historically, AISC has employed such simplified models in its design procedures to maintain design simplicity. Although some procedures are more truly descriptive of actual behavior, such as those for the design of flexural members, all result in design strengths that are reasonable and representative of those obtained from testing of the modeled components.
2. In several instances, the authors indicate that the AISC procedure may be unsafe. However, the authors also present substantial data that shows that the AISC procedure underestimates strength in many cases. Furthermore, the authors note that unstiffened seated connections have a very good historical performance record while stating that "... the overall LRFD procedures usually produce satisfactory and safe designs ..." and "... [the design strengths] generated from these procedures are not significantly off when compared with the results obtained using more rational and accurate models." Ultimately, the authors are assessing safety based upon comparisons of the results of the AISC procedure with those of another model that may be no more accurate. Safety is more appropriately assessed by comparing a model to physical test results.
3. The authors suggest that an interaction check is important to address the concurrent effects of shear and bending in determining the design strength of the seat angle. The Drucker criterion as recommended by the authors represents a lower-bound solution that is always conservative by some unknown amount. Without testing, its actual relevance, however, is unknown.

   An interaction check for concurrent shear and bending has not historically been made for unstiffened seated connections. Nor has it been made in other similar cases of combined shear and bending, including double-angle, single-angle, and single-plate shear connections and moment end-plate connections. It should be of surprise to no one that the actual distribution of stress in the outstanding leg of an unstiffened seated connection is much more complex than the idealized distribution that is fundamentally assumed to exist along the span of a

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beam. For this reason alone, such an interaction check is probably unwarranted. Furthermore, it must also be recognized that many beneficial aspects of the connection and system behavior are ignored in the AISC model. For example, in the AISC model, flexural design strength is assessed on the basis of the formation of a single plastic hinge in the outstanding leg, whereas true failure of the connection in flexure requires the formation of two hinges. Additionally, any contribution to the connection strength from the top flange stabilizing angle is ignored in the AISC model. Although small, this top angle does so contribute. These more than compensate for such design simplifications as the lack of an interaction check for combined shear and bending.

4. The authors assert that loading on the beam supported by the unstiffened seated connection tends to pull the seat angle away from the support, resulting in higher-order axial and flexural effects. In fact there is normally restraint in the structural system, such as that due to a floor slab, roof deck or other framing members, that prevents such deformations. The authors further contend that the deflection of the simple beam tends to pull the seat angle away from the support. This is incorrect because the bottom flange of the beam gets longer as the beam deflects, thereby pushing the seat angle into the support.
5. The authors discuss the effect of the presence of bolts connecting the beam to the seat angle and the installed tension when such bolts are present. While the strength assessment of the seat angle for the no-bolt case is technically acceptable, it should be noted that the AISC procedure requires that the beam be attached to the seat angle with two high-strength bolts. Any comparison of the AISC procedure to the no-bolt case strength is therefore inappropriate. From a practical standpoint, the no-bolt case is only of concern during erection when the connection normally is subject to an end reaction that is considerably lower than the in-service end reaction.

   Of greater concern is the authors claim that the actual pretension present in the installed high-strength bolts affects the strength of the connection. This claim is based upon the supposition that the toe of the seat angle is initially the only point of contact with the beam flange, which causes a bending moment as the plies are brought into contact during bolt installation. In fact, due to beam camber and/or angle cross-sectional tolerances, the end of the beam is more likely to be the point of initial contact. Regardless, the small rotation required to bring the plies into firm contact is insignificant and it can be stated emphatically that the performance of unstiffened seated connections is unaffected by the installed bolt tension as long as the connected plies are in contact as required in the RCSC *Specification for Structural Joints Using ASTM A325 or A490 Bolts*.

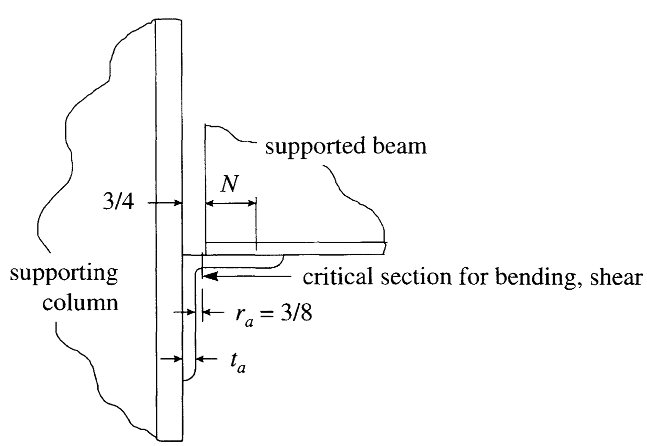

Fig. 1. Unstiffened seated connection.

The following design procedure alleviates the problem of negative bearing lengths and integrates both the local web yielding and web crippling checks into the seat-angle design. It is based upon the model and variables illustrated in Figure 1, where N is the bearing length at the beam end.

First determine the largest required bearing length for the limit states of local web yielding and web crippling of the beam. For local web yielding, from LRFD Specification Section K1.3,

$$\phi R_n = 1.0 \times (N + 2.5k) F_{yw} t_w$$

and the required bearing length N_{req} is,

$$N_{\min} = \frac{R_u}{F_{yw} t_w} - 2.5k = \frac{R_u - \phi R_1}{\phi R_2} \tag{1}$$

As indicated in LRFD Specification Section K1.3, as a lower bound,

$$N_{\min} = k \tag{2}$$

For web crippling, from LRFD Specification Section K1.4,

when $\frac{N}{d} \leq 0.2$

$$\phi R_n = 0.75 \times 68 t_w^2 \left[1 + \frac{3N}{d} \left(\frac{t_w}{t_f} \right)^{1.5} \right] \sqrt{\frac{F_{yw} t_f}{t_w}}$$

when $\frac{N}{d} > 0.2$

$$\phi R_n = 0.75 \times 68 t_w^2 \left[1 + \left(\frac{4N}{d} - 0.2 \right) \left(\frac{t_w}{t_f} \right)^{1.5} \right] \sqrt{\frac{F_{yw} t_f}{t_w}}$$

and the required bearing length N is,

when $\frac{N}{d} \leq 0.2$

$$N_{min} = \frac{d}{3}\left[\frac{R_u t_f}{0.75(68 t_w^3 \sqrt{F_{yw}})}\left(\frac{t_f}{t_w}\right)^{1.5}\right] = \frac{R_u - \phi_r R_3}{\phi_r R_4} \tag{3}$$

when $\frac{N}{d} \leq 0.2$

$$N_{min} = \frac{d}{4}\left[\frac{R_u t_f}{0.75(68 t_w^3 \sqrt{F_{yw}})}\left(\frac{t_f}{t_w}\right)^{1.5} + 0.2\right] = \frac{R_u - \phi_r R_5}{\phi_r R_6} \tag{4}$$

In the above equations,

F_{yw} = minimum specified yield stress of beam web, ksi
N = bearing length, in.
R_u = required strength (beam end reaction), kips
d = beam depth, in.
k = distance from outer face of flange to web toe fillet, in.
t_w = beam web thickness, in.
t_f = beam flange thickness, in.
ϕ = 1.0
ϕ_r = 0.75
ϕR_1, ϕR_2, $\phi_r R_3$, $\phi_r R_4$, $\phi_r R_5$, and $\phi_r R_6$ are constants tabulated in the factored uniform load tables in LRFD Manual Part 4

Because the value of N/d is initially unknown for the web crippling limit state, the larger value of N_{min} determined from Equations 3 and 4 is used (alternatively, an iterative approach can be used to determine which equation is appropriate). The required bearing length N_{req} is then the largest value of N_{min} determined from Equations 1, 2, 3, and 4. The outstanding leg dimension of the seat angle must be selected to meet or exceed the sum of the required bearing length N_{req} and ¾-in. (½-in. setback plus ¼-in. tolerance on beam length).

Next, select the angle length L_a and thickness t_a such that Equations 5 and 6 are satisfied. For flexural yielding of the angle,

$$t_a \geq \sqrt{\frac{4 R_u e}{\phi F_{ya} L_a}} \tag{5}$$

For shear yielding of the angle,

$$t_a \geq \frac{R_u}{\phi(0.6 F_{ya}) L_a} \tag{6}$$

In the above equations,

$$e = \frac{N_{req}}{2} + b_s - t_a - r_a = \frac{N_{req}}{2} + 3/8 - t_a$$

and

F_{ya} = minimum specified yield stress of seat angle, ksi
L_a = seat angle length, in.
N_{req} = largest required bearing length from Equations 1, 2, 3, and 4
R_u = required strength (beam end reaction), kips
b_s = ¾-in., (½-in. beam setback plus ¼-in. tolerance on beam length)
r_a = seat angle fillet radius taken as ⅜-in.
t_a = seat angle thickness, in.
ϕ = 0.90

The design of bolts or welds connecting the seat angle to the support is unchanged from that in the current AISC procedure.

Tables 1 and 2 simplify the selection of the unstiffened seated connections illustrated in Figure 2 according to the foregoing procedure. Table 1 is for unstiffened seated connections that are bolted to both the supported beam and the supporting member. Table 2 is for unstiffened seated connections that are welded to both the supported beam and the supporting member. Unstiffened seated connections that utilize a combination of bolting and welding can also be designed with the applicable sections of Tables 1 and 2.

With the value of N_{req} calculated as above, enter the appropriate table and select a combination of angle length L_a and angle thickness t_a that provides a design strength that equals or exceeds the required strength (beam end reaction). The

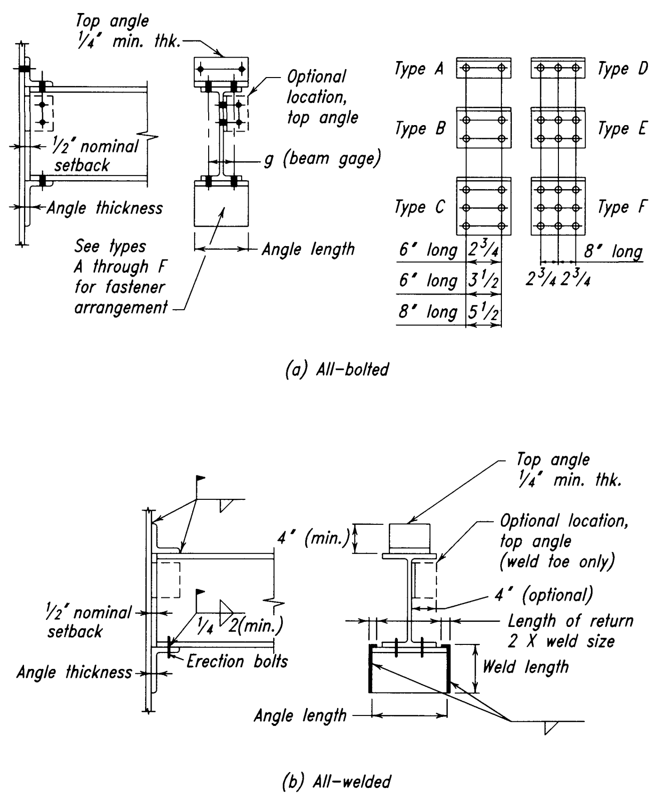

Fig. 2. Unstiffened seated connections.

outstanding angle leg size must be selected greater than the minimum angle leg tabulated in the right-hand column.

When a bolted connection to the supporting member is used, the design strength is tabulated in Table 1. When a welded connection to the supporting member is used, the design strength is tabulated in Table 2. The design strength of the supporting member must be checked independently.

Some common angle sizes with available ranges of thickness are indicated in both Tables. This is not intended to preclude the use of alternative angle sizes and thicknesses. The use of a longer outstanding angle leg than that indicated is permitted.

Table 1.
All-Bolted Unstiffened Seated Connections

Outstanding Angle Leg Design Strength, kips

Required Bearing Length N_{req}	Angle Length, in. 6 — Angle Thickness, in.					Angle Length, in. 8 — Angle Thickness, in.					Min. Angle Leg
in.	3/8	1/2	5/8	3/4	1	3/8	1/2	5/8	3/4	1	in.
1/2	27.3					36.5					
9/16	24.3					32.4					
5/8	21.9	58.3				29.2	77.8				
11/16	19.9	55.5				26.5	74.1				
3/4	18.2	48.6				24.3	64.8				
13/16	16.8	43.2				22.4	57.6				
7/8	15.6	38.9				20.8	51.8				
15/16	14.6	35.3				19.4	47.1				
1	13.7	32.4	72.9			18.2	43.2	97.2			
1 1/16	12.9	29.9	67.5			17.2	39.9	90.0			
1 1/8	12.2	27.8	60.8			16.2	37.0	81.0			
1 3/16	11.5	25.9	55.2			15.3	34.6	73.6			
1 1/4	10.9	24.3	50.6			14.6	32.4	67.5			
1 5/16	10.4	22.9	46.7			13.9	30.5	62.3			3 1/2
1 3/8	9.94	21.6	43.4	87.5		13.3	28.8	57.9	117		
1 7/16	9.51	20.5	40.5	79.5		12.7	27.3	54.0	106		
1 1/2	9.11	19.4	38.0	72.9		12.2	25.9	50.6	97.2		
1 5/8	8.41	17.7	33.8	62.5		11.2	23.6	45.0	83.3		
1 3/4	7.81	16.2	30.4	54.7		10.4	21.6	40.5	72.9		
1 7/8	7.29	15.0	27.6	48.6		9.72	19.9	36.8	64.8		
2	6.83	13.9	25.3	43.7	117	9.11	18.5	33.8	58.3	156	
2 1/8	6.43	13.0	23.4	39.8	111	8.58	17.3	31.2	53.0	148	
2 1/4	6.08	12.2	21.7	36.5	97.2	8.10	16.2	28.9	48.6	130	
2 3/8	5.76	11.4	20.3	33.6	86.4	7.67	15.2	27.0	44.9	115	
2 1/2	5.47	10.8	19.0	31.2	77.8	7.29	14.4	25.3	41.7	104	
2 5/8	5.21	10.2	17.9	29.2	70.7	6.94	13.6	23.8	38.9	94.3	
2 3/4	4.97	9.72	16.9	27.3	64.8	6.63	13.0	22.5	36.5	86.4	
2 7/8	4.75	9.26	16.0	25.7	59.8	6.34	12.3	21.3	34.3	79.8	4
3	4.56	8.84	15.2	24.3	55.5	6.08	11.8	20.3	32.4	74.1	
3 1/8	4.37	8.45	14.5	23.0	51.8	5.83	11.3	19.3	30.7	69.1	
3 1/4	4.21	8.10	13.8	21.9	48.6	5.61	10.8	18.4	29.2	64.8	

Bolt Design Strength, kips

Bolt Diameter, in.	ASTM Desig.	Thread Cond.	Connection Type from Figure 2a: A	B	C	D	E	F
3/4	A325	N	31.8	63.6	95.4	47.7	95.4	143
		X	39.8	79.5	119	59.6	119	179
	A490	N	39.8	79.5	119	59.6	119	179
		X	49.7	99.4	149	74.6	149	224
7/8	A325	N	43.3	86.6	130	64.9	130	195
		X	54.1	108	162	81.2	162	244
	A490	N	54.1	108	162	81.2	162	244
		X	67.6	135	203	101	203	304
1	A325	N	56.5	113	—	84.8	170	—
		X	70.7	141	—	106	212	—
	A490	N	70.7	141	—	106	212	—
		X	88.4	177	—	133	265	—

Available Angles

Connection Type	Angle Size	t, in.
A, D	4×3	3/8-1/2
	4×3 1/2	3/8-1/2
	4×4	3/8-3/4
B, E	6×4	3/8-3/4
	7×4	3/8-3/4
	8×4	1/2-1
C, F[b]	8×4	1/2-1

[b]Not suitable for use with 1-in. diameter bolts.

For tabulated values above the heavy line, shear yielding of the angle leg controls the design strength.

Table 2.
All-Welded Unstiffened Seated Connections

Outstanding Angle Leg Design Strength, kips

Required Bearing Length N_{req}	Angle Length, in. 6					8					Min. Angle Leg
	Angle Thickness, in.										
in.	3/8	1/2	5/8	3/4	1	3/8	1/2	5/8	3/4	1	in.
1/2	27.3					36.5					
9/16	24.3					32.4					
5/8	21.9	58.3				29.2	77.8				
11/16	19.9	55.5				26.5	74.1				
3/4	18.2	48.6				24.3	64.8				
13/16	16.8	43.2				22.4	57.6				
7/8	15.6	38.9				20.8	51.8				
15/16	14.6	35.3				19.4	47.1				
1	13.7	32.4	72.9			18.2	43.2	97.2			
1 1/16	12.9	29.9	67.5			17.2	39.9	90.0			
1 1/8	12.2	27.8	60.8			16.2	37.0	81.0			
1 3/16	11.5	25.9	55.2			15.3	34.6	73.6			
1 1/4	10.9	24.3	50.6			14.6	32.4	67.5			
1 5/16	10.4	22.9	46.7			13.9	30.5	62.3			3 1/2
1 3/8	9.94	21.6	43.4	87.5		13.3	28.8	57.9	117		
1 7/16	9.51	20.5	40.5	79.5		12.7	27.3	54.0	106		
1 1/2	9.11	19.4	38.0	72.9		12.2	25.9	50.6	97.2		
1 5/8	8.41	17.7	33.8	62.5		11.2	23.6	45.0	83.3		
1 3/4	7.81	16.2	30.4	54.7		10.4	21.6	40.5	72.9		
1 7/8	7.29	15.0	27.6	48.6		9.72	19.9	36.8	64.8		
2	6.83	13.9	25.3	43.7	117	9.11	18.5	33.8	58.3	156	
2 1/8	6.43	13.0	23.4	39.8	111	8.58	17.3	31.2	53.0	148	
2 1/4	6.08	12.2	21.7	36.5	97.2	8.10	16.2	28.9	48.6	130	
2 3/8	5.76	11.4	20.3	33.6	86.4	7.67	15.2	27.0	44.9	115	
2 1/2	5.47	10.8	19.0	31.2	77.8	7.29	14.4	25.3	41.7	104	
2 5/8	5.21	10.2	17.9	29.2	70.7	6.94	13.6	23.8	38.9	94.3	
2 3/4	4.97	9.72	16.9	27.3	64.8	6.63	13.0	22.5	36.5	86.4	
2 7/8	4.75	9.26	16.0	25.7	59.8	6.34	12.3	21.3	34.3	79.8	4
3	4.56	8.84	15.2	24.3	55.5	6.08	11.8	20.3	32.4	74.1	
3 1/8	4.37	8.45	14.5	23.0	51.8	5.83	11.3	19.3	30.7	69.1	
3 1/4	4.21	8.10	13.8	21.9	48.6	5.61	10.8	18.4	29.2	64.8	

Weld (70 ksi) Design Strength, kips

70 ksi Weld Size, in.	Seat Angle Size (long leg vertical) 4×3 1/2	5×3 1/2	6×4	7×4	8×4
1/4	17.3	25.8	32.7	42.8	53.4
5/16	21.5	32.3	41.0	53.4	66.8
3/8	25.8	38.7	49.1	64.1	80.1
7/16	30.2	45.2	57.3	74.7	93.5
1/2	—	51.6	65.4	83.4	107
5/8	—	64.5	81.8	107	134
11/16	—	71.0	90.0	117	—

Available Angle Thickness, in.

	4×3 1/2	5×3 1/2	6×4	7×4	8×4
Minimum	3/8	3/8	3/8	3/8	1/2
Maximum	1/2	3/4	3/4	3/4	1

For tabulated values above the heavy line, shear yielding of the angle leg controls the design strength.

ERRATA

Discussion of The Behavior and Load-Carrying Capacity of Unstiffened Seated Beam Connections

Discussion by C. J. CARTER, W. A. THORNTON, and T. M. MURRAY
(4th Quarter, 1997)

Page 153, left column, the line preceding Equation (4) should read:

when $\frac{N}{d} > 0.2$

Design Tables for Top- and Seat-Angle with Double Web-Angle Connections

Paper by YOSUK KIM and WAI-FAH CHEN
(2nd Quarter, 1998)

Revise the headings to the tables in Appendix II as follows:

Page 62, the table headings should be:

T & S l_t = 6-in. L6×4×t_t	Web 2L4×3.5 t_a	A36 ⅞-in. Dia.	Beam										
			W8	W10		W12		W14	W16		W18		
			2 row	2 row	3 row	2 row	3 row	3 row	3 row	4 row	3 row	4 row	5 row

T & S l_t = 6-in. L6×4×t_t	Web 2L4×3.5 t_a	A36 ⅞-in. Dia.	Beam										
			W21			W24				W27			
			3 row	4 row	5 row	4 row	5 row	6 row	7 row	5 row	6 row	7 row	8 row

Page 63, the table headings should be:

T & S l_t = 6-in. L6×4×t_t	Web 2L4×3.5 t_a	A36 ⅞-in. Dia.	Beam										
			W27	W30				W33				W36	
			9 row	6 row	7 row	8 row	9 row	6 row	7 row	8 row	9 row	6 row	7 row

T & S l_t = 6-in. L6×4×t_t	Web 2L4×3.5 t_a	A36 ⅞-in. Dia.	Beam									
			W36		W40				W44			
			8 row	9 row	6 row	7 row	8 row	9 row	6 row	7 row	8 row	9 row

Page 64, the table headings should be:

T & S l_t = 6-in. L6×4×t_t	Web 2L4×3.5 t_a	50 ksi ¾-in. Dia.	Beam										
			W8	W10		W12		W14	W16		W18		
			2 row	2 row	3 row	2 row	3 row	3 row	3 row	4 row	3 row	4 row	5 row

T & S l_t = 6-in. L6×4×t_t	Web 2L4×3.5 t_a	50 ksi ¾-in. Dia.	Beam										
			W21			W24				W27			
			3 row	4 row	5 row	4 row	5 row	6 row	7 row	5 row	6 row	7 row	8 row

Page 65, the table headings should be:

T & S l_t = 6-in. L6×4×t_t	Web 2L4×3.5 t_a	50 ksi ¾-in. Dia.	Beam										
			W27	W30				W33				W36	
			9 row	6 row	7 row	8 row	9 row	6 row	7 row	8 row	9 row	6 row	7 row

T & S l_t = 6-in. L6×4×t_t	Web 2L4×3.5 t_a	50 ksi ¾-in. Dia.	Beam									
			W36		W40				W44			
			8 row	9 row	6 row	7 row	8 row	9 row	6 row	7 row	8 row	9 row

ERRATA

DISCUSSION OF
The Behavior and Load-Carrying Capacity of Unstiffened Seated Beam Connections

Paper by W. H. YANG, W. F. CHEN, and M. D. BOWMAN
(Third Quarter, 1997)

Discussion by C. J. CARTER, W. A. THORNTON, and T. M. MURRAY
(Fourth Quarter, 1997)

On page 153 of the Discussion, replace Equations 3 and 4 with the following:

$$N_{\min} = \frac{d}{3}\left[\frac{R_u t_f}{0.75\left(68 t_w^3 \sqrt{F_{yw}}\right)} - \left(\frac{t_f}{t_w}\right)^{1.5}\right] = \frac{R_u - \phi_r R_3}{\phi_r R_4} \qquad (3)$$

$$N_{\min} = \frac{d}{4}\left[\frac{R_u t_f}{0.75\left(68 t_w^3 \sqrt{F_{yw}}\right)} - \left(\frac{t_f}{t_w}\right)^{1.5} + 0.2\right] = \frac{R_u - \phi_r R_5}{\phi_r R_6} \qquad (4)$$