# Semi-Rigid Composite Connections for Gravity Loads

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## INTRODUCTION

Double web angles, top and seat angles, and seat angle and web clips are some of the most common connections used for floor beams in braced steel frames. These connections are considered to be rotationally weak, and therefore design proceeds along the assumption that these connections can be idealized as pins. In reality, because these connections have a limited amount of moment transfer capability, they are semi-rigid connections. The economic and structural benefits of semi-rigid connections in braced frames are well known but seldom utilized.[3-5] The main advantages they provide over simple connections are lower mid-span moments for the beam and lower effective length of the columns. Depending on the geometry of the connection and the size of the angles used, the moment capacity of these simple connections may range up to 20 percent of the plastic capacity of the beam alone, and lead to saving up to 10 percent of the steel required in floor beams. This moment capacity, however, is associated with large, nonlinear rotations that make serviceability and drift computations difficult. Therefore the current design assumption of pinned ends seems justifiable and conservative.

Previous work by the authors[1,8] has shown that only a nominal amount of slab steel crossing the column lines is necessary to turn these simple connections into rather stiff, semi-rigid composite connections. This additional restraint can be provided at very low added cost, perhaps an increase in the size of the framing angles, an increase in the number of shear connectors, and a small number of reinforcing bars in the slab over the connections. These additions not only provide added restraint to the beams and columns but have significant serviceability benefits. Among their advantages are also the much larger initial stiffness and the bi-linear moment-rotation characteristics they possess. In general, addition of this slab steel will reduce the amount of cracking in the slab at the column lines and decrease the live load deflections.

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## DESIGN OF FLOOR FRAMING WITH SEMI-RIGID CONNECTIONS

If semi-rigid composite connections are going to be used in the typical unshored construction of building frames, it must be realized that the connection is going to behave differently under loads applied before the concrete hardens and those applied after the concrete hardens, as is the case with composite beams designed with simple supports in unshored construction.

For the design and analysis method for composite connections to be discussed in this paper, the following assumptions will be made:

1. The type of connection to be discussed is that shown in Fig. 1, consisting of web angles to transmit shear, and a seat angle and slab steel to transmit bending.
2. For this type of connection, design to two strength levels will be discussed. In what will be called a strong connection, the detailing will aim at providing enough restraint and stiffness to transmit a bending force less than or equal to 1.2 times the plastic capacity of the steel beam alone at a rotation less than 5.0 milliradians. This is near the maximum value of end moment possible with this type of connection when typical beam sections are used. The so-called intermediate connection will aim to transmit about one-half of the beam plastic moment capacity at a rotation less than 10 milliradians.
3. Unshored construction will be assumed. This means that the connections behave as pinned before the concrete hardens, and that the total dead load is applied at this stage. The weight of the wet concrete is treated as a dead load. This implies that the design must satisfy

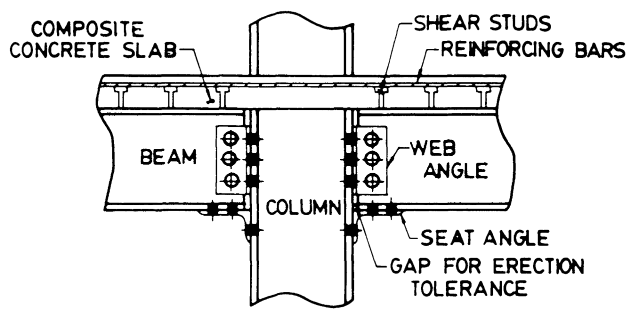


*Fig. 1. Typical connection configuration.*

| **Table 1.** **Range of End Moments for Different Loadings** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Load case** | $M_{efe}$ | $M_{cfe}$ | $M_{ss}$ | $\theta_{ss}$ | $M_{sre}$ | $\Delta_{ss}$ | $\Delta_{fe}$ |
| ↓↓↓↓↓↓↓↓↓↓ w | $\frac{wL^2}{12}$ | $\frac{wL^2}{24}$ | $\frac{wL^2}{8}$ | $\frac{wL^3}{24EI}$ | $\frac{wL^2}{14}$ to $\frac{wL^2}{24}$ | $\frac{5wL^4}{384EI}$ | $\frac{wL^4}{384EI}$ |
| ↓P | $\frac{PL}{8}$ | $\frac{PL}{8}$ | $\frac{PL}{4}$ | $\frac{PL^2}{16EI}$ | $\frac{PL}{10}$ to $\frac{PL}{16}$ | $\frac{PL^3}{48EI}$ | $\frac{PL^3}{192EI}$ |
| ↓P ↓P | $\frac{2PL}{9}$ | $\frac{PL}{9}$ | $\frac{PL}{3}$ | $\frac{PL^2}{9EI}$ | $\frac{PL}{6}$ to $\frac{PL}{9}$ | $\frac{23PL^3}{648EI}$ | $\frac{5PL^3}{648EI}$ |
| ↓P ↓P ↓P | $\frac{5PL}{16}$ | $\frac{3PL}{16}$ | $\frac{PL}{2}$ | $\frac{5PL^2}{32EI}$ | $\frac{PL}{4}$ to $\frac{PL}{8}$ | $\frac{19PL^3}{384EI}$ | $\frac{PL^3}{96EI}$ |
| ↓P↓P↓P↓P | $\frac{2PL}{5}$ | $\frac{PL}{5}$ | $\frac{3PL}{5}$ | $\frac{PL^2}{5EI}$ | $\frac{7PL}{20}$ to $\frac{PL}{10}$ | $\frac{63PL^3}{1000EI}$ | $\frac{13PL^3}{1000EI}$ |

both the ultimate loads as a composite section as well as the dead and construction loads as a non-composite section.

4. The same connection will be used at both ends of the beam. While this is not strictly necessary, it will simplify the design and construction. In the case of connections to exterior columns it is usually beneficial in braced frames to use non-composite connections with low rotational stiffness to limit the amount of moment transferred to these columns. Even if connections with high rigidity are used the stiffness of the exterior support of the beam will be less than that of the interior support due to column flexibility.
5. The connections will frame into the strong axis of the column. While similar connections can be hypothesized for the weak axis and for connections to girders, not enough experimental data is available to verify the design procedure for those connections.
6. A very important step in the design procedure is the arbitrary selection of the degree of fixity desired at the columns. Two common cases are illustrated in Figs. 2 and 3. Figure 2 deals with two equally spaced point loads, and Fig. 3 with a distributed load. If full fixity is chosen, then the moments will be $2PL/9$ at the supports and $PL/9$ at the centerline for point loads, and $WL^2/12$ at the supports and $WL^2/24$ at the centerline for distributed loads. If the ends are unrestrained, then the moments will be zero at the supports and $PL/3$ or $WL^2/8$ at the centerline. For a semi-rigid case, the values of moments will vary between these two extremes. For optimum semi-rigid design the moments at the support and at the centerline should be the same ($PL/6$ or $WL^2/16$) if the section is symmetrical and homogeneous. Unfortunately, in the design of composite beams the section is neither symmetrical nor homogeneous, and therefore the capacity of the member and moments of inertia in positive and negative bending are different. This leads to a range of best values for the support moment. For the type of connection used here, the practical limits to this moment, labeled $M_{u,e}$, are between $PL/6$ and $PL/9$ for the two point load case and $WL^2/14$ and $WL^2/24$ for the distributed load case. In both of these cases the loads are those induced by live loads only, as the connections are assumed to be pinned for the dead loads. Table 1 shows upper and lower bounds and suggested values for some common loading cases.
7. The design, to some extent, will be controlled by the number of bolts that can be attached to the bottom flange of the beam. It will be assumed here that for practical reasons that limit is 6 bolts (3 on each side of the web). These will be slip-critical connections, so they will need to be checked as slip-critical connections for service loads and as bearing connections for ultimate. The maximum practical size of bolts to be used in these connections is 1 in. and the friction capacity for service loads will always govern (the ratio between friction capacity and bearing capacity is slightly over 2.0 and the ratio between service load and ultimate load is 1.6). This means the total force in the bottom flange at service loads cannot exceed 80 kips for A325 bolts or 99 kips for A490 bolts, or multiplying these values by 1.6 gives a maximum flange force at ultimate of 128 kips for A325 bolts and 158 kips for A490 bolts.
8. If the desired force in the bottom flange is larger than 158 kips, then welding a flange plate instead of bolting an angle seems the logical solution. A single experimental test on a semi-rigid composite connection with a welded bottom plate indicated very ductile and stiff behavior. Extension of this procedure to connections incorporating a welded flange plate is straightforward.
9. An effective width criteria is needed to determine the width of the strip over which the slab steel has to be placed. While tests indicate that at ultimate all the steel in a slab will be effective, and thus the effective width

equals the spacing of the beams, this only happens at very large rotations. The tests and analysis on which this procedure is based indicate that reinforcing steel over a width of 4 to 7 column flanges is effective in resisting moments at the connection. For design, therefore, the slab steel needed has to be placed within 7 column flange widths. In many cases the slab outside of this width will also have some reinforcing in it; this reinforcing should not be considered as acting to resist moments in the beam.

10. The connection to exterior columns in these frames will be designed and detailed as a simple connection. In braced frames a moment connection to the exterior column will increase the moments in the column, resulting in an increase of column size. The decrease in effective length obtained because of the connec-

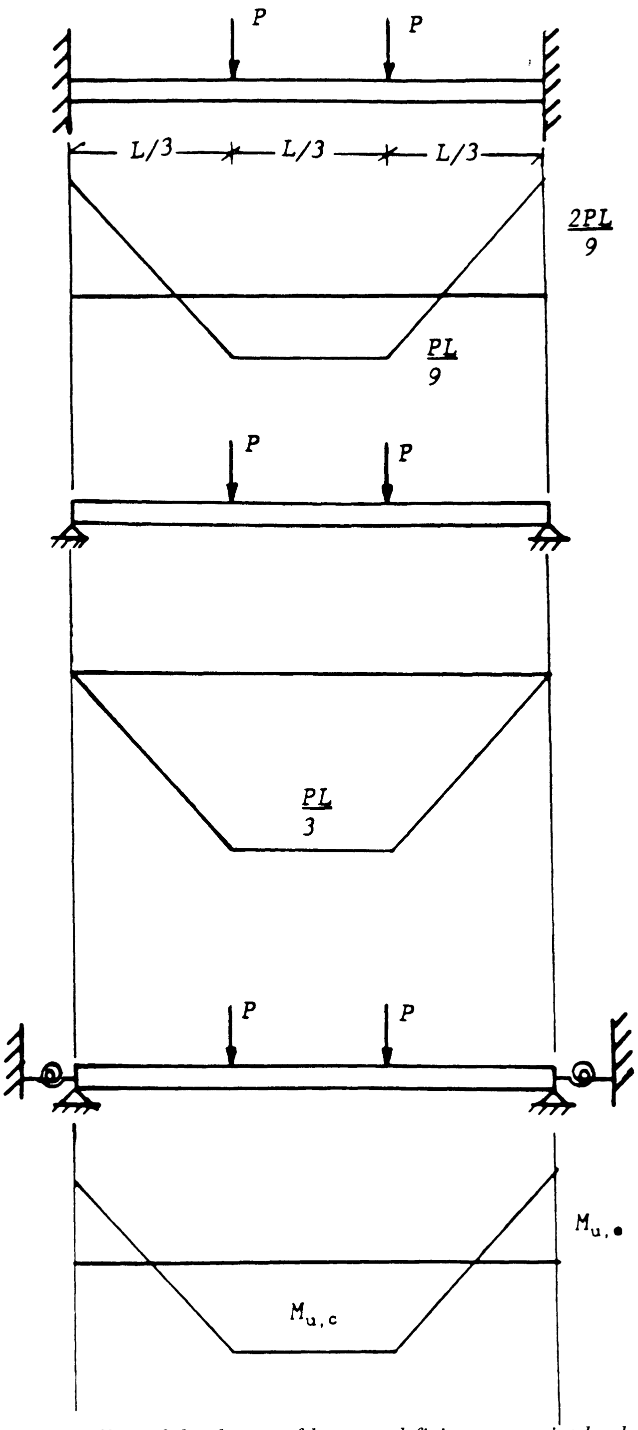


*Fig. 2. Effect of the degree of beam-end fixity—two point load.*

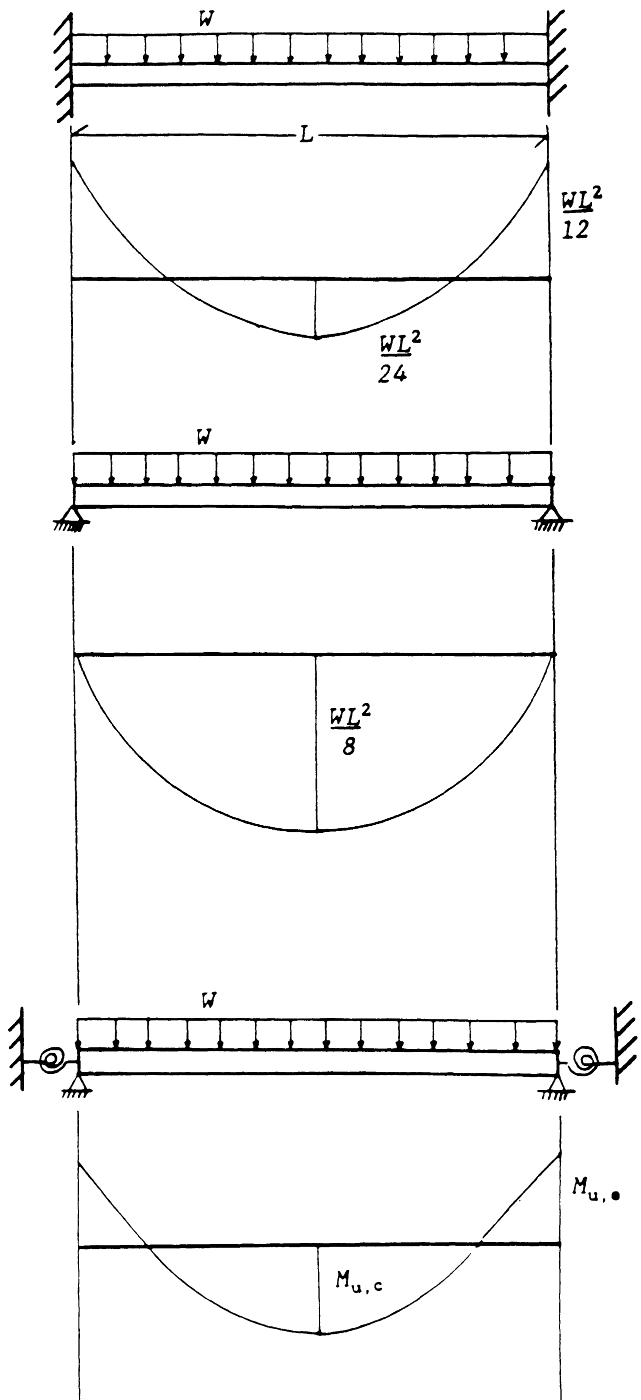


*Fig. 3. Effect of the degree of beam-end fixity—distributed load.*

tion restraint is generally not sufficient to offset the additional strength requirements due to this moment. Thus the exterior spans will have a pin at one end and a semi-rigid composite connection at the other.

## DESIGN METHOD: EXAMPLE 1

The design procedure proposed here will be discussed with the aid of a numerical example for a braced frame utilizing composite connections of intermediate strength. A similar example is discussed following the section on column design for the case of strong connections.

The general topology of the structure is given by Fig. 4. The typical girder to be designed is that along column line B and is limited in depth to 18 in. The span is 30 ft and the beams are spaced at 10 ft. The materials will be assumed as A572 ($F_y$ = 50 ksi) for the girder, A36 ($F_y$ = 36 ksi) for the connection angles, Grade 60 ($F_y$ = 60 ksi) for the reinforcing bars, and a lightweight concrete with a compressive strength $f'_c$ of 3.5 ksi. The beams will have a tributary area of 240 ft$^2$, and the loads on the girders will be simulated by point loads at the third points.

The following data is given for the design of a typical interior bay:

$S_{dl}$ = service dead load = 60 psf
$F_{dl}$ = factored dead load = 1.2 × 60 = 72 psf
$S_{ll}$ = service live load = 100 psf

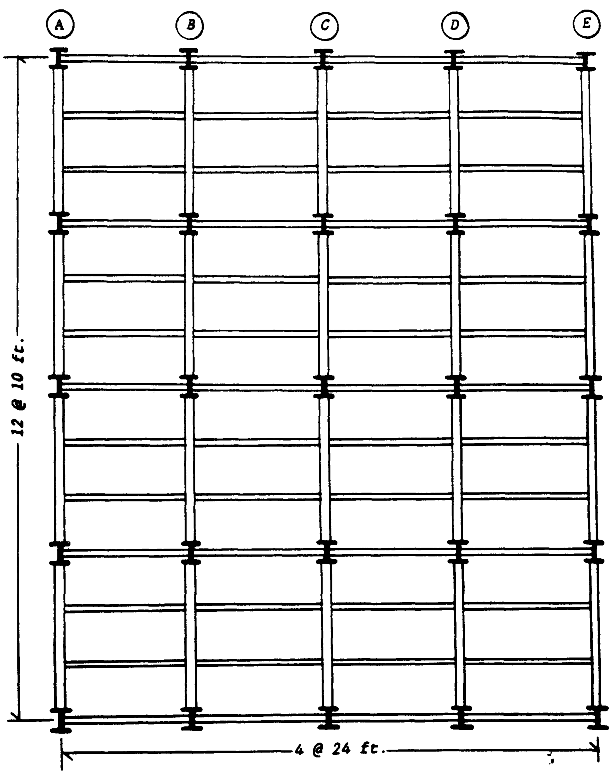


*Fig. 4. General topology of the floor framing—Example 1.*

$F_{ll}$ = factored live load = 1.6 × 100 = 160 psf
$P_D$ = point load on girder due to service dead load = 14.40 kips
$P_L$ = point load on girder due to service live load = 24.00 kips
$P_{Df}$ = factored $PD$ = 17.28 kips
$P_{Lf}$ = factored $PL$ = 38.40 kips
$P_f$ = factored point load on girder = 55.68 kips

For comparison purposes, the design of a simply supported composite beam to carry this loading will be performed. Assume that the floor slab will be made of lightweight concrete (110 psf), and that it will be 5 in. deep, including 2 in. formed metal deck. Thus the weight of wet concrete will be about 40 psf, and the construction loads will be:

$S_{cdl}$ = construction dead load = $S_{dl}$ = 60 psf
$F_{cdl}$ = factored construction dead load = 1.2 × $S_{cdl}$ = 72 psf
$S_{cll}$ = construction live load = 20 psf
$F_{cll}$ = factored construction live load = 20 × 1.6 = 32 psf
$F_{cl}$ = factored construction loads = $F_{cdl}$ + $F_{cll}$ = 104 psf
$P_{cf}$ = factored point load on girder (construction) = 24.96 kips

The maximum moment $M_{cf}$ during construction will be 249.6 kip-ft, and the maximum centerline moment at ultimate $M_{u,c}$ will be 556.8 kip-ft. A W18×46 is the lightest W18 shape that can resist these moments. For this section with full composite action $\phi M_{pc}$ = 611 kip-ft., so partial composite action could be used with $\Sigma Q_n$ = 492 kips and $\phi M_{pc}$ = 569 kip-ft. With this choice of partial composite action the required number of ¾ in. shear studs will be 50 for the entire span. The service load deflection for this beam is 0.82 in. or about $L$/441.

The design of a braced steel frame utilizing semi-rigid composite connections will entail the following steps:

**Step 1:** Calculate design moments $M_{cf}$ due to factored construction loads, and the design dead load moments $MD_u$ assuming simple supports. The self weight of the girder and floor beams is assumed to be included in the overall dead loads, and the $\phi$ factor is assumed as 0.90 for the steel section alone and 0.85 for composite sections.

$$M_{cf} = (P_{cf}L)/(3) = 2{,}995 \text{ kip-in.} \quad (1)$$

$$MD_u = (P_{Df}L)/(3) = 2{,}074 \text{ kip-in.} \quad (2)$$

**Step 2:** Select degree of restraint desired at the connections by assuming the amount of the factored live load moment $M_{u,e}$, desired at the supports. We will assume the capacity at the support as $P_{Lf}L/(9)$.

$$M_{u,e} = (P_{Lf}L)/(9) = 1{,}536 \text{ kip-in.} \quad (3)$$

Once $M_{u,e}$ has been chosen, the factored moment at the center $M_{u,c}$ can be calculated as the difference between the ultimate simply supported factored moment $M_{u,s}$ and $M_{u,e}$:

$$M_{u,s} = (P_f L)/(3) = 6{,}682 \text{ kip-in.} \quad (4)$$

$$M_{u,c} = M_{u,s} - M_{u,e} = 5{,}146 \text{ kip-in.} \quad (5a)$$

For beams with one end simply supported Eq. 5a becomes:

$$M_{u,c} = M_{u,s} - M_{u,e}/2 \quad (5b)$$

**Step 3:** Select a composite beam to carry $M_{u,c}$, based on a steel beam capable of carrying $M_{cf}$ without reaching its plastic capacity and $MD_u$ without yielding. Assuming A572 steel ($F_y = 50$ ksi), the required elastic section modulus $S_s$ and plastic section modulus $Z_s$ for the steel beam are:

$$S_s = MD_u/F_y = 41.5 \text{ in.}^3 \quad (6)$$

$$Z_s = M_{cf}/\phi_b F_y = 66.56 \text{ in.}^3 \quad (7)$$

The lightest W18 which meets these criteria is a W18×35, with $S_s = 57.6$ in.$^3$, $Z_s = 66.5$ in.$^3$ and $\phi M_{pc} = 5{,}628$ kip-in.

As for most cases of unshored construction, the construction loads are going to determine the beam size. The case discussed here has assumed reasonable values for construction loads, but still we end with a composite section much stronger than needed.

An alternative method for design is to find the steel section required for the construction loads and provide sufficient end restraint so the composite section with this steel shape is sufficient for the full loads. This can be expressed as:

$$M_{u,e} = P_f L/3 = \phi M_{cp} \quad (8)$$

This equation is subjected to the limit $M_{u,e} < 1.2M_p$. If this limit is exceeded a larger beam section is required, and construction loads do not control the design. This will be the case if the ratio of full load to construction load is greater than the ratio of $\phi M_{cp}/\phi_b M_p$.

**Step 4:** Choose a seat angle based on the area of the angle leg $A_{sl}$ being capable of transmitting the total required horizontal force $H_a$ on the bottom angle assuming a lever arm equal to the beam depth plus the distance from the top of the beam to the center of the slab force ($Y2$, this may be different for positive and negative moment) and the yield strength, $F_{ysl}$, of the seat angle:

$$H_a = M_{u,e}/(d + Y2) = 70.5 \text{ kips} \quad (9)$$

Recall the limit on $H_a$ imposed by the friction force capable of being transmitted by bolts to the angles is 158 kips.

$$A_{sl} = 1.33\ H_a/F_{ysl} = 2.61 \text{ in.}^2 \quad (10)$$

The 1.33 factor will ensure that the slab steel, which will also be based on the horizontal force $H_a$, will yield first. The connection will be a slip-critical-type connection for the unfactored live load moment. Using 6 bolts the required shear strength of each bolt is:

$$V_b = H_a/(\text{L.F.} = 1.6) \times 6 = 7.34 \text{ kips} \quad (11)$$

This will require ¾ in. A325 bolts.

Since 6 bolts are required for the connection, the angle leg along the beam should be probably 9 in. For connection purposes the angle needs to be at least as wide as the beam flange (W18×35, $b_f = 6.00$ in.); therefore select an angle width ($\ell_a$) of 6.5 in. The required thickness will be:

$$t_a = A_{sl}/\ell_a = 0.435 \text{ in.} \approx 7/16 \text{ in.} \quad (12)$$

The smallest thickness available in a 9 in. angle is ½ in., therefore the seat angle will be a L9×4×½, 6.5 in. long connected with six ¾ in. A325 fully tightened bolts. Alternatively we can use an L6×4×7/16, 6.5 in. long with four ⅞ in. A490 bolts.

**Step 5:** Calculate the amount of slab reinforcement $A_r$ based on the force $H_a$ and a slab steel yield strength $F_{yr}$ of 60 ksi.

$$A_r = H_a/F_{yr} = 1.18 \text{ in.}^2 \quad (13)$$

This requires six #4 bars (1.20 in.$^2$) or four #5 bars ($A$ = 1.24 in.$^2$). Because the intent is to yield the slab steel, care must be taken not to provide an excess amount of slab steel. The six #4 bars will be selected here. The total area is 1.20 in.$^2$ For best efficiency these should be grouped within a strip equal to 7 times the column flange width, or 84 in. if a W12 column is assumed.

**Step 6:** Calculate the moment capacity for the connection designed above at service and ultimate by using the formulas proposed by Kulkarni for negative moments.[7] For service loads the moment capacity $M_{\theta s}$ is given by:

$$M_{\theta s} = 0.17[(4)(A_r)(F_{yr}) + (A_{sl})(F_{ysl})]\ (d + Y2) \quad (14)$$

while for ultimate the moment capacity $M_{\theta u}$ is given by:

$$M_{\theta u} = 0.245[(4)(A_r)(F_{yr}) + (A_{sl})(F_{ysl})](d + Y2) \quad (15)$$

Substituting into Eqs. 14 and 15 and using $\phi = 0.85$, we have:

$$\phi M_{\theta s} = 1{,}270 \text{ kip-in.} > M_{u,e}/(\text{L.F.}=1.6) = 956 \text{ kip-in.} \quad \textbf{O.K.}$$

$$\phi M_{\theta u} = 1{,}830 \text{ kip-in.} > M_{u,e} = 1{,}536 \text{ kip-in.} \quad \textbf{O.K.}$$

Since both the capacity at service and ultimate exceed the required moments, the design is adequate. If both conditions are not satisfied, then we must either go back to step 2 and assume a smaller value for $M_{u,e}$, or increase the strength of the connection in steps four and five.

**Step 7:** To check the compatibility condition, several techniques can be used. The two simplest will be discussed here:

(1) Assume that the serviceability condition will be reached at a rotation of 2.5 milliradians, and that the ultimate will be reached at 10 milliradians. For these values of $\theta$ calculate the moment given by Eq. 16:

$$M(\theta) = C1(1 - e^{-c2\theta}) + C3\theta \qquad (16)$$

where:

$C1 = A_r F_{yr}(d + Y2)$
$C2 = 32.9(A_{sl}/A_r)^{0.15}(d + Y2)$
$C3 = 24F_{ysl}A_{sl}(d + Y2)$
$\theta$ = rotation in radians

Therefore,

$$\phi M(\theta{=}0.00025) = 1{,}290 \text{ kip-in.} > 956 \text{ kip-in. } \textbf{O.K.}$$

$$\phi M(\theta{=}0.0010) = 1{,}846 \text{ kip-in} > 1{,}536 \text{ kip-in. } \textbf{O.K.}$$

If the moments obtained by Eq. 16 exceed those required, the connection is O.K.

(2) Develop a beam line for the beam (see Fig. 5) with the given loading conditions by calculating the fixed end moment, $M_{fe}$, and simply supported rotation, $\theta_{ss}$, for both service and factored live loads (for exterior spans with the exterior connection pinned use the moment at the end of a propped cantilever in place of the fixed end moment to achieve compatibility at the interior connection):

$$M_{fef} = 2P_{Lf}L/9 = 3{,}072 \text{ kip-in.} \qquad (17)$$

$$\theta_{ssf} = P_{Lf}L^2/(9EI_{cb}) = 17.7 \times 10^{-3} \text{ rad} \qquad (18)$$

$$M_{fe} = 2P_L L/9 = 1{,}920 \text{ kip-in.} \qquad (19)$$

$$\theta_{ss} = P_L L^2/(9EI_{cb}) = 11.1 \times 10^{-3} \text{ rad} \qquad (20)$$

where $E$ is Young's modulus and $I_{cb}$ is the moment of inertia of the composite section. An estimate of $I_{cb}$ can be obtained from the tables of $I_{LB}$ given in the LRFD Manual by using a weighted average of moments of inertia in the positive moment region and negative moment region. For interior spans approximately 60 percent of the span is experiencing positive moment. Therefore it is suggested to use:

$$I_{cb} = 0.6I_{LBp} + 0.4I_{LBn} \qquad (21)$$

Where $I_{LBp}$ is the lower bound moment of inertia for positive moment and $I_{LBn}$ is the lower bound moment of inertia for negative moment. For this case $I_{cb}$ = 0.6 × 1,360 + 0.4 × 655 = 1,078 in.[4]

Find the rotations ($\theta_{sr}$ or $\theta_{srf}$) at the intersection of the beam line with the connection moment-rotation curve for service and factored loads. This can be done graphically (see Fig. 5) or analytically. The latter involves solving the equations:

$$M(\theta_{sr}) = M_{fe}(1 - \theta_{sr}/\theta_{ss}) \qquad (22)$$

$$M(\theta_{srf}) = M_{fef}(1 - \theta_{srf}/\theta_{ssf}) \qquad (23)$$

From the graph, $\theta_{sr}$ = 0.0024 radians and $\theta_{srf}$ = 0.0065 radians. Find the factored and service live load moments by:

$$M_{sr} = M_{fe} - M_{fe}\theta_{sr}/\theta_{ss} = 1500 \text{ kip-in.} \qquad (24)$$

$$M_{srf} = M_{fef} - M_{fef}\theta_{srf}/\theta_{ssf} = 1949 \text{ kip-in.} \qquad (25)$$

and recalculate the factored moment at centerline:

$$M_{u,c} = (P_f L)/(3) - M_{srf} = 4{,}733 \text{ kip-in.} \qquad (26)$$

This is less than the 5,628 kip-in. that the W18×35 with a 5 in. slab can carry as a composite section, and thus the section is satisfactory.

**Step 8:** Check the stresses in the section due to unfactored loads:

$$\sigma_D = P_D L/3\ S_s = 30.0 \text{ ksi} \qquad (27)$$

$$\sigma_L = (P_L L/3 - M_{sr})/S_{tr} = 15.5 \text{ ksi} \qquad (28)$$

$S_{tr}$ can be approximated from the tables of $I_{LB}$ in the LRFD Manual for full composite action by the equation:

$$S_{tr} = I_{LB}/(3d/4 + Y2/2) \qquad (29)$$

This gives a total stress under unfactored loads of 45.5 ksi, which is less than the nominal yield stress of 50 ksi. The live load stresses given are for the full live load. Under the arbitrary point in time (APT) concept[5] the expected live load is half of the full live load. Therefore a reasonable stress check, rather than full live plus dead load is:

$$1.2\sigma_d + 0.5\sigma_l < 0.9F_y \qquad (30)$$

For our case the W18×35 is satisfactory because the stress from Eq. 30 would be 43.8 ksi, less than the 45 ksi allowed.

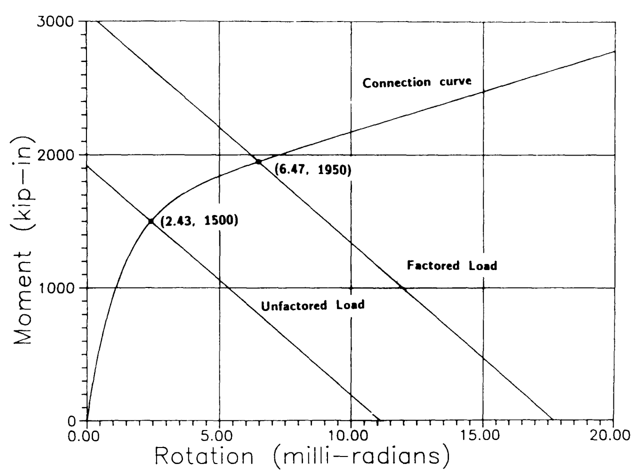


*Fig. 5. Intersection of the beam line with the connection curve.*

**Step 9:** Design web angles for maximum factored shear as a bearing-type connection. Select the size of the bolts in the web connections to match those in the beam flange to avoid confusion during erection. For this case we will use ¾ in. A325 bolts with threads in the shear plane. The maximum shear is:

$$V_u = P_f/\phi_v = 74.2 \text{ kips} \quad (31)$$

Checking for bearing and shear on the beam web:

$$N = V_u/2.4F_u'dt = 2.12 \quad (32)$$

$$N = V_u\phi/31.0 \text{ (kips/bolt)} = 1.80 \quad (33)$$

Therefore 3 bolts are required. Checking bearing on the angles, we can find the angle thickness $t_a$.

$$2t_a = V_u/2.4\ F_uNd = 0.237 \text{ in.} \quad (34)$$

Therefore a ¼ in. angle is sufficient. Based on clearance and minimum distances, select a pair of L4×4×¼ angles, 8.5 in. long for the web.

The tests on which this method is based all had a minimum of 3 bolts for the connection of the web angles to the beam, therefore it is not recommended to use a lower amount of bolts in this connection.

**Step 10:** Calculate dead load deflection for cambering, and service load deflections:

$$\delta_d = 23(PD)(L^3)/648EI_s = 1.61 \text{ in.} \quad (35)$$

$$\delta_l = 5(PL)(L^3)/648EI_{cp} + (L/4)\theta_{sr} = 0.49 \text{ in.} \quad (36)$$

This live load deflection is equal to $L/731$, much lower than the allowable $L/360$, which is expected for beams using semi-rigid composite connections. This small service load deflection will probably mean that vibration problems will be minimized.

**Step 11:** To ensure complete composite action, supply the required number of shear connectors. The negative moment at the support is 1,949 kip-in. and the positive moment under the point loads is 4,733 kip-in. This implies that the inflection point will be 35 in. from the end. From the Manual $\Sigma Q_n = 515$ kips in the positive moment region which will require 52 studs over the center 290 in. and $\Sigma_{Qn} = 72$ kips in the negative moment region, which will require 4 studs in the end 35 in. A stud every 6 in. over the entire span will be sufficient. This system should also work with partially composite sections, which would allow a reduction in the number of studs required in the positive moment region, but no experimental data has been gathered on the effect of partial shear connection on the connection moment-rotation characteristics.

For this example the use of semi-rigid composite connections resulted in a beam selection of W18×35 with sixty ¾ in. shear studs, as opposed to the W18×46 with fifty ¾ in. shear studs required if the beam was designed as simply supported. The savings of 24 percent in weight which results is offset by a 20 percent increase in the number of shear studs required and the addition of 1.20 in.$^2$ of reinforcing at each end of the span. This small amount of reinforcing may have been provided in the simply supported beam design in order to reduce the amount of cracking in the concrete slab at the columns. The major difference will be in the service load deflections, for which the semi-rigid composite beam will give $L/731$ versus $L/441$ for the simply supported one.

This design method was developed for the typical floor framing of buildings and has several limits of applicability. Most of these limits come from uncertainty of the connection moment-rotation behavior. The method should not be used for very long spans (greater than 48 ft), for beams deeper than W27, and for beams with flange thicknesses greater than 0.8 in. This last requirement is mostly for a practical reason, as it would be difficult to develop the force in the bottom flange of these beams with a seat angle bolted to the flange. Welding the seat angle to the beam flange would perhaps allow a slightly larger flange to be used, but 0.8 in. is still very near the upper limit of applicable beam flange thickness.

In unshored construction the beam size is frequently controlled by the construction loading, and in that case the use of semi-rigid composite connections will have no effect on the beam size, but it may still be desirable for the added restraint it provides for the columns or to stiffen the beam to reduce deflections or vibration problems. Generally construction loading will govern the beam selection using simply supported beams if the ratio of factored construction load to factored service load is greater than 0.55. When this ratio is less than 0.40, the beam size will generally be determined by the full loading even with semi-rigid composite connections.

The calculations for $\delta_l$, $\theta_{ss}$, $\theta_{ssf}$, and $\sigma_2$ require elastic section properties $I_{cp}$, $I_{cb} = (0.6I_{cp} + I_{cn})$, and $S_{cp}$ of the composite section to be calculated. The designer may choose to estimate these properties using the tabulated values for $I_{LB}$ in the LRFD manual for initial design, but if the design is close to any limit states the values should be calculated rigorously.

If the floor framing is to be designed with an ASD approach, computation of the elastic section modulus of the composite section in positive moment with respect to the concrete and the section moduli in the negative moment region is necessary, and the design checks are replaced with limiting the concrete force to $0.45f_c'$ in compression, the force in the reinforcing steel in the negative moment region to $0.66F_{yr}$, the bottom flange stress in the composite beam to $0.89F_y$, and the bottom flange stress in the steel beam before the concrete hardens to $0.66F_y$.

## DESIGN OF COLUMNS IN BRACED FRAMES WITH SEMI-RIGID CONNECTIONS

The effect of semi-rigid connections on column design is to reduce the effective length of the column to less than 1.0. The design method proposed by Bjorhovde[2] is used here with modifications due to the increased connection stiffness.

**Step C1:** Design beams and connections as described above.

**Step C2:** Determine the connection stiffness $C_\theta$ for all connections framing into the column using Eq. 37. When the column buckles some connections are going to load (i.e., rotations increase) while others are going to unload (i.e., rotations decrease). For this reason at both ends of an interior column the stiffness for one connection is the tangent stiffness at $\theta = \theta_{srf}$ and the stiffness for the other connection is the unloading stiffness, which is equal to the stiffness at $\theta = 0$. For an exterior column the connection on one end is going to load and the connection on the other end is going to unload. Figure 6 shows an example of an exterior and an interior column before and after buckling, indicating which connections load and which ones unload.

$$C_\theta = C1 \times C2[e^{-C2\theta}] + C3 \quad (37)$$

**Step C3:** Determine the effective end restraint, taking into account both the stiffness of the connection and the stiffness of the beam for each beam connected to the column:

$$C^* = (2EI_{cb}/L)/(1 + 2EI_{cb}/LC) \quad (38)$$

**Step C4:** Determine the ratio of beam to connection stiffness $\beta$ and the ratio of effective end restraint to beam stiffness $\beta^*$ for all beams connected to the column.

$$\beta = C/(EI_{cb}/L) \quad (39)$$

$$\beta^* = C^*/(EI_{cb}/L) = 2\beta/(\beta + 2) \quad (40)$$

**Step C5:** Determine the ratio of effective end restraint to end restraint with no connection $\beta_b$.

$$\beta_b = C^*/(2EI_{cb}/L) = \beta^*/2 \quad (41)$$

**Step C6:** Compute the elastic stiffness distribution factors for the braced frame effective length nomograph in the LRFD Manual,[9] using:

$$G = (\Sigma EI_{col}/H)/(\Sigma\beta_b EI_{cb}/L) \quad (42)$$

**Step C7:** Determine the inelastic stiffness distribution factors as suggested by Disque.[3] These are determined by multiplying the elastic $G$ by a stiffness reduction factor, tabulated for LRFD and ASD in their respective manuals.[9,10]

**Step C8:** Using these distribution factors determine the effective length using the nomographs and design the column in the usual way. It should be noted that exterior columns or interior columns with unequal beam spans or loadings will have a moment on their ends, determined by $M_{u,e}$ from the adjoining beams and must be designed as beam-columns.

## DESIGN METHOD: EXAMPLE 2

The following example will illustrate the method for designing both beams and columns in a braced frame. The beam and loading are the same as given in Example 2 of the chapter on composite beams of the LRFD Manual. The following data is given:

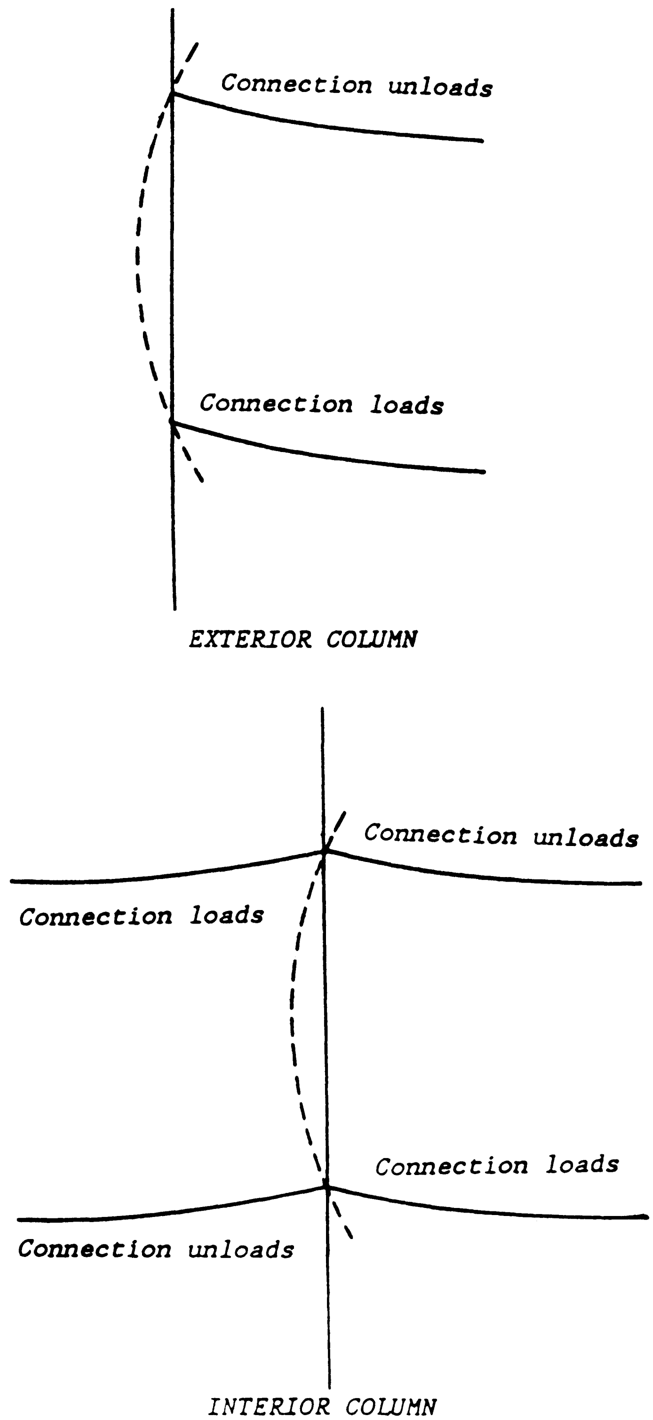


*Fig. 6. Behavior of the connections when the column buckles.*

Beam span = 40 ft
Beam spacing = 10 ft
Service dead load = 90 psf
Service live load = 250 psf
Construction live load = 20 psf
Column height = 13 ft
Interior column load = 1,000 kips
Exterior column load = 500 kips
4.5 in. slab ($w_c$ = 145 pcf) on 3 in. metal deck
Use $f'_c$=4 ksi, $F_y$=50 ksi (steel beam), $F_{yr}$=60 ksi (rebar)
Deck ribs are oriented perpendicular to the beam
Unshored construction
Use 6.5 in. long ¾ in. studs
$W_d$ = 0.0750 kip/in.
$W_{df}$ = 0.0900 kip/in.
$W_L$ = 0.2083 kip/in.
$W_{Lf}$ = 0.3333 kip/in.
$W_{cf}$ = 0.0267 kip/in.

The major departure from available test data in this example is that the beams frame into the columns weak axis. The behavior of the connections framing into the column weak axis has not been tested, but is included here as an example, as this is the axis of the column to which the connections will typically be attached in braced frames.

The beam design calculations follow those of Example 1 and will be abbreviated here:

**Step 1:**
$MD_u = w_{df}L^2/8 = 2{,}592$ kip-in.
$M_{cf} = (w_{df} + w_{cf})L^2/8 = 3360$ kip-in.

**Step 2:**
$M_{u,e} = w_{Lf}L^2/14 = 5486$ kip-in.
$M_{u,s} = (w_{Lf} + w_{df})L^2/8 = 12{,}192$ kip-in.
$M_{u,c} = M_{u,s} - M_{u,e} = 6706$ kip-in.

**Step 3:** This moment requires a W18×35, but a W18×40 is required for construction loads. Using a W18×40, and assume that we will try for the maximum possible moment ($1.2M_p$) at the supports. Thus:

$M_{u,e}$ = 4704 kip-in.
$M_{u,c}$ = 7488 kip-in. W18×40 still **ok**

Try a W18×40.

**Step 4:**
$H_a = M_{u,e}/(d + Y2) = 193$ kips

This is larger than the 158 kips which can be transmitted by a bolted slip-critical connection, but if the angle is welded to the beam flange this force can still be transferred.

$A_{sl} = 1.33H_a/F_{ysl} = 7.12$ in.$^2$

Use L8×4×1 7 in. long ($A_{sl}$ = 7.00 in.$^2$)

**Step 5:**
$A_r = H_a/F_{yr}$ =3.22 in.$^2$ – try 10 #5 bars $A_r$=3.10 in.$^2$

**Step 6:**
$M_{\theta s} = 0.17(4A_rF_{yr} + A_{sl}F_{ysl})(d+Y2) = 4131$ kip-in.
$M_{\theta u} = 0.245(4A_rF_{yr} + A_{sl}F_{ysl})(d+Y2) = 5954$ kip-in.
$\phi M_{\theta s}$ = 3511 kip-in. > 4704/1.6 = 2940 kip.in.
$\phi M_{\theta u}$ = 5061 kip-in. > 4704 kip-in.

**Step 7:** Using beams lines,

$M_{fef}$ = 6400 kip-in.
$M_{fe}$ = 4000 kip-in.
$\theta_{ssf}$ = 0.0192 Rad
$\theta_{ss}$ = 0.0120 Rad

This yields,

$\theta_{srf}$ = 0.0041 Rad
$M_{srf}$ = 5034 kip-in.
$\theta_{sr}$ = 0.0014 Rad
$M_{sr}$ = 3521 kip-in.

Therefore,

$M_{u,c}$ = 7158 kip-in. < $\phi M_{pp}$ = 7887 kip-in.
$M_{u,e}$ = 5034 kip-in. < $\phi M_{pn}$ = 5319 kip-in.

**Step 8:**
$\sigma_d$ = 31.6 ksi
$\sigma_L$ = 18.9 ksi
$\sigma_{d+L}$ = 50.5 ksi > $F_{ys}$
$1.2\sigma_d + 0.5\sigma_L$ = 47.4 ksi > 45 ksi (see note in Step 10)

**Step 9:** Estimation of elastic section properties using $I_{LB}$:
(a) Positive moment:

$a$ = 11.8 × 50/(0.85 × 4 × 120) = 1.446 in.
$Y2 = 7.5 - a/2$ = 6.777 in.

Interpolating, $I_{LB}$ = 2070 in.$^4$ (exact $I_{cp}$ = 2753 in.$^4$)

$Y_{ENA} = 3d/4 + Y2/2$ = 16.8 in.
$S_{cp} = I_{LB}/(Y_{ENA})$ = 123 in.$^3$ (exact $S_{cp}$ = 130.8 in.$^3$)

(b) Negative moment:
$\Sigma Q_n$ = 186 kips => $Y1$ = 3.06 in.
$Y2$ = 6.5 in.
$I_{LB}$ = 1280 in.$^4$ (exact $I_{cn}$ = 1198 in.$^4$)
$I_{cb}$ = 1754 in.$^4$ (exact $I_{cb}$ = 2131 in.$^4$)

Using the exact properties,

$\delta_d$ = 2.92 in.
$\delta_L$ = 0.69 in. < 1.33 in. = $L/360$

Note: The AISC LRFD Specification does not require a check for yielding of the tension flange for composite beams, and views this yielding as a serviceability issue. If the deflection including the yielding is less than the allowable deflection, then the design is adequate. For this case the live load deflection is $L/695$. However, both the stresses under full dead plus live load exceeds yield, and under the proposed equation for serviceability exceeds its allowable value. Thus some yielding is likely.

**Step 10:** The required shear connectors are:
(a) Inflection point location is at 51 in. For negative moment region we have:

$$\Sigma Q_n = 3.1 \times 60 = 186 \text{ kips}$$
$$N = \Sigma Q_n / Q_n = 186/26.1 = 7.1$$

We need at least 8 in. and 51 in. at each end

(b) For positive moment region we have:

$$\Sigma Q_n = 11.8 \times 50 = 590 \text{ kips}$$
$$N = 2\Sigma Q_n / Q_n = 1180/26.1 = 45.2$$

We need at least 46 in. center 378 in.

Total number of studs required = 62

The final design is thus for a W18×40. This same beam is designed as simply supported as Example 2 in the LRFD Manual and a W24×55 is required. The significant savings in steel encountered in Example 1 are repeated here.

The column design proceeds as follows, with the subscript $l$ used for the connection that continues loading and subscript $u$ for the connection that is unloading:

**Step C2:** Determine the connection stiffness:
$C_u$ = 4,264,000 kip-in./Rad
$C_l$ = 247,000 kip-in./Rad

**Step C3:** Determine the effective end restraint:
$C_u^*$ = 242,800 kip-in.
$C_l^*$ = 126,100 kip-in.

**Step C4:**
$\beta_u$ = 33.121
$\beta_l$ = 1.9187
$\beta_u^*$ = 1.8861
$\beta_l^*$ = 0.9793

**Step C5:**
$\beta_{bu}$ = 0.9431
$\beta_{bl}$ = 0.4896

**Step C6:**
(a) Interior Column: try W12×87
$A$ = 25.6 in.$^2$
$I_y$ = 241 in.$^2$
$r_y$ = 3.07 in.

$$G_T = G_B = [2\times241/156]/[(0.4896+0.9431)\times2131/480] = 0.4858$$
$$P_u/A = 1000/25.6 = 39.1 \text{ ksi} \geq SRF = 0.525$$
$$G_T = G_B = 0.525 \times 0.4858 = 0.26 \geq K = 0.62$$
$$KL/r_y = 0.62\times156/3.07 = 31.5 \geq \phi_c F_{cr} = 39.5 \text{ ksi} > 39.1 \text{ ksi}$$
Use W12×87

(b) For exterior columns in braced frames, the connections should be designed as simply supported, and detailed so as to provide as little end restraint as possible, leading to no moments on the columns and an effective length factor of 1. This is the same as the design of columns in simply supported construction. For simply supported construction, the required exterior column is a W12×58.

## WEIGHT COMPARISON

For comparison a similar braced frame assuming simply supported construction was designed. The interior columns for this case would be either a W12×106 or a W14×99. Recalling that for simply supported construction the required beam depth was 6 in. more than for semi-rigid construction, these columns were designed using a column height of 13 ft 6 in.

The following table offers a comparison of weights per floor:

**Table 2.**
**Weight Comparison**

| | Semi-Rigid | Simply Supported |
|---|---|---|
| Interior Beam | W18×40—1600 lbs | W24×55 —2200 lbs |
| Exterior Beams | W18×50—2000 lbs | W24×55 —2200 lbs |
| Interior Column | W12×87—1131 lbs | W12×106—1431 lbs |
| Exterior Column | W12×58— 754 lbs | W12×58 — 783 lbs |

For a building with four bays the total weight for each frame per floor would be:

| | |
|---|---|
| Semi-Rigid Composite | 12,101 lbs |
| Simply Supported Composite | 14,659 lbs |

In this example, the use of semi-rigid composite connections creates a 17 percent savings in the weight of steel required.

## CONCLUSIONS

The two examples given in this paper show how the use of semi-rigid composite connections can lead to substantial weight savings for braced frames. The design method required to obtain these benefits is relatively simple, and can easily be programmed into a personal computer. A method is given in this procedure to calculate the deflections due to live load, which incorporates the non-linear connection behavior. This design procedure assumes unshored construction, which leads to smaller weight savings in the beams, as the dead loads are carried by rotationally weak connections instead of by the semi-rigid composite connections. In shored construction the entire load will be carried by the semi-rigid composite connections, further reducing the weight of beams required, and expanding the range for which this method would provide savings.

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## APPENDIX A: LIST OF SYMBOLS

$S_{dl}$ = service dead load
$F_{dl}$ = factored dead load
$S_{ll}$ = service live load
$F_{ll}$ = factored live load
$P_D$ = point load on girder due to service dead load
$P_L$ = point load on girder due to service live load
$P_{Df}$ = factored *PD*
$P_{Lf}$ = factored *PL*
$P_f$ = factored point load on girder
$S_{cdl}$ = construction dead load
$F_{cdl}$ = factored construction dead load
$S_{cll}$ = construction live load
$F_{cll}$ = factored construction live load
$F_{cl}$ = factored construction loads
$P_{cf}$ = factored point load on girder (construction)
$M_{cf}$ = maximum construction moment
$M_{u,c}$ = maximum centerline moment at ultimate
$M_p$ = plastic moment capacity of steel section alone
$M_{pc}$ = plastic moment capacity of composite section
$MD_u$ = design dead load moment
$M_{u,e}$ = ultimate live load moment at the connection
$M_{u,s}$ = ultimate live load moment assuming simple supports
$S_s$ = elastic section modulus of the steel beam
$Z_s$ = plastic section modulus of the steel beam
$H_a$ = ultimate horizontal force in seat angle
$d$ = depth of the steel beam
$Y_2$ = distance from top of the beam to center of slab force
$A_{sl}$ = required area of seat angle leg
$V_b$ = shear force on an individual bolt
$l_a$ = width of seat angle
$t_a$ = thickness of seat angle
$F_{yr}$ = yield strength of slab reinforcing steel
$F_{ysl}$ = yield strength of seat angle steel
$A_r$ = area of slab reinforcement required
$M_{\theta s}$ = moment capacity at service load rotation
$M_{\theta u}$ = moment capacity at ultimate load rotation
$M_{fef}$ = fixed end moment for factored loads
$\theta_{ssf}$ = support rotation for simply supported beam under factored loads
$M_{fe}$ = fixed end moments for service loads
$\theta_{ss}$ = support rotation for simply supported beam under service loads
$I_{cb}$ = moment of inertia for composite section
$I_{LBp}$ = lower bound moment of inertia for positive moment
$I_{LBn}$ = lower bound moment of inertia for negative moment
$\theta_{sr}$ = rotation at intersection of service beam line and moment-rotation curve
$\theta_{srf}$ = rotation at intersection of factored beam line and moment-rotation curve
$M_{sr}$ = service load moment at support
$M_{srf}$ = factored load moments at support