An Analytical Criterion for Buckling Strength of Built-up Compression Members

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INTRODUCTION

In current design practice, North American and European specifications differ significantly regarding the rules governing the design of stitched built-up struts. The British and German specifications treat such members essentially like battened columns and apply more stringent provisions to limit their shear flexibility. Consequently, built-up struts designed according to these specifications must be closely stitched. Furthermore, the strength of built-up members are reduced due to shear flexibility. The American and Canadian specifications (except for recent AISC-LRFD), on the other hand, do not treat stitched built-up struts in a way similar to battened columns. They only require that minimum individual slenderness ratio between stitches not exceed the governing slenderness ratio of the built-up strut as a whole. Thus, in strength calculation, the detrimental effect of shear flexibility is not taken into account.

Recently, AISC-LRFD Specification[1] introduced an empirical equation to modify the overall slenderness ratio of built-up struts, including stitched members, in order to take into account the detrimental effect of shear flexibility on their strength (see Reference 1, p. 6-40, Section E4). The criterion is based on experimental study by Zandonini[3] on stitched double channel struts and verification by Astaneh and Goel[4] on double angle bracing members. In 1952, Bleich[5] proposed a similar but analytical criterion to modify the overall slenderness ratio of battened columns. The equation has the limitation of being strictly applicable to battened columns because of simplifications made in the derivation process. Another limitation of Bleich's equation is that it was derived strictly for hinged-end members.

This paper presents a generalized version of the analytical equation proposed by Bleich. Thus, it does not have the limitations of Bleich's equation. Therefore, the proposed analytical equation is applicable to all built-up struts with general end conditions. The equation is verified both analytically and experimentally. For analytical verification, a parametric study is performed in which, for different overall and individual slenderness ratios, the buckling load is calculated according to the proposed analytical equation, Bleich's approximate analytical equation, and the LRFD empirical equation. The results are compared and discussed. For experimental verification, test results from another study by the authors[6,7] are compared with the calculated buckling load according to the proposed analytical equation and the LRFD empirical equation. All comparisons verify the validity of the proposed analytical equation.

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NOTATION

Following notations are used in this paper:

a	= The distance between batten plates or stitches
A_i	= Cross sectional area of each individual component
A	= Cross sectional area of the member = $2A_i$
α	= Separation ratio = $h/2r_{ib}$
E	= Young's Modulus of elasticity
F_r	= Axial force in the chord between panel points $r-1$ and r caused by the flexure of the column in the state of buckling
h	= Distance between the centroids of individual components.
I_o	= Moment of inertia of the section about the buckling axis, neglecting the moment of inertia of individual components about their own centroidal axis = $2[A_i(h/2)^2]$ = $A_ih^2/2$
I_{ib}	= Moment of inertia of individual components about their own centroidal axis parallel to the axis of buckling = $A_i(r_{ib})^2$
I_t	= Moment of inertia of the integral section, about the axis of buckling = $2A_ir^2$
K	= Effective length factor for the overall member
L	= Length of the member
$\left(\frac{L}{r}\right)_o$	= Overall slenderness ratio of the integral member
$\left(\frac{KL}{r}\right)_o$	= Effective overall slenderness ratio of the integral member

$\left(\frac{KL}{r}\right)_m$	=	Modified effective slenderness ratio of the member
n	=	Number of panels
P_c	=	Total axial force in the column
r	=	The index of panel between two adjacent stitches
r_{ib}	=	Radius of gyration of individual components about their own centroidal axis parallel to the axis of buckling
r_{min}	=	Least radius of gyration of the individual components
V	=	Strain energy due to deflection
W	=	Work done by the external axial force P_c

ANALYTICAL MODIFICATION OF SLENDERNESS RATIO

Bleich[5] developed an equation to calculate the modified slenderness ratio of hinged-end battened columns. The derivation is based on the energy condition that the transition from stable to unstable equilibrium of any elastic system is characterized by,

$$V - W = 0$$

where V is the strain energy due to deflection and W is the work done by the external axial force P_c. The elastic strain energy of a deformed battened column (Fig. 1) consists of the following three terms:

a. The energy due to axial force F_r in the two individual components.
b. The energy due to local bending of the two individual components.
c. The energy due to local bending of the battened plates.

It should be noted that the first term can be interpreted as the strain energy due to overall bending of the column. Bleich points out that the third term is small and can be neglected compared to the other two terms. This assumption is more accurate in the case of stitched columns because the stitches can be considered as more rigid compared to the batten plates. Considering each segment of the member between two stitches as a panel, Bleich summed the strain energy of all panels to calculate the total strain energy. As shown in Fig. 1, the classical assumption of inflection points being located at the center of the transverse members and mid point of each component is used in the derivation process.

Based on the above approach, Bleich (see Ref. 5, p. 178, Eq. 350) derived the following equation for modification of the slenderness ratio of a hinged-end battened column:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{L}{r}\right)_o^2 + \frac{\pi^2 I_o}{24 I_{ib}}\left(\frac{a}{r}\right)^2} \qquad (1)$$

Replacing $(1/r)^2$ by $2A_i/I_t$, the above equation becomes:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{L}{r}\right)_o^2 + \frac{\pi^2 I_o}{24 I_{ib}}(a^2)\frac{2A_i}{I_t}} \qquad (2)$$

The individual slenderness ratio r_{ib} is introduced in the above equation by substituting $I_{ib} = r_{ib}^2 A_i$:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{L}{r}\right)_o^2 + \frac{\pi^2 I_o}{12 I_t}\left(\frac{a}{r_{ib}}\right)^2} \qquad (3)$$

Bleich indicated that in a battened column section the distance between the centroids of individual components (h in Fig. 2a) is large compared to the radius of gyration of the individual components. Thus, in calculation of the total moment of inertia (I_t), the individual moment of inertia may be neglected in comparison with $2A_i(h/2)^2$. This approximation leads to:

$$I_t = 2I_{ib} + 2A_i(h/2)^2 = 2A_i(r_{ib})^2 + 2A_i(h/2)^2$$
$$\approx 2A_i(h/2)^2 \approx I_o$$

or,

$$\frac{I_o}{I_t} = 1.0 \qquad (4)$$

Bleich introduced the above approximation into Eq. 3 and simplified it as:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{L}{r}\right)_o^2 + \frac{\pi^2}{12}\left(\frac{a}{r_{ib}}\right)^2} \qquad (5)$$

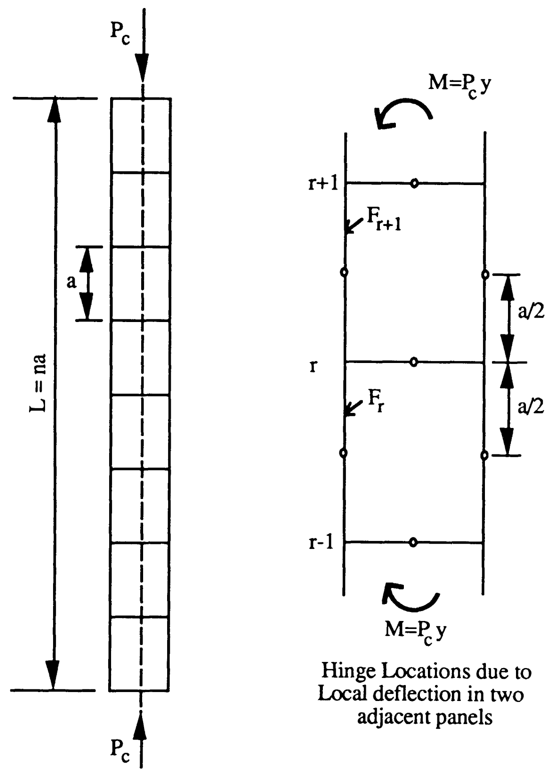

Fig. 1. Hinge locations in the local deflection of battened columns.

This is the equation suggested by Bleich for modification of the slenderness ratio of *hinged-end* battened columns to account for the detrimental effect of shear flexibility on the member strength (see Ref. 5, p. 178, Eq. 351).

GENERALIZATION OF BLEICH'S EQUATION

Two modifications are needed to generalize Eq. 5 in order to make it applicable to general end conditions as well as to all built-up compression members and not just limited to battened columns. First, an equation needs to be derived for the fixed-end case from which general end conditions could be treated. Second, approximation of the ratio I_o/I_t by unity should be replaced by the exact expression since, as shown in Fig. 2b, even for some battened column sections this ratio might be off by as much as 20%.

End Fixity

Bleich derived Eq. 3 for hinged-end members based on the buckling shape of such members which can be represented as:

$$y = f \sin \frac{\pi x}{L}$$

where f is the deflection at mid span as shown in Fig. 3a. For a fixed-end strut, however, the exact buckling shape can be represented by the following equation:

$$y = \frac{f}{2}\left(1 - \cos \frac{2\pi x}{L}\right)$$

The above equation satisfies the geometric boundary conditions. Again, f is the deflection at mid span as shown in Fig. 3b. Using the above equation in the procedure followed by Bleich leaves all strain energy terms identical to those for a hinged-end case except for one term. The strain energy associated with overall bending of the member is given in Eq. 345 in Bleich.[5] The overall bending moment denoted by M is different here since end fixity causes a different buckling shape and also causes non-zero moments at the two ends. The equation for bending moment is:

$$M = P_c y + M_e = P_c \frac{f}{2}\left(1 - \cos \frac{2\pi x}{L}\right) + M_e \qquad (6)$$

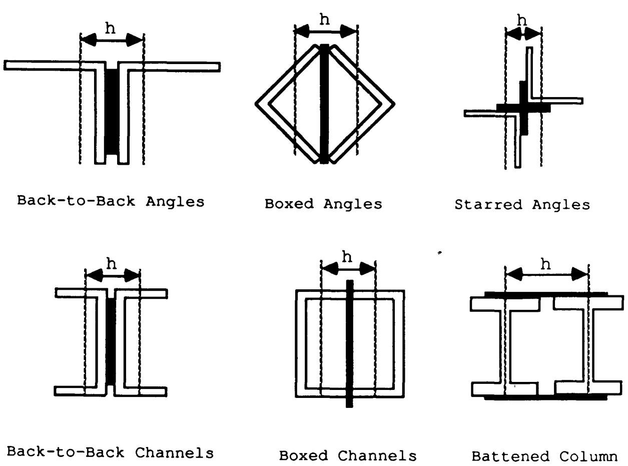

Fig. 2a. Some common types of built-up sections.

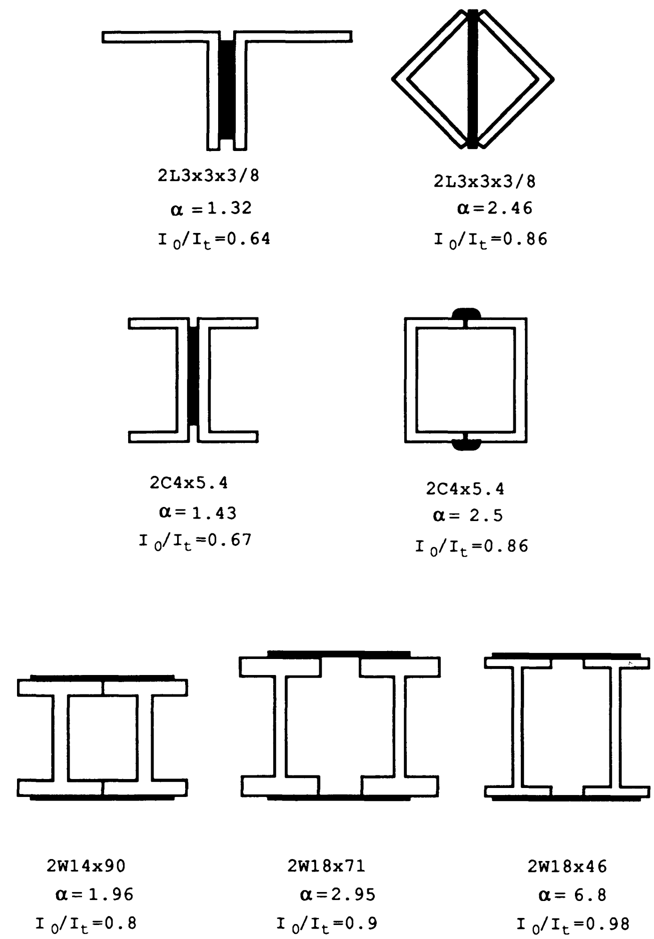

Fig. 2b. Separation ratio of some built-up sections.

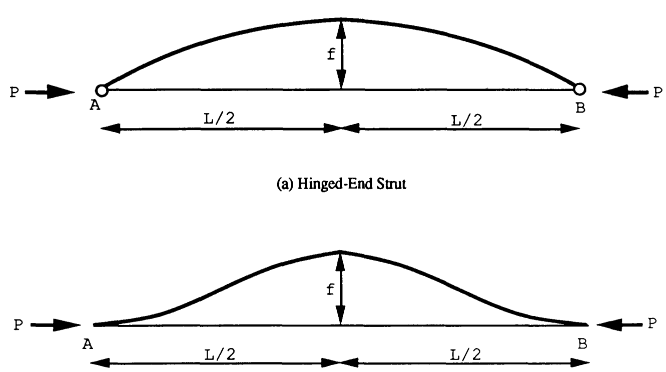

Fig. 3. Symmetric buckling shape of fixed-end struts.

The term M_e can be determined by basic mechanics approach in which the moment equation is integrated twice to derive the expression for the deflection curve; then, the boundary conditions are imposed to determine the constants of integration as well as the end moment M_e. Such a procedure results in:

$$M_e = -P_c\frac{f}{2}$$

Substitution of M_e in Eq. 6 results in:

$$\begin{aligned} M &= P_c\frac{f}{2}\left(1 - \cos\frac{2\pi x}{L}\right) - P_c\frac{f}{2} \\ &= -P_c\frac{f}{2}\cos\frac{2\pi x}{L} \end{aligned} \qquad (7a)$$

The above equation gives the moments at the end and at mid span as:

$$M(x{=}0) = -P_c\, f/2$$

$$M(x{=}L/2) = P_c\frac{f}{2}(1 - \cos\pi) - P_c\, f/2 = +P_c\, f/2$$

The above two expressions for moment at the ends and at mid span can be verified by checking the equilibrium equation of the free body diagram shown in Fig. 4. Thus,

$$\Sigma M_A = 0\ ;\ -P_c\, f/2 - P_c\, f/2 + P_c f = 0 \quad \textbf{O.K.}$$

Equation 7a can be used to derive the strain energy term associated with overall bending of the member. The bending moment at the mid-height of r^{th} panel can be expressed as,

$$M = -P_c\frac{f}{2}\cos\frac{2\pi x}{L} = -P_c\frac{f}{2}\cos\frac{2r-1}{2n}2\pi \qquad (7b)$$

Therefore, for a fixed-end strut, the energy term in Eq. 345 of Ref. 5 becomes,

$$\sum_{r=1}^{n}\frac{F_r^2 a}{EA} = \sum_{r=1}^{n}\left(\frac{M}{h}\right)^2\frac{a}{EA} = \frac{P_c^2 f^2}{4EA}\frac{a}{h^2}\sum_{r=1}^{n}\cos^2\frac{(2r-1)}{2n}2\pi$$

The following identity can be used:

$$\sum_{r=1}^{n}\cos^2\frac{(2r-1)}{2n}2\pi = \frac{n}{2} = \frac{L}{2a}$$

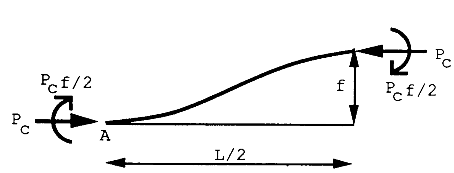

Fig. 4. Free body diagram of a fixed-end strut after buckling.

Substitution results in the following equation:

$$\sum_{r=1}^{n}\frac{F_r^2 c}{EA} = \frac{P_c^2 f^2}{4EA}\frac{a}{h^2}\frac{L}{2a} = \frac{P_c^2 f^2 L}{16EI_o}$$

The above term is similar to the term given in Eq. 345 of Ref. 5 with the exception that it is multiplied by a factor of ¼ which accounts for the end fixity of a column. It should be noted that the above term is due to the summation of the strain energy in the panels caused by overall bending of the member at the state of buckling. However, this term also could be derived from the simpler approach of calculating the strain energy due to the overall bending of the member:

$$\begin{aligned} V_{Bending} &= \int_o^L \frac{M^2}{2E_t I_o}dx = \frac{P_c^2 f^2}{8E_t I_o}\int_o^L \cos^2\frac{2\pi x}{L}dx = \frac{P_c^2 f^2}{8E_t I_o}\frac{L}{2} \\ &= \frac{P_c^2 f^2 L}{16E_t I_o} \end{aligned}$$

Interestingly, the other energy term in Eq. 340 of Ref. 5 and the work done by the external force remain intact and are not affected by the use of new moment expression given by Eq. 7a. Consequently, the only change in Eq. 3 is the appearance of a factor of ½ in the first term of the right side of the equation as:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{L}{2r}\right)_o^2 + \frac{\pi^2}{12}\frac{I_o}{I_t}\left(\frac{a}{r_{ib}}\right)^2} \qquad (8)$$

Equations (3) and (8) give the modified slenderness ratio for hinged- and fixed-end struts, respectively. A comparison of the above two equations leads to the conclusion that the equation for general end conditions can be given as:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \frac{\pi^2}{12}\frac{I_o}{I_t}\left(\frac{a}{r_{ib}}\right)^2} \qquad (9)$$

The ratio I_o/I_t

Bleich approximated the ratio I_o/I_t by 1.0 to derive Eq. 5. However, it will be shown that the ratio I_o/I_t decreases as the distance between the two components becomes smaller. For a general case, the exact expression for the ratio of I_o/I_t can be derived in terms of separation ratio α. The total moment of inertia I_t can be expressed as:

$$I_t = 2I_{ib} + 2[A_i(h/2)^2]$$

Replacing A_i by $I_{ib}/r_{ib}^{\,2}$,

$$\begin{aligned} I_t &= 2I_{ib} + 2\left[\frac{I_{ib}}{r_{ib}^{\,2}}(h/2)^2\right] \\ &= 2I_{ib}[1 + (h/2r_{ib})^2] \end{aligned}$$

In the literature, the ratio $h/2r_{ib}$ is called separation ratio,

$$\alpha = \frac{h}{2r_{ib}}$$

Thus,

$$I_t = 2I_{ib}(1 + \alpha^2) \quad (10)$$

Using $I_o = A_i h^2/2$ the ratio I_o/I_t can be expressed as:

$$\frac{I_o}{I_t} = \frac{A_i h^2/2}{2I_{ib}(1 + \alpha^2)}$$

Substitution of I_{ib} by $A_i r_{ib}{}^2$ results in:

$$\frac{I_o}{I_t} = \frac{(h/2r_{ib})^2}{1 + \alpha^2} \quad (11)$$

or,

$$\frac{I_o}{I_t} = \frac{\alpha^2}{\alpha^2 + 1} \quad (12)$$

The plot in Fig. 5 shows the variation of I_o/I_t with α. The figure shows that the above ratio asymptotically approaches unity with increasing α. Therefore, Bleich's approximation appears acceptable for battened columns with large separation ratio. However, Fig. 2b shows that even for some battened columns, Bleich's approximation may be in error by as much as 20%. Such cases could occur for battened columns made from relatively heavy sections with small depth, e.g., 2-W14×90. Furthermore, Fig. 2b presents some sample cases indicating that Bleich's approximation is in error by up to 36% for stitched compression members.

Substitution of Eq. 12 into Eq. 9 results in:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \frac{\pi^2}{12}\frac{\alpha^2}{(1 + \alpha^2)}\left(\frac{a}{r_{ib}}\right)^2} \quad (13)$$

Finally, using the numerical value of $\pi = 3.14$ results in:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + 0.82\frac{\alpha^2}{(1 + \alpha^2)}\left(\frac{a}{r_{ib}}\right)^2} \quad (14)$$

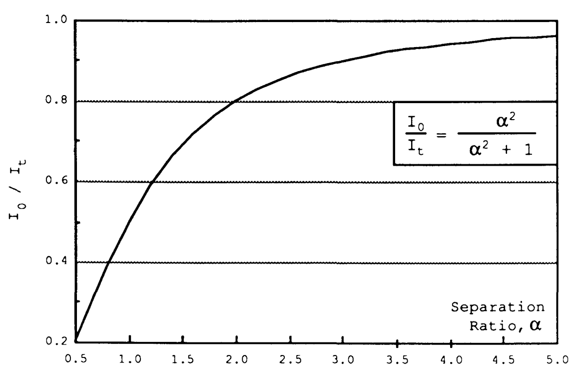

Fig. 5. Variation of I_o/I_t with α.

The above analytical equation is proposed to be used for calculating the modified slenderness ratio of both battened as well as stitched struts since the approximation used by Bleich is not used in Eq. 14. The equation takes into account the detrimental effect of shear flexibility on the overall strength of built-up struts; yet it is simple to use in practice.

LRFD EMPIRICAL EQUATION

The LRFD Specification[1] uses the following empirical equation (see Ref. 1, p. 6-40, Sect. E4) for the modification of the slenderness ratio of built-up struts:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{a}{r_{min}} - 50\right)^2} \quad (15)$$

The equation is applicable to built-up struts with component slenderness ratio larger than 50, i.e., $a/r_{min} > 50.0$. The criterion is based on an experimental study by Zandonini[3] on stitched double channel struts and some results of the experimental study by Astaneh and Goel[4] on double angle bracing members. There are two major differences between the proposed analytical Eq. 14 and the LRFD empirical Eq. 15.

First, the effect of separation ratio is not accounted for in the LRFD equation since it was based on a set of tests with specific separation ratio. In contrast, analytical Eq. 14 includes the effect of separation ratio through the coefficient $\alpha^2/(1+\alpha^2)$. Second, the proposed analytical equation uses the component radius of gyration corresponding to an axis parallel to the plane of buckling, that is r_{ib}. The use of r_{ib} is rational since the detrimental effect on buckling strength is due to local bending about an axis parallel to the plane of buckling. On the contrary, the LRFD empirical equation uses the minimum individual radius of gyration r_{min} which may not be necessarily corresponding to an axis parallel to the plane of buckling, e.g., in back-to-back angles.

VERIFICATION OF EQUATION 14

Two approaches were taken in this study to verify the proposed analytical Eq. 15. First, a parametric study was performed in which, for different overall and individual slenderness ratios, the buckling load was calculated according to the LRFD empirical Eq. 15, Bleich's approximate analytical Eq. 5, and the proposed analytical Eq. 14. The results are compared and discussed below. Second, test results were compared with the calculated buckling load according to the AISC-ASD,[2] LRFD Specification,[1] and Eq. 15). The results are also presented here.

Parametric Verification

The overall slenderness ratio is modified by using the LRFD empirical, Bleich's approximate analytical, and the proposed analytical equation. Then, using LRFD method, the buckling loads of built-up struts are calculated using the modi-

Table 1.
Modified Strength of Built-Up Struts According to LRFD Empirical, Bleich's Approximate Analytical, and Proposed Exact Analytical Equations for Small Slenderness Ratio of 50

1	2	3	4	5	6	7	8	9
$\frac{a}{r_{Min}}$ (b)		$\left(\frac{KL}{r}\right)_{Modified}$			$\frac{(P_{cr})_{AISC}}{(P_{cr})_{LRFD}}$ (a,f,g)	$\frac{(P_{cr})_{Modified}}{(P_{cr})_{Unmodified}}$ (h)		
	α	LRFD(c)	Bleich(d)	Proposed(e)		LRFD(g)	Bleich(g)	Proposed(g)
	0.0			50.0				1.00
	1.0			59.4				.947
	2.0			64.3				.917
	3.0			65.9				.907
50.0	4.0	50.0	67.5	66.6	1.05	1.0	.897	.903
	5.0			66.9				.901
	6.0			67.0				.900
	7.0			67.1				.900

a. F_y = 36 ksi
b. r_{ib} and r_{Min} are assumed equal, like in double channel
c. LRFD empirical Eq. 15
d. Bleich's approximate analytical Eq. 5
e. Proposed exact analytical Eq. 5
f. AISC method using KL/r = 50, without using factor of safety
g. LRFD[1] method using corresponding modified KL/r, without using the resistance factor
h. Unmodified strength calculated from LRFD using KL/r = 50

fied slenderness ratio. Also, the buckling load is calculated by AISC method using the unmodified overall slenderness ratio in order to evaluate the AISC-ASD approach which does not consider any strength modification for built-up struts.

The calculations are performed for three different unmodified overall slenderness ratios of 50, 100, and 150, which are representative of small, medium, and large overall slenderness ratios, respectively. Individual slenderness ratio of 50 is investigated for overall slenderness ratio of 50 while individual slenderness ratios of 50, and 100 are investigated for overall slenderness ratios of 100 and 150. The results are presented in the following three sections.

Unmodified overall slenderness ratio of 50

The numerical results for the case of unmodified overall slenderness ratio of 50 are presented in Table 1. Column 6 of the table shows that the AISC ASD Specification[2] overestimates the strength only by 5% since it does not consider the shear flexibility of built-up struts. The overestimation is not significant due to small individual slenderness ratio of 50.0. The results are graphically presented in Fig. 6. The figure indicates that, regardless of the value of separation ratio, LRFD empirical criterion considers no modification since the individual slenderness ratio is not greater than 50. On the other extreme, regardless of the value of separation ratio, Bleich's approximate equation results in a strength modification of almost 10%. Bleich's estimation may be too conservative for the case of small separation ratio since shear flexibility is not considerable due to the closeness of the two individual components. However, the proposed analytical criterion results in a more reasonable strength modification which is identical to LRFD results for $\alpha = 0$ and asymptotically approaches Bleich's result as separation ratio increases.

Unmodified overall slenderness ratio of 100

The numerical results for the case of unmodified overall slenderness ratio of 100 are presented in Table 2. Column 6 of the table shows that the AISC ASD Specification[2] overestimates the strength by 16% for the case of individual slenderness ratio of 50. The overestimation is as high as 32% for the case of large individual slenderness ratio of 100. The results are graphically presented in Fig. 7. Figure 7a shows that the LRFD criterion does not result in any strength

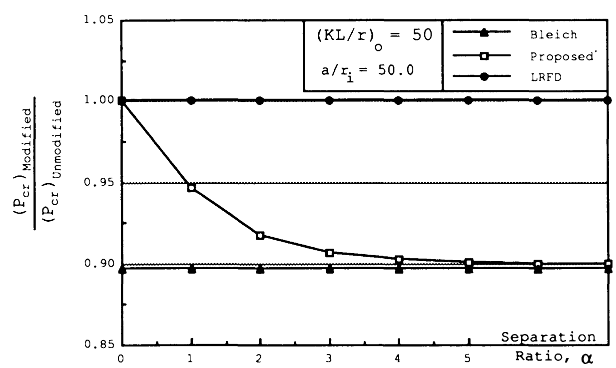

Fig. 6. Comparisons of the calculated buckling load according to LRFD empirical, Bleich's approximate analytical, and proposed analytical equations for unmodified slenderness ratio of 50.0.

Table 2. Modified Strength of Built-Up Struts According to LRFD Empirical, Bleich's Approximate Analytical, and Proposed Exact Analytical Equations for Medium Slenderness Ratio of 100								
1	2	3	4	5	6	7	8	9
		$\left(\frac{KL}{r}\right)_{Modified}$				$\frac{(P_{cr})_{Modified}}{(P_{cr})_{Unmodified}}$		
$\frac{a}{r_{Min}}$	α	LRFD	Bleich	Proposed	$\frac{(P_{cr})_{AISC}}{(P_{cr})_{LRFD}}$	LRFD	Bleich	Proposed
50.0	0.0	100.0	109.8	100.0	1.16	1.00	.897	1.00
	1.0			105.0				.947
	2.0			107.9				.917
	3.0			108.8				.907
	4.0			109.2				.903
	5.0			109.4				.901
	6.0			109.5				.900
	7.0			109.6				.900
100.0	0.0	111.8	135.0	100.0	1.32	.877	.648	1.00
	1.0			118.7				.806
	2.0			128.7				.708
	3.0			131.8				.678
	4.0			133.1				.666
	5.0			133.7				.660
	6.0			134.1				.657
	7.0			134.3				.654

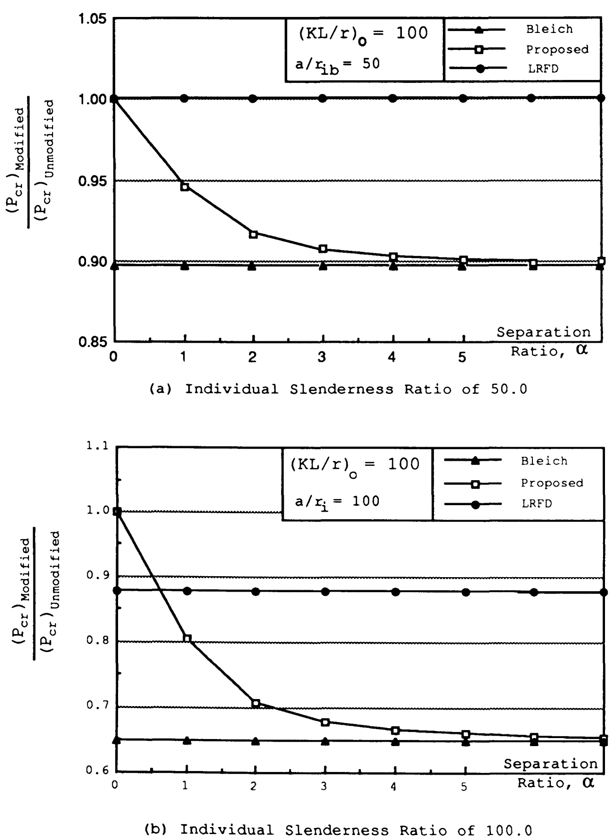

Fig. 7. Comparisons of the calculated buckling load according to LRFD empirical, Bleich's approximate analytical, and proposed analytical equations for unmodified slenderness ratio of 100.0.

modification, regardless of the value of separation ratio. However, Fig. 7b shows that LRFD criterion results in a constant 12% modification of strength, but again it does not change with separation ratio. In contrast, the proposed analytical Eq. 14 causes a 0% strength modification for $\alpha = 0$, with decreasing strength as the separation between the two components increases. At large α, the proposed analytical Eq. 14 and Bleich's approximation give an identical result of 35% strength modification.

Unmodified overall slenderness ratio of 150

Numerical results for the case of unmodified overall slenderness ratio of 150 are presented in Table 3. Column 6 of the table shows that the AISC ASD Specification[2] overestimates the strength by 14% for individual slenderness ratio of 50. The overestimation is as high as 27% in case of large individual slendemess ratio of 100. The results are graphically presented in Fig. 8 and the trend is similar to that for the case of unmodified overall slenderness ratio of 100.

Conclusions from the Parametric Study

It can be concluded that the proposed analytical equation results in 0% strength modification when no shear flexibility is involved, i.e., $\alpha = 0$. As the separation ratio increases, the strength drop is rather steep in the beginning. The modified strength quickly settles down near the value given by Bleich's approximate equation.

Experimental Verification

To further verify the proposed equation, test results from

Table 3. Modified Strength of Built-Up Struts According to LRFD Empirical, Bleich's Approximate Analytical, and Proposed Exact Analytical Equations for Large Slenderness Ratio of 150

1	2	3	4	5	6	7	8	9
$\frac{a}{r_{Min}}$	α	$\left(\frac{KL}{r}\right)_{Modified}$ LRFD	$\left(\frac{KL}{r}\right)_{Modified}$ Bleich	$\left(\frac{KL}{r}\right)_{Modified}$ Proposed	$\frac{(P_{cr})_{AISC}}{(P_{cr})_{LRFD}}$	$\frac{(P_{cr})_{Modified}}{(P_{cr})_{Unmodified}}$ LRFD	$\frac{(P_{cr})_{Modified}}{(P_{cr})_{Unmodified}}$ Bleich	$\frac{(P_{cr})_{Modified}}{(P_{cr})_{Unmodified}}$ Proposed
50.0	0.0	150.0	156.7	150.0	1.14	1.00	.916	1.00
	1.0			153.4				.956
	2.0			155.4				.932
	3.0			156.0				.924
	4.0			156.3				.921
	5.0			156.4				.919
	6.0			156.5				.919
	7.0			156.6				.918
100.0	0.0	158.1	175.3	150.0	1.27	.900	.732	1.00
	1.0			163.1				.846
	2.0			170.5				.774
	3.0			172.9				.753
	4.0			173.0				.745
	5.0			174.3				.741
	6.0			174.6				.738
	7.0			174.7				.737

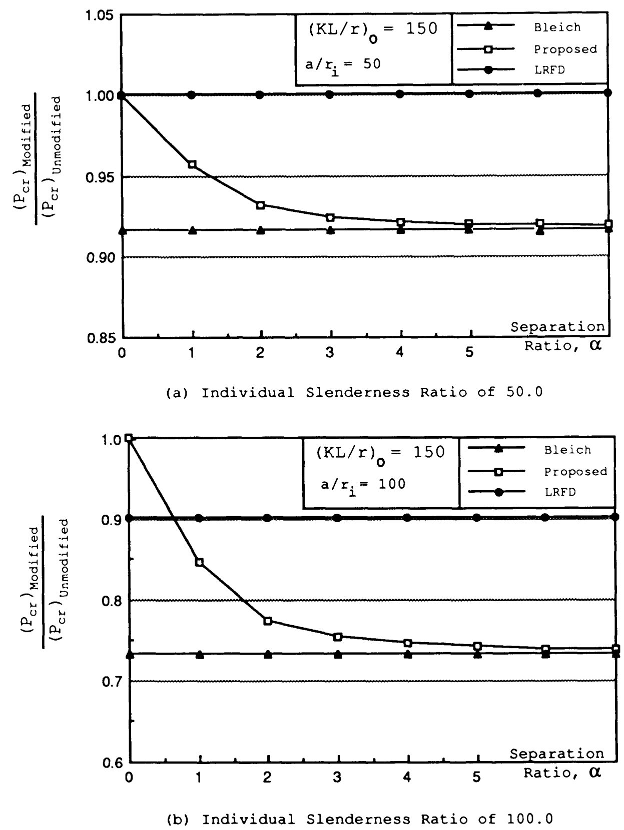

Fig. 8. Comparisons of the calculated buckling load according to LRFD empirical, Bleich's approximate analytical, and proposed analytical equations for unmodified slenderness ratio of 150.0.

another study by the authors[6,7] are compared with the calculated buckling load according to the proposed analytical equation. Only specimens with individual slenderness ratio greater than 50 are included in this comparison. The overall slenderness ratio of the test specimens was modified according to LRFD empirical and the proposed analytical Eq. 14. As mentioned earlier, r_{ib} is used in Eq. 14 whereas r_{min} is used in Eq. 15. The numerical values are presented in Columns 7 and 8 of Table 4. Using the LRFD method, the buckling load was calculated based on the modified slenderness ratio. Columns 10, 11, and 12 present the ratio of the buckling load from test result, LRFD modification, and the proposed modification to the unmodified buckling load, respectively.

The same results are presented graphically in Figs. 9 through 12. In general, the figures indicate that the buckling load calculated from both methods are close to the one measured from the test. However, it should be noted that these specimens were part of a study in which buckling strength of the built-up struts was not the center of focus. Therefore, specimens with different separation ratio were not tested to further verify the proposed analytical equation and examine its differences with LRFD empirical equation for a wide range of separation ratio which would cover stitched struts as well as battened columns. This could be the subject of future study.

CONCLUSIONS AND DESIGN RECOMMENDATIONS

1. Built-up struts have shear flexibility which reduces the overall strength of the member. Equation 14 is proposed

Table 4.
Comparisons of the Measured Buckling Load with the Calculated Buckling Load According to LRFD Empirical, Bleich's Approximate Analytical, and Proposed Exact Analytical Equations

1	2	3	4	5	6	7	8	9	10
	F_y [(a)]	$\left(\frac{KL}{r_y}\right)_o$	$\frac{a}{r_{ib}}$	$\frac{a}{r_{min}}$	$\left(\frac{KL}{r_y}\right)_{Modified}$		$(P_{cr})_{Modified} / (P_{cr})_{Unmodified}$ [(b)]		
Spec	(ksi)				Proposed	LRFD	Test	LRFD	Proposed
AB1	46.4	113	74.8	101.1	123.5	124.0	0.899	0.837	0.843
AB2	47.0		50.3	68.0	117.9	114.4	0.962	0.978	0.924
AXH13	43.7	83	63.0	63.0	97.5	84.0	0.939	0.989	0.834
AXH14	43.7		93.7	93.7	112.5	93.8	0.686	0.885	0.670

a. Measured F_y used in calculation
b. LRFD[1] method without using the resistance factor

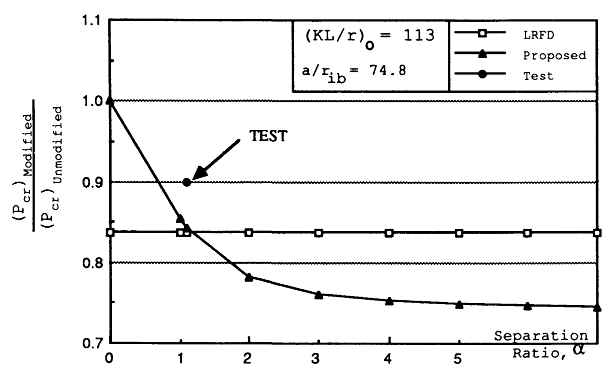

Fig. 9. Comparisons of the calculated buckling load according to LRFD empirical and the proposed analytical equations with test result for Specimen AB1.

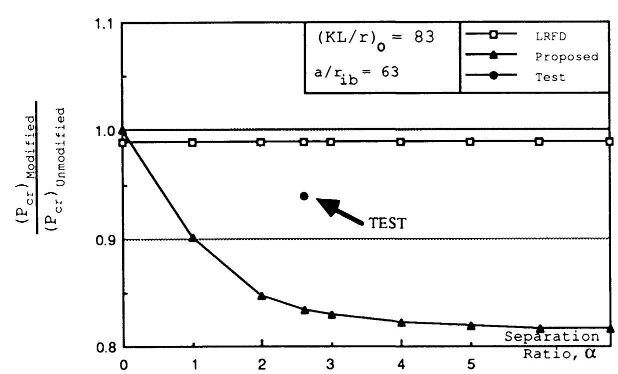

Fig. 11. Comparisons of the calculated buckling load according to LRFD empirical and the proposed analytical equations with test result for Specimen AXH13.

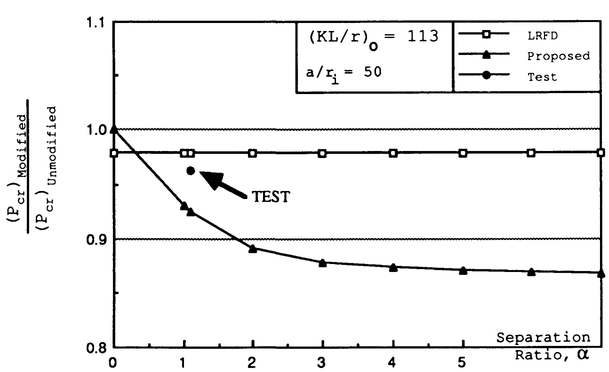

Fig. 10. Comparisons of the calculated buckling load according to LRFD empirical and the proposed analytical equations with test result for Specimen AB2.

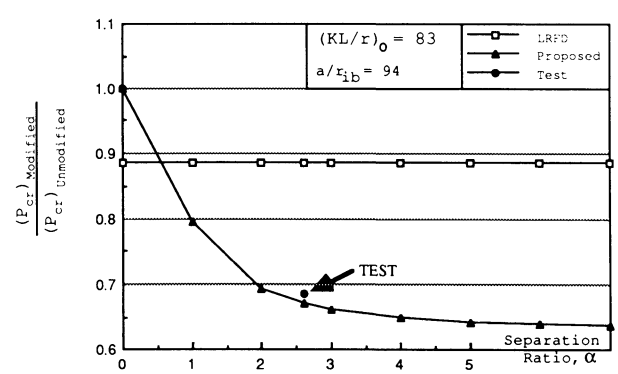

Fig. 12. Comparisons of the calculated buckling load according to LRFD empirical and the proposed analytical equations with test result for Specimen AXH14.

for use in practice to modify the overall slenderness ratio to take into account the detrimental effect of shear flexibility. Unlike the LRFD empirical equation, the proposed analytical equation considers the effect of separation ratio.

2. The proposed analytical equation results in 0% strength modification when no shear flexibility is involved (i.e., when $\alpha = 0$) and it gives results identical to those from Bleich's approximation at the other extreme of large separation between individual components. For intermediate separation ratios, the proposed equation results in a strength modification which falls in between the two extremes. Test result verify the validity of the proposed analytical equation.

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REFERENCES

1. American Institute of Steel Construction, "Load and Resistance Factor Design," First Edition, Chicago, Illinois, 1986.
2. American Institute of Steel Construction, "Steel Construction Manual," 8th ed., Chicago, Illinois, 1980.
3. Zandonini, R., "Stability of Compact Built-Up Struts: Experimental Investigation and Numerical Simulation," *Cotrouzione Mettalliche,* November 4, 1985.
4. Astaneh-Asl, A., Goel, S. C., Hanson, R. D., "Cyclic Out-of-Plane Buckling of Double-Angle Bracing," *Journal of the Structural Division,* ASCE, Vol. 111, No. 5, May 1985.
5. Bleich, F., "Buckling Strength of Metal Structures," McGraw-Hill Book Company, 1952, pp. 176–179.
6. Aslani, F. and Goel, S. C., "Stitch Spacing and Local Buckling in Seismic Resistant Double-Angle Braces," Submitted for Publication in *Journal of the Structural Division,* ASCE, 1990.
7. Aslani, F., and Goel, S. C., "Experimental and Analytical Study of the Inelastic Behavior of Double Angle Bracing Members Under Cyclic Loading," Report No. UMCE 89-5, Department of Civil Engineering, University of Michigan, Ann Arbor, Michigan, February 1989.