

# Effective Length Factor for the Design of X-bracing Systems

ADEL A. EL-TAYEM and SUBHASH C. GOEL

X-bracing systems made with single angles are quite common in steel structures. In current practice, the design of X-bracing members may be performed in one of two ways. The first method is to ignore the strength of the compression diagonal in resisting the imposed loads. Conversely, the second method recognizes the contribution of the compression diagonal. This method requires overall buckling of the full diagonal in the out-of-plane direction as well as buckling of one half diagonal about the weak principal axis to be investigated in calculating the effective slenderness ratio. However, for all the hot-rolled single equal leg angles (listed in Ref. 4), the radius of gyration about the weak axis, Z-axis, is greater than half of that in the out-of-plane direction. Accordingly, buckling of full diagonal in the out-of-plane direction governs the strength of single-angle compression diagonals.

The purpose of this paper is to provide designers with some recommendations regarding the effective length factor to be considered in the design of X-bracing systems. The recommendations were drawn from experimental and theoretical study of full-scale X-bracing specimens. For more details, refer to the original report by the authors.<sup>2</sup>

## EXPERIMENTAL STUDY

### Test Specimens

Five full-scale single-angle specimens and one double-angle X-bracing specimen made of A36 steel were included in this study. The test setup is shown in Fig. 1, and the important geometrical properties of the test specimens are given in Table 1. All of the single-angle specimens were made from equal-leg angles, whereas unequal-leg angles were used in the double-angle specimen. End gusset plates were provided to connect the bracing members to the testing frame. Fillet welds were used for all the connections. The working stress design method per the AISC Specification,<sup>4</sup> together with Whitmore's concept<sup>5</sup> and Astaneh's recommenda-

*Adel A. El-Tayem formerly Graduate Student, Department of Civil Engineering, University of Michigan, Ann Arbor, Michigan.*

*Subhash C. Goel is Professor of Civil Engineering, University of Michigan, Ann Arbor, Michigan.*

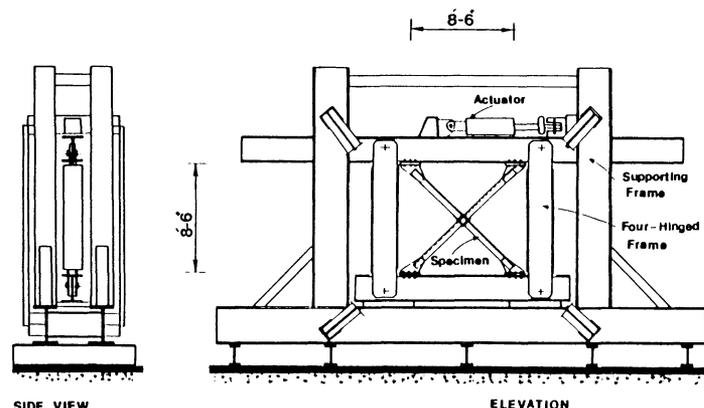


Fig. 1. Test set-up

tions,<sup>1</sup> were used to design and detail the test specimens. Typical details of a single-angle specimen are shown in Fig. 2. Quasi-static cyclic loading was used for the tests with the small early deformation amplitudes used to help study the first buckling load of the specimens.

### Test Results

During the first cycle, and at small deformation levels, the compression diagonal showed symmetrical lateral deformations in the out-of-plane direction over its entire length without any drop in load-carrying capacity. But, as the displacement level was increased, drastic changes in behavior were noticed. These changes involved both the buckling configuration and the axis of buckling. Only one half of the compression diagonal buckled about its weak axis (Fig. 3). It was after this that a drop in the load-carrying capacity was observed in these specimens (Fig. 4). This biased buckling can be explained by the interaction between the two cross diagonals. The interconnection provides an elastic restraint against both lateral and rotational deformations of the compression diagonal at the point of intersection. The restraint is sensitive to the force and sag in the tension diagonal. Therefore, the tension member, at small displacement level, was incapable of preventing the lateral deformation of the full compression diagonal. The increase in the force and sagging of the tension member

**Table 1. Geometrical Properties of Test Specimens**

Specimen	Cross Section	A in. <sup>2</sup>	$\frac{L^a}{r_z}$	$\frac{b^b}{t}$	$t_g^c$
AW0	L2½ x 2½ x ¼	1.19	142.6	10.0	.438
AW1	L4 x 4 x ¼	1.94	88.6	16.0	.375
AW2	L4 x 4 x ⅜	2.86	87.6	10.7	.625
AW3	L2½ x 2½ x 5/16	1.46	143.0	8.0	.438
AW4	L2½ x 2½ x 3/16	.902	141.4	13.3	.313
DAW1	2L 3 x 2½ x ¼	2.63	(62.0) <sup>d</sup>	10.0	.563

<sup>a</sup>L = 71 in. (1.81m)

<sup>b</sup> $\frac{b}{t}$  stands for width-thickness ratio

<sup>c</sup>t<sub>g</sub> stands for thickness of gusset plate

<sup>d</sup>The value in parenthesis stands for out-of-plane buckling

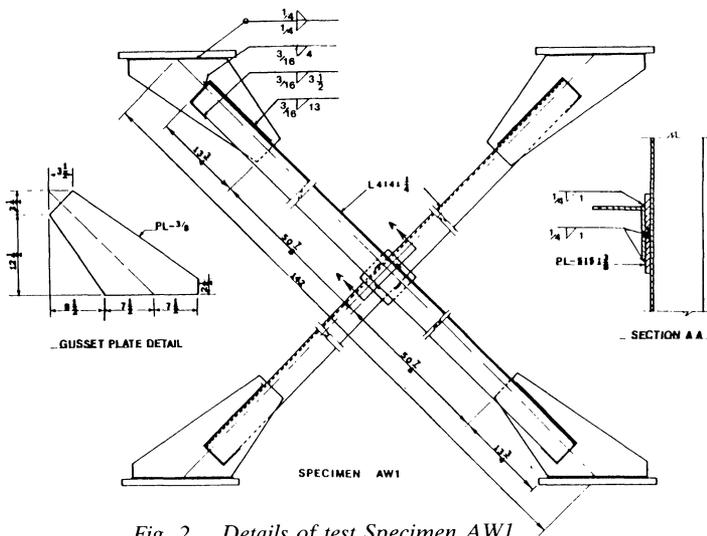


Fig. 2. Details of test Specimen AW1

associated with the increase in the displacement level produce a restraining interaction force large enough to cause biased buckling in one-half length of the compression diagonal.

The measured buckling loads at the instant of occurrence of biased buckling for each of the diagonals, which alternately sustained the compressive load, were measured by the strain gages. The measured buckling loads in the first cycle (normalized by  $P_y$ ) are plotted against the calculated normalized buckling loads per the AISC formulas (safety factor removed) (Fig. 5). Results show that despite early display of lateral deformation over entire length, the buckled halves of the system were capable of developing buckling loads (in the first cycle) equivalent to those per the AISC formulas safety with the value of effective length being equal to about .85 times the half diagonal length.

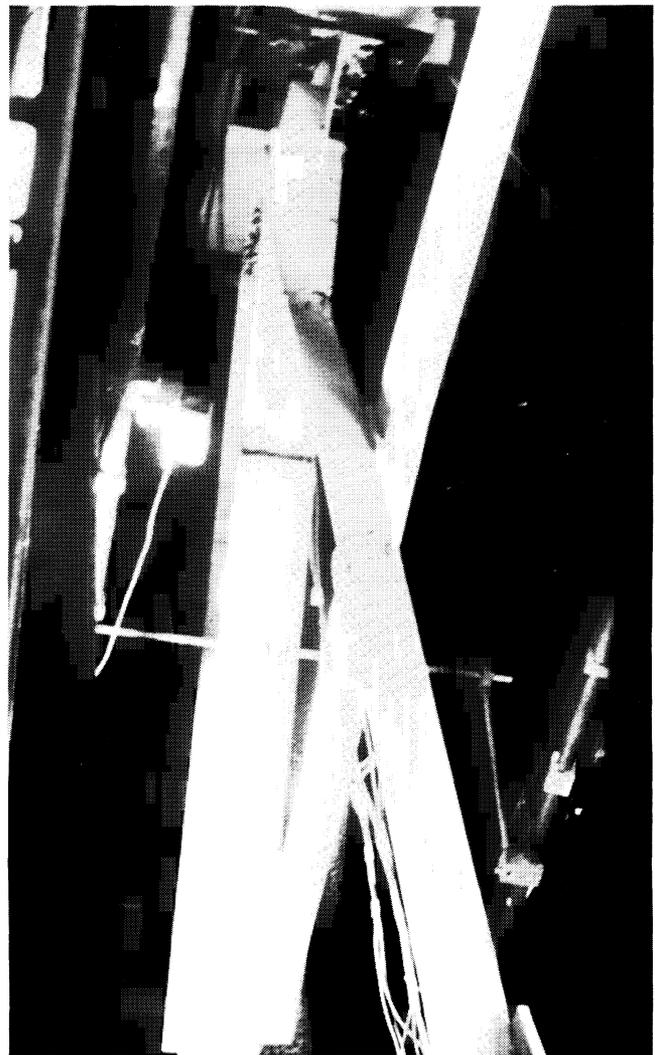


Fig. 3. Biased buckling of Specimen AW4

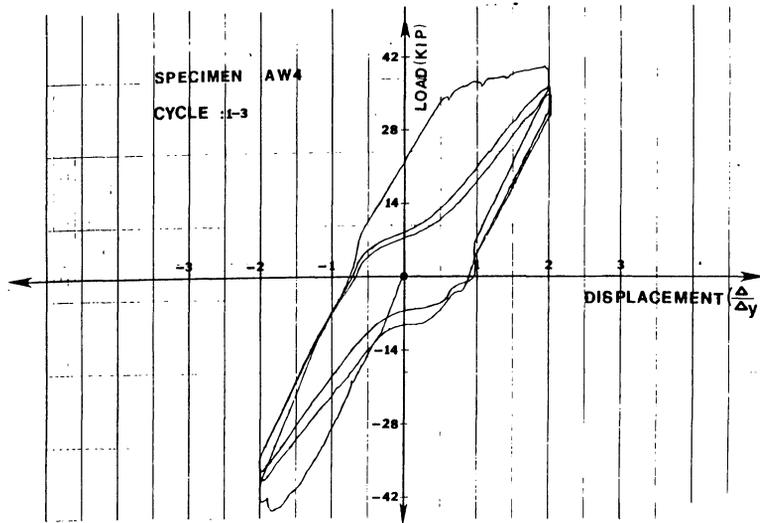


Fig. 4. Recorded hysteresis loops of Specimen AW4, first three cycles

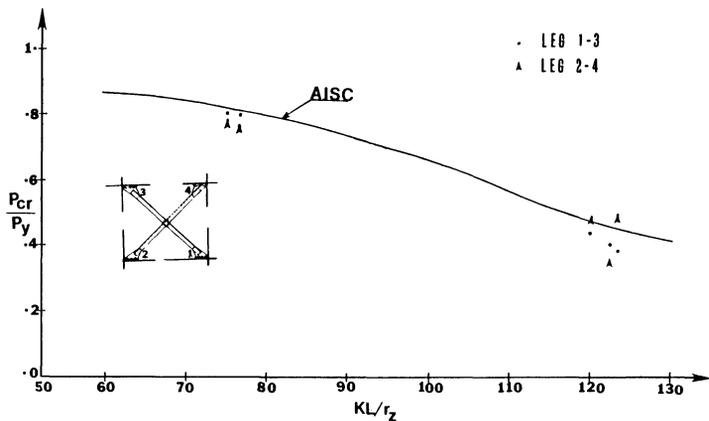


Fig. 5. Normalized first buckling loads compared to AISC values

## ANALYTICAL STUDY

### Model

The biased buckling mode in which only one half of the compression member deformed about its weak axis was the prevalent mode for all of the tested specimens. As a result, only one half of the compression member is represented by a model shown in Fig. 6. This model incorporates the imposed elastic restraints against both rotational and translational deformations. End connections and the interaction at the intersection joint provide these restraints.

The differential equation of equilibrium in the biased buckling mode for this member can be written as follows:

$$EI \frac{d^4 y}{dx^4} = P \frac{d^2 y}{dx^2} = 0 \quad (1)$$

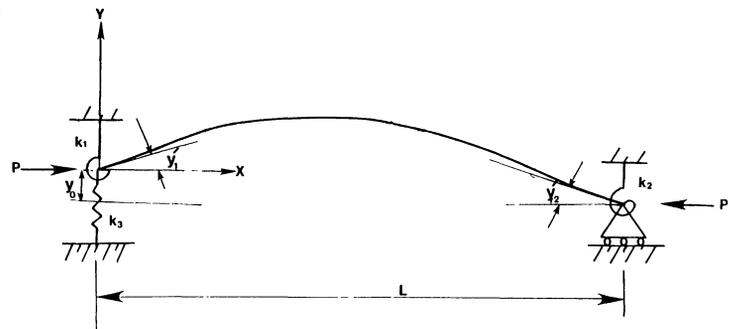


Fig. 6. Model for buckled half to determine K factor

where,

$EI$  is the flexural rigidity of the cross section about buckling axis,  $P$  is the applied compressive load,  $x$  is the distance measured from the left end of the member and  $y$  is the lateral displacement transverse to the buckling axis of the cross section. Introduction of the general boundary conditions applicable to this model and seeking the non-trivial solution leads to the following equation:

$$[1 - \eta_1 - \eta_2 - \eta_1 \eta_2 \Phi^2 + Z(\eta_1 + \eta_2)] \Phi \sin \Phi + [2 - 2Z + \Phi^2(\eta_1 + \eta_2)] \cos \Phi - 2 + 2Z = 0 \quad (2)$$

where,

$k_1, k_2$  and  $k_3$  are the stiffnesses of the attached springs,  $L$  is half diagonal length and the non-dimensional parameters  $\eta_1, \eta_2, \Phi$  and  $Z$  are defined by the relations:

$$\eta_1 = \frac{EI}{k_1 L}, \quad \eta_2 = \frac{EI}{k_2 L}, \quad \Phi = \frac{\sqrt{P^2 L}}{EI} \quad \text{and} \quad Z = \frac{P}{P - k_3 L}$$

Equation 2 is to be solved for discrete values of the load parameter  $\Phi$ . Consequently, the effective length factor  $K$  is determined by the following equation:

$$K = \frac{\pi}{\Phi} \quad (3)$$

Complete details of underlying assumptions and derivation of Eq. 2 are given in the report.<sup>2</sup>

### Special Cases

1. Pin connected at one end and fixed at the other

*Solution:*

For the above boundary conditions, the values of  $k_1, k_2$  and  $k_3$  are  $0, \infty$  and  $\infty$ , respectively. Consequently,  $\eta_1$  and  $\eta_2$

become equal to  $\infty$  and  $Z$  equal to  $O$ . After substitution for the above values, Eq. 2 becomes:

$$- \sin\Phi + \Phi \cos\Phi = 0$$

This equation has a solution at  $\Phi = 4.488$  and from Eq. 3  $K$  is found to be equal to 0.70.

2. Member fixed at both ends but free to sway at one end only

*Solution:*

For such conditions the fixed end implies that  $k_1$  is equal to  $\infty$  and, therefore,  $\eta_1$  is equal to 0. At the other end, restraint against rotation makes  $\eta_2$  equal to 0 while the free-to-sway condition implies that  $k_3$  is equal to 0, thus,  $Z$  becomes equal to 1. Substitution for the values of  $\eta_1$ ,  $\eta_2$  and  $Z$  in to Eq. 2 gives:

$$\Phi \sin\Phi = 0$$

This equation has a solution at  $\Phi = \pi$ , and from Eq. 3  $K$  is found to be equal to 1.

### Test Specimens

To calculate the effective length factor  $K$  for each of the tested specimens, the values of  $k_1$ ,  $k_2$  and  $k_3$  need to be determined first. The rotational spring stiffness  $k_2$  depends on the geometrical and mechanical properties of the end gusset plates, calculated as follows:

$$k_2 = \frac{EI_g}{L_g} \quad (4)$$

where,

$E$  is the modulus of elasticity,  $I_g$  is the moment of inertia of gusset plate at Sect. a-a as shown in Fig. 7, and  $L_g$  is the distance between the free and fixed edges of the gusset plate. The stiffnesses of the rotational and linear springs,  $k_1$  and  $k_3$ , at the intersection joint are functions of the torsional and flexural properties of the two cross diagonals as well as the magnitude of the applied force. These stiffnesses are determined by using the conventional stiffness approach.

Thus:

$$k_1 = \frac{EI}{L_m} \left( S_1 - r S_2^2 / S_1 \right) + \frac{2GJ}{L_m} \quad (5)$$

and

$$k_3 = \frac{EI}{L_m^3} \sum_{i=1}^3 \left( S_1 - r S_2^2 / S_1 \right)_i \quad (6)$$

where,

$i$  refers to the other three half diagonals,  $L_m$  is half length of the bracing angle,  $J$  is the torsion constant of the section and  $r$  is the degree of rotational fixity of end connections taken

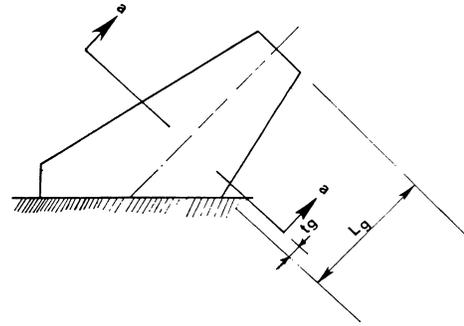


Fig. 7. Length and critical section of end gusset plate

to be equal to

$$\frac{S_1 \frac{EI}{L}}{S_1 \frac{EI}{L} + k_2}$$

The values of  $S_1$  and  $S_2$  are adopted from Ref. 3 and expressed as:

$$S_1 = \begin{cases} \frac{1 - \Phi \cot\Phi}{\frac{2 \tan\Phi/2}{\Phi} - 1} & \text{if, } P \leq 0 \\ \frac{\Phi \coth\Phi - 1}{1 - \frac{2 \tanh\Phi/2}{\Phi}} & \text{if, } P \geq 0 \end{cases}$$

and

$$S_2 = \begin{cases} \frac{\Phi \operatorname{cosec}\Phi - 1}{\frac{2 \tan\Phi/2}{\Phi} - 1} & \text{if, } P \leq 0 \\ \frac{1 - \Phi \coth\Phi}{1 - \frac{2 \tanh\Phi/2}{\Phi}} & \text{if, } P \geq 0 \end{cases}$$

A short computer program was developed to solve for the effective length factor  $K$  by an iterative procedure. The output of this program is compared with the experimental results of Table 2, where the latter were determined by back substitution in the AISC buckling formulas. The comparison shows that the experimental and theoretical values are in good agreement.

**Table 2. Theoretical and Experimental Values for Effective Length Factor, K**

Specimen	Stiffness			$K_{theo.}$	$K_{exp.}$
	$k_1$ (kip-in./rad.)	$k_2$ (kip-in./rad.)	$k_3$ (kip/in.)		
AW0	35.5	159.0	0.278	0.862	0.892
AW1	797.0	107.8	1.3	0.838	0.836
AW2	1405.1	516.0	2.048	0.806	0.873
AW3	37.7	161.9	0.359	0.863	0.887
AW4	34.5	72.9	0.216	0.875	0.846
DAW1	3620.0(3274.5) <sup>a</sup>	354.3(5. × 10 <sup>5</sup> )	3.400(3.11)	0.799(.577)	0.805

<sup>a</sup>The values in parentheses for specimen DAW1 are for in-plane buckling

**CONCLUSIONS**

The experimental and theoretical studies show the current design procedure underestimates the contribution of compression diagonal in resisting applied loads. Based on the experimental and theoretical results, the following conclusions can be drawn:

1. Design of X-bracing systems should be based on exclusive consideration of one half diagonal only.
2. For X-bracing systems made from single equal-leg angles, an effective length of .85 times the half diagonal length is reasonable.
3. The proposed theoretical model can be used for estimating the effective length factor in any direction and for any cross-sectional shapes.

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**REFERENCES**

1. *Astaneh-Asl, A., S. C. Goel and R. D. Hanson* Cyclic Behavior of Double Angle Bracing Members with End Gusset Plates *Report No. UMEE 82R7, Department of Civil Engineering, The University of Michigan, Ann Arbor, Mich., August 1982.*
2. *El-Tayem, A.A. and S.C. Goel* Cyclic Behavior of Angle X-Bracing with Welded Connections *Report No. UMCE 85-4, Department of Civil Engineering, The University of Michigan, Ann Arbor, Mich., April, 1985.*
3. *Maugh, L.C.* Statically Indeterminate Structures *J. Wiley and Sons, 1946, New York, N.Y.*

4. *American Institute of Steel Construction, Inc.* Specification for Design, Fabrication and Erection of Structural Steel for Buildings 1978, Chicago, Ill.
5. *Whitmore, R.E.* Experimental Investigation of Stresses in Gusset Plates *University of Tennessee Engineering Experiment Station Bulletin, No. 16, May, 1952.*

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