Unbraced Frames with Semi-Rigid Composite Connections

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The benefits of semi-rigid connections are well known[1,2,3] and much has been written about their use in braced frames[4,5] and in Type 2 construction.[6,7] One of the reasons these methods are not being used frequently by designers is that most semi-rigid connections are highly nonlinear, and analysis of the behavior of frames using them is difficult. One type of connection which may be able to overcome this difficulty is the semi-rigid composite connection. This is a connection made from a typical pinned end connection (e.g., seat angle and web clips or double web angles) and continuous slab reinforcement across the connection. Tests on the connection with seat angle (Fig. 1) by the authors [8,9] have shown this connection type has a high degree of linearity in the service load region and that its ultimate capacity is easy to predict. This paper describes a design procedure for unbraced frames utilizing semi-rigid composite connections such as those described above. Two design examples, one for a four-story structure and one for a ten-story structure, and comparisons of their behavior with that of rigidly designed frames are included.

PRELIMINARY DESIGN METHOD

A preliminary design method for unbraced frames utilizing semi-rigid composite connections is described in this section. The procedure is based partially on a method suggested by the authors[10] for braced frames and consists of six steps. The steps are:

Step 1: Determine initial beam sizes and connection details by doing a gravity analysis of the floor system, similar to that described in an earlier paper by those authors.[10] If the ratio of factored service load to factored construction load is low enough so small, or no, end restraint is required for the composite beam, detail the connections for the higher of $M_{fe}/2$ or $M_p/2$, where M_{fe} is the fixed end moment for the given loading and M_p is the plastic moment of the steel beam.

In frames in which the lateral loading is large when compared to the gravity loading, the connections will experience moment reversal and be loaded under positive moment. In this case it is desirable to detail the connection so it has similar stiffness in the positive moment region as in the negative moment region. This will require increasing the area (and especially the thickness) of the seat angle and increasing the area of the web angles. It is recommended to provide a seat angle area of 1.5 times that required for negative moments and increasing the size of the web angles by 50 percent from that required for shear. These increases are needed because the connection angles have different behavior when loaded in tension from that when loaded in compression, as illustrated in Fig. 2.

Step 2: Determine preliminary column sizes. Typically for frames constructed with semi-rigid connections, stiffness under service load conditions is going to control the column sizes for all but the uppermost floors. For this reason it is best to determine initial column sizes based on a stiffness calculation. One method for obtaining approximate column moments of inertia for semi-rigid frames is as follows:

The lateral deflection of one floor (assume this is equal to the drift limit times the floor height) can be expressed by:

$$\Delta = PH^3/\alpha E\Sigma I_{col} \tag{1}$$

where

P = the total lateral load on the floor
H = the story height
α = varies between 3 and 12
E = Young's modulus
ΣI_{col} = the sum of the moments of inertia for all of the columns on that floor

if the base of the floor is assumed to be fixed. In Eq. 1 α equals 12 if the top of the columns do not rotate with respect to the floor and 3 if the tops are free to rotate (see Fig. 3). For most buildings the value of α will be between these two extremes, $\alpha = 12$ would be the case if the connections and beams are rigid and $\alpha = 3$ when the connections and beams provide no restraint for the top of the columns. To determine α for buildings with restraint provided to the column tops by beams and connections with intermediate rigidity a convenient equation which provides good results is:

$$\alpha = 3 + 9e^{-GF} \tag{2}$$

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where

GF = $(E\Sigma I_{col}/H)/(\Sigma\beta_\beta EI_{cb}/L)$
ΣI_{col} = *sum of I* for all columns on the floor
I_{cb} = moment of inertia of the composite beam
β_b = connection stiffness factor, defined by:

$$\beta_b = 1/(1 + 2EI_{cb}/LC) \quad (3)$$

C = connection tangent stiffness

The expression for α requires $E\Sigma I_{col}/H$, which is still an unknown. Therefore, solving Eq. 1 for $E\Sigma I_{col}/H$ will require an iterative technique, or else, Eq. 2 can be substituted into Eq. 1 and $E\Sigma I_{col}$ can be obtained numerically.

If the columns from the stiffness analysis have much larger moments of inertia than the beams determined from the gravity load analysis of the floor, it may be desirable to increase the size of the beams to have a moment of inertia of approximately $0.5I_{col}$ and the strength of the connections so that $M_{u,e}$ remains about $0.5M_p$ of the beam in order to improve the stiffness of the frame for lateral loads. If the beams or connections are too flexible the columns will behave independently, producing large drifts, instead of as a semi-rigid frame.

Step 3: Check the columns chosen in Step 2 for the loading condition $1.2D + 0.5L + 1.3W$ (or $1.3E$). In general the strength of the composite beam at mid-span will be much larger than the strength of the connections, so the mechanism which will govern lateral load carrying capacity is the sway mechanism. The preliminary column section for lateral load should be determined by assuming a single story sway mechanism, which will give a required ΣM_p for the columns. The value of ΣM_p does not include any contribution from P-Δ moments, which are generally significant in frames with semi-rigid connections, so it is recommended to add to it the moment caused by the total gravity load of the floor acting at an eccentricity equal to the desired drift limit:

$$M_{P\text{-}\Delta} = \Sigma P \times \Delta_{all} \quad (4)$$

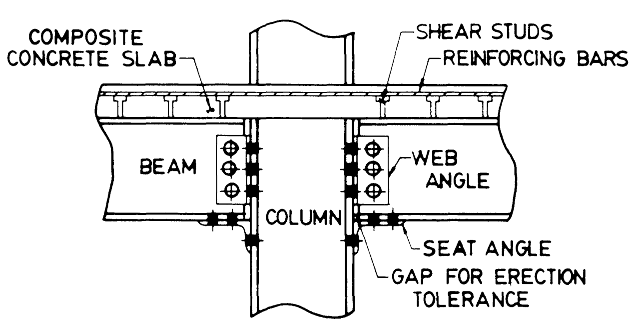

Fig. 1. Typical semi-rigid composite connection.

where

$M_{P\text{-}\Delta}$ = the moment due to the P-Δ effect
ΣP = the total gravity load acting at this level
Δ_{all} = the allowable story drift

This summation of moments must be resisted by the moments at the top and bottom of the columns of the floor in consideration.

It is not necessary to apportion this moment evenly among all of the columns and the designer may choose to assign more of it to the interior columns than to the exterior ones. In general the column end moments at the base of a building with fixed bases will be much larger than the moment at the top end of these columns, that is, the inflection point for the base columns will not be in the middle of the story, but rather, they will be closer to the top of the floor. For this reason it is recommended that a conservative assumption for preliminary design is to assume the base moment is equal to three times the top moment for columns rigidly attached to a foundation. This is the same as assuming the

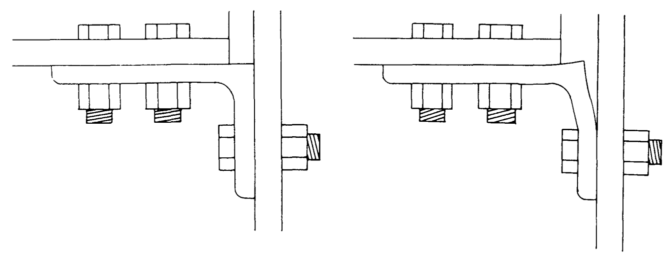

Fig. 2. Behavior of the connection angles when loaded in compression or tension.

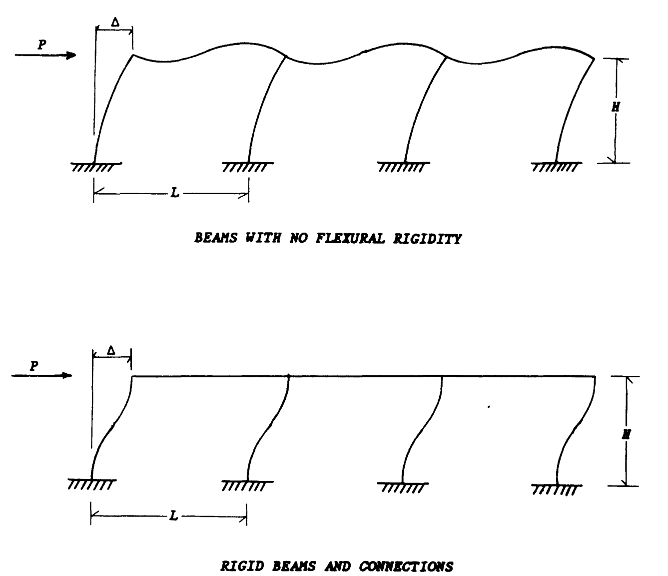

Fig. 3. Effect of beam rigidity on story sway mechanism.

inflection point is at $0.75H$ from the base of the columns. For other floors the moments at the two ends of the columns will not necessarily be equal, and it is recommended to assume the higher column end moment is equal to 1.67 times the smaller end moment. This is especially true at floors where there is a change in column sections or beam sections, as the relative rigidity of one end will be greater than the other. Using the resulting values as M_{col} and the axial load determined from the applied gravity loads (using $1.2D + 0.5L$) plus the axial load determined from the applied lateral loading, assigned to the columns assuming pins at the bases, a column can be chosen based on:

$$M_{col} = 1.18M_p(1 - P/P_y) \quad (5)$$

and the interaction equations for combined moment and axial load (Eq. H-1a and H1-1b of the LRFD Specification):

for $P_u/\phi_c P_n \geq 0.2$,

$$\frac{P_u^2}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \quad (6)$$

for $P_u/\phi_c P_n < 0.2$,

$$\frac{P_{u2}}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \quad (7)$$

If the columns chosen for stiffness are not adequate for the combined gravity and lateral loading, stronger sections must be chosen which will meet the required strength criteria.

Step 4: Check the columns chosen from Steps 2 and 3 for the loading condition $1.2D + 1.6L$ using K-factors from the sway permitted alignment charts of the LRFD Manual. The G factors should be determined from:

$$G = (\Sigma EI_{col}/H)/(\Sigma \beta_b EI_{cb}/L) \quad (8)$$

where the summations are for all beams and columns framing into the joint at either the top or bottom of the column.

The moments at the interior columns are zero, while the moments at the exterior columns should be assumed as $M_e/2$, where M_e is the moment for which the connection is designed. In general the flexibility of the exterior columns will reduce this moment, but this is a good value for initial design.

Step 5: Once the preliminary member sizes have been determined a more exact analysis of the frame should be done. This will require taking into account second order moments and semi-rigid connections.

Semi-rigid composite frames have much better behavior than other semi-rigid frames when subjected to combined gravity and lateral loads. Generally semi-rigid connections are very nonlinear, but semi-rigid composite connections have nearly linear behavior in the service region, and the complete moment rotation curve can be modelled quite accurately with a tri-linear curve. This tri-linear curve is obtained from an exponential moment rotation curve developed for this type of connection using a finite element model developed by Lin[11] and implemented by Kulkarni[12] (see Fig. 4). The exponential curve is:

$$M(\theta) = C1[1 - e^{-C2 \times \theta}] + C3 \times \theta \quad (9)$$

where

$C1 = A_r F_{yr}(d + Y2)$
$C2 = 32.9(A_{sl}/A_r)^{0.15}(d + Y2)$
$C3 = 24F_{ysl}A_{sl}(d + Y2)$
θ = rotation in radians

The tri-linear curve can be obtained from this exponential curve by assuming the first linear portion has a slope equal to 0.8 times the initial slope of the exponential curve, that is $Sl = 0.8(C1 \times C2 + C3)$. This linear portion of the curve continues until it intersects the exponential curve. This will occur when:

$$Sl \times \theta 1 = C1[1 - e^{-C2\theta 1}] + C3 \times \theta 1 \quad (10)$$

where $\theta 1$ is the rotation at the intersection and $Sl \times \theta 1$ is the moment at the intersection. Finding this point will involve solving Eq. 10, which is nonlinear; therefore, the solution must be obtained numerically or graphically. The second linear portion goes from this point until the point where the exponential term has reached 10 percent of its initial value, this occurs when $e^{-C\theta 2} = 0.1$, or $\theta 2 = ln(0.1)/-C2$. The moment at this point is $0.9C1 + C3 \times \theta 2$. The third linear portion extends from point $(\theta 2, M2)$ to $\theta 3 = 0.02$ and $M3 = C1 + 0.02C3$. The value of 0.02 for $\theta 3$ was chosen as a limit because it is unlikely that a connection will be able to obtain this rotation in a frame which is serviceable. The tests conducted on these types of connections have shown they have sufficient ductility to easily surpass this rotation.

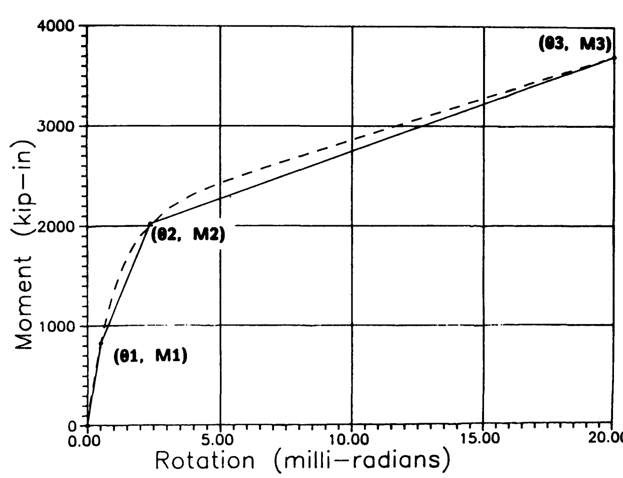

Fig. 4. Exponential moment-rotation curve with tri-linear approximation.

The calculation of drift for serviceability requires a second order analysis because of the semi-rigid nature of the connections. Usually for frames less than ten stories the P-Δ moments are small, and many designers do not perform second order analysis for these structures. When the connections are semi-rigid, however, the drifts associated with even these low-rise structures can be substantial and the P-Δ moments can become significant. For this reason it is recommended that a second order analysis be used for all structures with semi-rigid connections.

If this second order analysis results in combinations of moment and axial load for the columns and mid-span moment for the beams less than those assumed for the preliminary design, the frame is adequate for strength. If the lateral deflection for each story of the frame is less than the story drift limit (and if the top story drift is less than the frame drift limit, if these are different) the frame is adequate for lateral stiffness.

A check should also be made here for the adequacy of the connections. If any connection rotation is greater than 0.020 radians, the connection should be stiffened. It is very unlikely a frame which meets all serviceability criteria could have connection rotations this large. If the moment at any connection is greater than the assumed moment from Step 1, that connection must be strengthened. If the frame is adequate, the final design of the connections can be done. The number of bolts required can be determined from the largest moment at the connection, as the stiffness of the connection, prior to slip, is not affected by the number of bolts.

If the preliminary design proves to be too flexible, or fails due to stability the frame must be stiffened. This can be done in several ways. The least expensive way is to increase the stiffness of the connections by increasing the size of the connection angles and the amount of reinforcing which is continuous across the slab. If the connections have reached their maximum size and the drift is still too large then the framing members must be strengthened. If the depth of the beams is increased this will also increase the strength and stiffness of the connections, or else the stiffness of the columns can be increased, either by using deeper columns of the same nominal weight, or by using heavier columns of the same depth.

Step 6: The analysis procedure described in Step 5 is very lengthy, and requires a large amount of computer time. An alternative, although less exact method, is to assume the connection behaves as a linear spring with stiffness equal to the secant stiffness to point ($\theta 2$, $M2$). This rotation is generally greater than the actual connection rotation, so this will provide conservative values of frame stiffness. Another way to make the analysis easier to eliminate the P-Δ terms from the stiffness matrix. These two assumptions reduce the problem to a linear analysis. However, care must be taken to

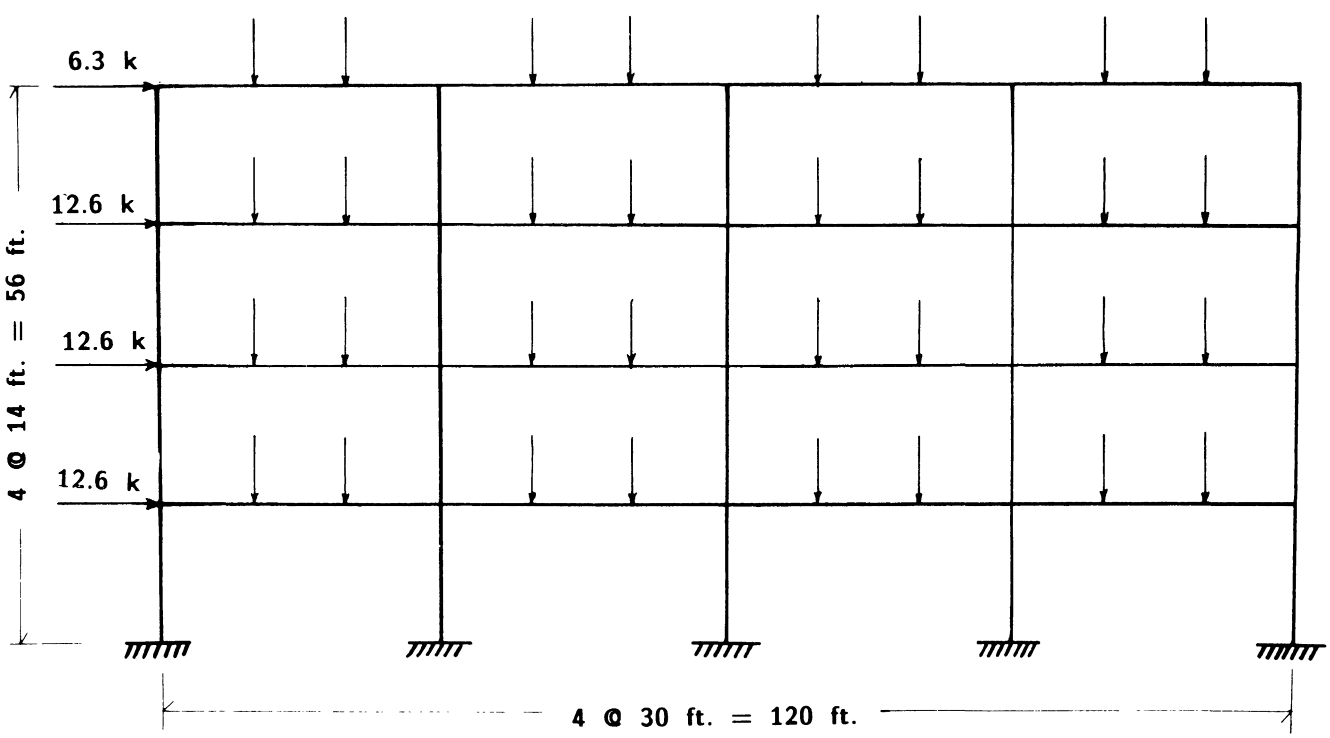

Fig. 5. Elevation of 4-story building of Example 1.

ensure the results are conservative, and a degree of engineering judgment is required.

DESIGN EXAMPLE 1

To illustrate how this method works for a frame, the 4-story frame of Fig. 5 will be designed using W18 beams and W10 columns. The girders have beams framing into them at the third points, and it will be assumed that all of the floor load is transferred to the girders from these beams. The frames are spaced at 30 ft, have a bay spacing of 30 ft, and a story height of 14 ft. The floor slab is 5 in. thick with 2 in. metal deck, using 3000 psi lightweight (110 pcf) concrete. Use the same columns for the first and second floors and for the third and fourth floors. The wind loading is assumed to be constant over the height of the building, and is reduced to point loads at the floor levels. The beams, columns, and connection angles are A36 steel and the reinforcing bars are Grade 60. The drift limit for any floor of the structure is $H/400$.

Loading: floor dead load = 50 psf
floor live load = 50 psf
roof dead load = 50 psf
roof live load = 50 psf
lateral load = 30 psf
construction load = 20 psf

Reducing these to point loads gives:

P_D = 15.0 kips
P_{Df} = 18.4 kips
P_L = 15.0 kips
P_{Lf} = 24.0 kips
P_{Lw} = 7.5 kips
P_C = 6.0 kips
P_{Cf} = 9.6 kips
W_f = 12.6 kips
W_r = 6.3 kips

Step 1: A gravity load analysis requires a W18×55 for the factored construction loading. This beam is adequate for the factored service loading without any end moment, but provides end restraint of $M_{u,e} = M_p/2 = 2016$ kip-in. This requires:

A_{sl} = 5.07 in.^{2}—use ⅝ in. thick × 8 in. long—A_{sl} = 5.0 in.^{2}
A_r = 1.52 in.^{2}—use 8 #4 bars—A_r = 1.6 in.^{2}

Calculate the beam end moments and rotations for P_{Lf} and for P_{Lw}:

(a) For 1.6 live load:

$M_{u,e}$ = 1471 kip-in.
θ_{srf} = 0.001183 Rad.

which gives:

βb = 0.74

(b) For 0.5 live load:

$M_{u,e}$ = 491.5 kip-in.
θ_{srf} = 0.000286 Rad.

which gives:

β_b = 0.85

Step 2: Get preliminary column sizes by designing for drift. Let $Sl = E\Sigma I_{col}/H$ and $S2 = \Sigma\beta_b EI_{cb}/L$. Substituting these values into Eqs. 1 and 2 and substituting Eq. 2 into Eq. 1 gives:

$$\Delta = PH_2/[(3 + 9_e^{-Sl/S2})Sl$$

for this building and loading, at the first floor:

P = (3 × 12.6 + 6.3) = 44.1 kips
H = 168 in.
$S2$ = 4 × 0.85 × 29000 × 1630/360 = 503,000 kip-in.
Δ = 168/400 = 0.42 in.

Solving numerically gives:

Sl = 438,000 kip-in. and ΣI_{col} = 2535 in.^{4}

For the second floor columns a similar analysis gives:

Sl = 250,000 kip-in.

For these columns, however, the bases are not fixed so Eq. 1 will yield a value of ΣI_{col} which is too low. An analysis similar to the one for the base columns with differing rigidity at the tops could be done for this partial rigidity at the bottom, but for preliminary design this is not necessary, and increasing the required column stiffness by a factor of 1.5 seems to give good results. This gives ΣI_{col} = 2169 in.^{4} Therefore for this example the first floor governs.

Assuming the interior columns are 1.5 times as stiff as the exterior columns gives:

I_{Int} = 585 in.^{4}
I_{Ext} = 390 in.^{4}

A similar analysis for the third floor gives:

I_{Int} = 255 in.^{4}
I_{Ext} = 170 in.^{4}

The required columns for drift are:

Base — Int. W10×100
— Ext. W10×68
Third — Int. W10×49
— Ext. W10×33

Step 3: Calculate column axial loads and moments for gravity plus lateral load case.

(a) The axial loads from gravity loading:

Third — Int. P = 153 kips
— Ext. P = 76.5 kips
Base — Int. P = 306 kips
— Ext. P = 153 kips

To determine the axial load on the column assume the overturning moment of the building at that level is entirely resisted by axial loads in the columns. Assume the load in each column is proportional to the distance the column is from the center of the building (cantilever method). For this frame, this yields $P_{Ext} = 2P_{Int}$, and the center column has no additional axial load. Adding these axial loads to those from the gravity analysis gives:

Third — Int. P = 155 kips
— Ext. P = 79.5 kips
Base — Int. P = 312 kips
— Ext. P = 165 kips

(b) Moments: For bottom floor columns assume first floor sway mechanism:

$$\Sigma M = 1.3 \times (3 \times 12.6 + 6.3) \times 168 = 9631 \text{ kip-in.}$$
$$M_{P\Delta} = (2 \times 153 + 3 \times 306)168/400 = 313 \text{ kip-in.}$$

Assume the moment in the interior columns is 1.5 times the moment in the exterior columns, and the base moment is three times the bottom moment. This yields:

$$M_{Int} = 1721 \text{ kip-in.}$$
$$M_{Ext} = 1147 \text{ kip-in.}$$

For the third floor columns assume single floor sway mechanism:

$$\Sigma M = 1.3(12.6 + 6.3)168 = 4128 \text{ kip-in.}$$
$$M_{P\Delta} = (2 \times 76.5 + 3 \times 153)168/400 = 257 \text{ kip-in.}$$

Assume the moment in the interior columns is 1.5 times the moment in the exterior columns, and the moment at one end of the columns is 1.67 times the moment at the other end. This yield:

$$M_{Int} = 607 \text{ kip-in.}$$
$$M_{Ext} = 405 \text{ kip-in.}$$

Check the columns required for the drift analysis using Eq. 6. This procedure will be shown for the top exterior column as an example.

For W10×33:

$$A = 9.71 \text{ in.}^2$$
$$I = 170 \text{ in.}^4$$
$$Z = 38.8 \text{ in.}^3$$

The loading is: P = 79.5 kips $\quad M$ = 405 kip-in.

From Eq. 5:

$$M_{col} = 1.18\times8.8\times36[1-79.5/(9.71\times36)]=1273 \text{ kip-in.}$$
$$G_{top} = (2 \times 170/168)/(0.85 \times 1630/360) = 0.53$$
$$G_{bot} = ([170 + 394]/168)/0.85 \times 1630/360) = 0..87$$

which yields K = 1.17, and from the LRFD Manual $\phi_c P_{cr}$ = 264 kips.

Eq. 6 becomes:

$$79.5/264 + (8/9)405/(0.9 \times 1273) = 0.62 < 1.0 \quad \textbf{OK}$$

Similarly, all of the other columns are OK for this loading.

Column	P	M
Third — Int.	252 kips	0.0
— Ext.	126 kips	736 kip-in.
Base — Int.	504 kips	0.0
— Ext.	252 kips	736 kip-in.

Checking the columns from the drift analysis for this loading using Eq. 6, shows the third floor exterior column needs to be a W10×39, the rest of the columns are OK.

Step 5: The results from a second order analysis of the frame taking into account the semi-rigid connections gives the following lateral deflections:

Top floor: Δ = 1.247 in. drift = 0.128 in.
Third floor: Δ = 1.119 in. drift = 0.360 in.
Second floor: Δ = 0.759 in. drift = 0.390 in.
First floor: Δ = 0.369 in.

All of these are less than the allowable drift of 0.42 in. Therefore the frame is slightly too stiff and lighter columns can be used. For a second iteration try lighter interior columns. Try:

Base — Int. W10×88
Third — Int. W10×45

These columns lead to the following deflections:

Top floor: Δ = 1.328 in. drift = 0.134 in.
Third floor: Δ = 1.194 in. drift = 0.378 in.
Second floor: Δ = 0.816 in. drift = 0.415 in.
First floor: Δ = 0.401 in.

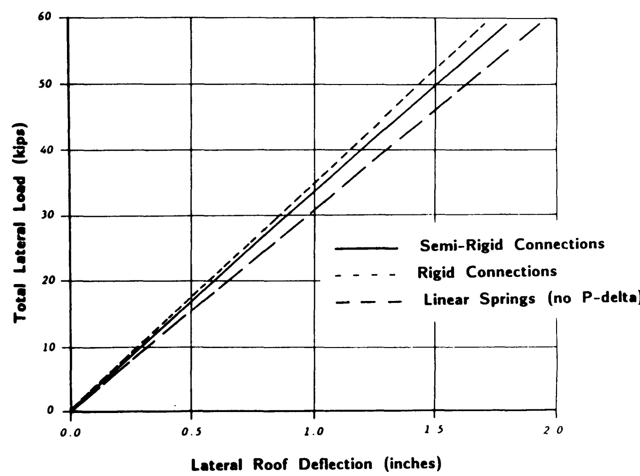

Fig. 6. Load-deflection curve for the top of the 4-story building.

The value for drift between the first and second floor is $L/405$, which is slightly less than the allowable $L/400$. The total drift for the frame is $L/506$, much less than the required $L/400$. The loads and moments for all of the members are less than their capacities. Figure 6 shows the load-deflection curve of this frame with semi-rigid composite connections. Also shown in the figure are the load-deflection curves for the same frame with rigid connections and the results of the simplified analysis which neglects the nonlinear nature of the moment-rotation curve and the P-Δ effects.

The maximum connection rotation achieved under $1.3W$ was 0.015 radians, much less than the 0.020 radian rotation limit for this type of connection. This value of rotation is typical for frames designed with this procedure, because larger rotations will result in excessive lateral drifts. The final connection design for this example is given in Appendix B.

Step 6: The analysis of this frame using linear springs and ignoring second order (P-Δ) effects gives the following results:

Top floor:	Δ = 1.449 in.	drift = 0.153 in.
Third floor:	Δ = 1.296 in.	drift = 0.416 in.
Second floor:	Δ = 0.880 in.	drift = 0.460 in.
First floor:	Δ = 0.420 in.	

This analysis gives large deflections and story drifts for each floor than the more exact method, and the drift of the second floor is greater than the allowable 0.420 in. If this method was used for the design of this frame, the frame would have to be strengthened slightly to reduce the second story drift. Therefore, for this example, using the simplified method of analysis would result in a conservative design.

COMPARISON WITH RIGID FRAME DESIGN

If the same sections are used with rigid connections the resulting deflections are:

Top floor:	Δ = 1.104 in.	drift = 0.113 in.
Third floor:	Δ = 0.991 in.	drift = 0.320 in.
Second floor:	Δ = 0.671 in.	drift = 0.323 in.
First floor:	Δ = 0.348 in.	

The use of rigid connections results in a maximum story drift of $L/520$, and a total building drift of $L/609$. The forces in the members are very similar for both cases, with slightly smaller column moments in the rigid frame due to decreased P-Δ moments.

Alternatively, the frame can be redesigned with rigid connections so the drifts are comparable to those with semi-rigid connections. If the column sizes are kept the same and the beam sizes are reduced until the deflections are similar, the resulting composite beams are W18×35, which are approximately 33 percent less stiff than those required using semi-rigid connections. This is as expected, because the effect of the semi-rigid composite connections in this example is to reduce the effective beam stiffness by about 33 percent. The weight savings (36 percent) for the rigidly connected frame will be offset by the added cost of the fully welded rigid connections. Also the rigid connections will generally require column stiffeners, while the semi-rigid composite connections generally do not. If the frame is to be designed without composite beams, and with rigid connections, the required beam would be a W18×76 or a W21×62 or a W24×55.

DESIGN FOR CYCLIC LOADS

If the semi-rigid composite frame is subjected to cyclic load at a level equal to $1.3W$ and then checked at a level of load equal to $1.0W$ the deflections are:

Top floor:	Δ = 1.336 in.	drift = 0.134 in.
Third floor:	Δ = 1.202 in.	drift = 0.380 in.
Second floor:	Δ = 0.822 in.	drift = 0.418 in.
First floor:	Δ = 0.404 in.	

These are only slightly larger than those from the monotonic loading to this level, showing the excellent behavior semi-

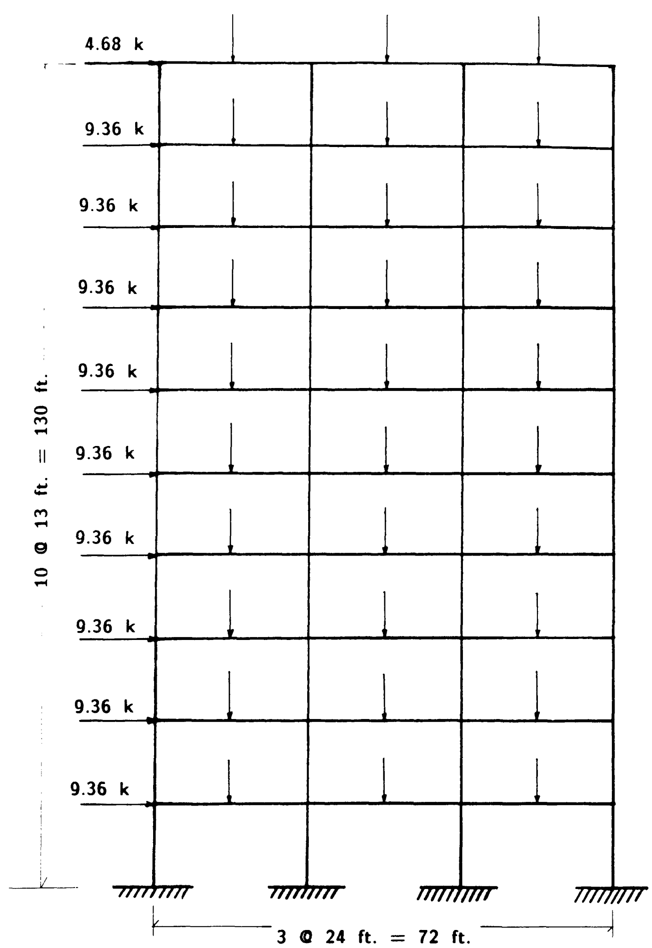

Fig. 7. Elevation of 10-story building of Example 2.

rigid composite connections have when subjected to cyclic loads.

DESIGN EXAMPLE 2

This example will briefly describe the design of the 10-story frame shown in Fig. 7. The bay length is 24 ft and the column height is 13 ft. The frame spacing is also 24 ft. The floor girders have beams framing into them at midspan, and this is assumed to be the only load acting on them. The bottom four floors have 3 in. metal deck with 3 in. of concrete cover. The other floors have 2 in. deck with 3 in. of concrete. The concrete is 3500 psi lightweight (110 pcf). Use the same columns for the first four floors and for floors 5–7 and 8–10. Use the same beams for the first four floors and for all other floors. The beams, columns, and connection angles are A36 steel, and the reinforcing bars are Grade 60. The drift limit for any floor of the structure is $H/400$.

Loading:	floor dead load	= 70 psf (bottom 4 floors
	floor dead load	= 60 psf (all other floors)
	floor live load	= 100 psf (bottom 4 floors)
	floor live load	= 40 psf
	roof live load	= 40 psf
	construction load	= 20 psf (all floors)
	lateral load	= 30 psf (uniform)

As point loads:

all floors:
CL = 5.76 kips
CLF = 9.22 kips
bottom 4 floors:
DL = 20.16 kips
DLF = 24.19 kips
LL = 28.80 kips
LLF = 46.08 kips
other floors:
DL = 17.28 kips
DLF = 20.74 kips
LL = 17.28 kips
LLF = 27.65 kips
roof:
LL = 11.52 kips
LLF = 18.43 kips
lateral:
W = 9.36 kips

Step 1: Required beam for gravity loads:

Bottom floors — W18×40
Other floors — W18×40

Step 2: Assume $\beta_b = 0.80$:

Allowable drift = 156/400 = 0.39 in.
Required moments of inertia:

Base Ext. — 2610 in.^{4}
Int. — 3910 in.^{4}

These moments of inertia are much larger than those of the beams, so it will be beneficial to increase the sizes of the beams and connections. Try using W18×55 beams (I_{cb} = 1700 – bottom floors and I_{cb} = 1570 – other floors) with $\beta_b = 0.85$:

The required column moments of inertia become:

Base	Ext.	— 2490 in.^{4}
	Int.	— 3730 in.^{4}
Mid	Ext.	— 1170 in.^{4}
	Int.	— 1760 in.^{4}
Top	Ext.	— 264 in.^{4}
	Int.	— 395 in.^{4}

Using W14 columns, the required sections are:

Base	Ext.	— W14×211
	Int.	— W14×283
Mid	Ext.	— W14×109
	Int.	— W14×159
Top	Ext.	— W14×30
	Int.	— W14×43

Steps 3 & 4: Check for strength: All columns except top exterior are OK.

This column needs to be W14×43 also.

Step 5: The results of the second order analysis for this frame taking into account the semi-rigid connections gives a maximum story drift of 0.58 in. for the second floor, which is too flexible. A warning of this outcome was the relatively large ratio of column moment of inertia to beam moment of inertia for the bottom floors. To improve this behavior

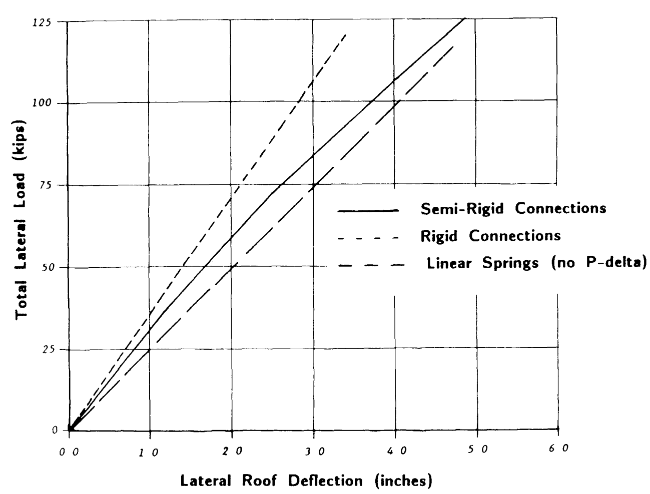

Fig. 8. Load-deflection curve for the top of the 10-story building.

try using W21×50 beams (I_{cb} = 2210 in.^{4}) for the bottom 4 floors.

The results of the second order analysis with these beams has a maximum story drift of 0.425 in. = L/367, which is slightly larger than the target L/400. The total building drift is L/484, much less than the allowed L/400, call the design OK. If the drift is unacceptable try increasing the connections, beams, or columns slightly.

The load-deflection curve for this frame is shown in Fig. 8, along with the curves for the same frame with rigid connections, and the results of the simplified analysis.

Step 6: If the frame designed in Step 5 is analyzed using the simplified method the maximum story drift is L/326, which is much more than the allowable L/400, and considerably greater than L/367 determined with the more exact analysis. For this frame using the simplified method of analysis would result in a conservative design.

COMPARISON WITH RIGID FRAME DESIGN

If the same frame is analyzed using rigid connections the maximum story drift is L/589, which is 62 percent of the drift for the semi-rigid frame. If the frame is designed so the drift is L/400, keeping the same column and reducing the beam size, the required beams are W18×50 for the bottom floors and W18×35 for the other floors. The moments of inertia of these beams are 1690 in.^{4} for the bottom floors and 1227 in.^{4} for the top beams. These values are about 78 percent of the stiffnesses for the beams required for the semi-rigid frame, which is approximately the value of β_b.

DESIGN FOR CYCLIC LOADS

When the semi-rigid composite frame is subjected to cyclic loads at a level equal to 1.3W, and then checked for deflections at a level of loading equal to 1.0W, the maximum story drift is L/346 on the fifth floor. The drift for this floor not considering cyclic loads was L/367. The increased damage due to cycling at 1.3W caused a 6 percent increase in the deflection of this floor. The total building deflection after cycling is L/460, which is 5 percent greater than the building drift not considering cyclic loading.

CONCLUSIONS

A straightforward procedure for the design of unbraced frames with semi-rigid composite connections has been developed in this paper. The method gives good results, and usually will converge to the final design in very few iterations. The method takes into account both the strength and the lateral stiffness of the designed frame. Using this method, frames up to 10 stories have been successfully designed. Due to the large connection stiffnesses which are possible with semi-rigid composite connections, the frames designed are very comparable to ones with rigid connections. The frame in Example 1 has a total building drift 20 percent more than the same frame with rigid connections, and the frame of Example 2 had a drift 35 percent more than the same frame with rigid connections. The very low cost of construction with semi-rigid composite connections will offset any increase in cost derived from the larger sections which may be required to limit this extra drift.

Several simplifications and arbitrary assumptions made in the procedure stem from trial designs made by the authors. It is left to the judgment of the structural engineer to accept or modify these values. In general the selection of these terms was made on the conservative side to eliminate the need for many iterations. The design of these frames using the simplified procedure should probably be limited to four to six stories; taller frames should include a more exact analysis (including second-order effects) as the last step of the procedure.

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APPENDIX A: LIST OF SYMBOLS

S_{dl} = service dead load
F_{dl} = factored dead load
S_{ll} = service live load
F_{ll} = factored live load
P_D = point load on girder due to service dead load
P_L = point load on girder due to service live load
P_{Df} = factored PD
P_{Lf} = factored PL
P_f = factored point load on girder
S_{cdl} = construction dead load
F_{cdl} = factored construction dead load
S_{cll} = construction live load
F_{cll} = factored construction live load
F_{cl} = factored construction loads
P_{cf} = factored point load on girder (construction)
M_{cf} = maximum construction moment
$M_{u,c}$ = maximum centerline moment at ultimate
M_p = plastic moment capacity of steel section alone
M_{pc} = plastic moment capacity of composite section
MD_u = design dead load moment
$M_{u,e}$ = ultimate live load moment at the connection
$M_{u,s}$ = ultimate live load moment assuming simple supports
S_s = elastic section modulus of the steel beam
Z_s = plastic section modulus of the steel beam
H_a = ultimate horizontal force in seat angle
d = depth of the steel beam
Y_2 = distance from top of the beam to center of slab force
A_{sl} = required area of seat angle leg
V_b = shear force on an individual bolt
l_a = width of seat angle
t_a = thickness of seat angle
F_{yr} = yield strength of slab reinforcing steel
F_{ysl} = yield strength of seat angle steel
A_r = area of slab reinforcement required
$M_{\theta s}$ = moment capacity at service load rotation
$M_{\theta u}$ = moment capacity at ultimate load rotation
M_{fef} = fixed end moment for factored loads
θ_{ssf} = support rotation for simply supported beam under factored loads
M_{fe} = fixed end moments for service loads
θ_{ss} = support rotation for simply supported beam under service loads
I_{cb} = moment of inertia for composite section
I_{LBp} = lower bound moment of inertia for positive moment
I_{LBn} = lower bound moment of inertia for negative moment
θ_{sr} = rotation at intersection of service beam line and moment-rotation curve
θ_{srf} = rotation at intersection of factored beam line and moment-rotation curve
M_{sr} = service load moment at support
M_{srf} = factored load moments at support

APPENDIX B

Design of the connection angles for Example 1:

The seat angle has A_{sl} = 5.0 in.^{2}
For a W18×55, d = 18.11 in.
For the slab configuration of this example, $Y2$ = 4.0 in.

The maximum connection moment for lateral loading is 1451 kip-in.

The maximum connection moment for gravity loading is 1530 kip-in.

The bolts of the seat angle to the beam flange are slip-critical, so they must be designed for slip at service load levels. Therefore, the gravity loading moment which must be resisted by the slip capacity of the bolts is 1530/1.6 = 956 kip-in. and the maximum for lateral loading is 1451/1.3 = 1116 kip-in. The bolt design is governed by the lateral loading.

$$H_a = 1116/(18.11 + 4.0) = 50.48 \text{ kips}$$

If there are 4 bolts, the capacity of each bolt must be 12.6 kips. This requires 1 in. A325 bolts.

The maximum shear at the beam ends is 44 kips, which would require three ¾ in. A325 bolts with the threads included in the shear plane. Bearing on the beam web is OK. The minimum thickness for an L4×4 angle is ¼ in., which would be adequate for the shear. SInce this connection will experience positive moment, it is recommended to increase this thickness by 1.5 so use 2*L*4×4×⅜in. 8.5 in. long with three ¾ in. A325 bolts.