Multiple Bolt Anchorages: Method for Determining the Effective Projected Area of Overlapping Stress Cones

M. LEE MARSH and EDWIN G. BURDETTE

To determine the resistance concrete offers to tensile pull-out, the concept of the stress cone has been developed. When a tensile load is applied to an anchor bolt, failure may occur by pulling out a cone of concrete which radiates outward from the embedded head to the surface of the concrete.[1] The internal angle of the cone's apex is approximately 90°. The calculation of the concrete's resistance to pull-out is accomplished by applying a nominal concrete stress to the horizontal projected area of the potential failure cone. This is illustrated in Fig. 1.

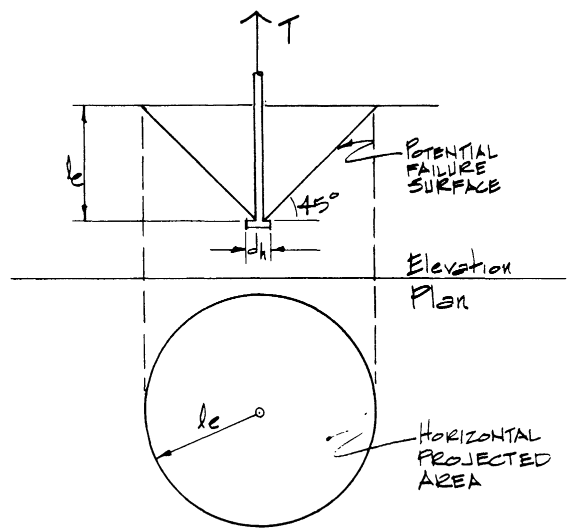

Fig. 1. Tensile pull-out cone

The calculation of the projected area of the stress cone is easily performed for a single anchor. The equation for this area is given by:[2]

$$A_p = \pi l_e(l_e + d_h)$$

However, when anchors are placed closely enough together that their individual stress cones overlap, calculation of the effective projected area becomes more difficult. As anchorage configurations become more irregular or complex, calculation of effective stress areas can become extremely difficult. Consider, for example, the anchorage configuration and projected areas shown in Fig. 2. The projected areas overlap threefold. Simple geometric manipulations are not adequate to solve the problem, and thus calculation of the effective area requires considerable effort.

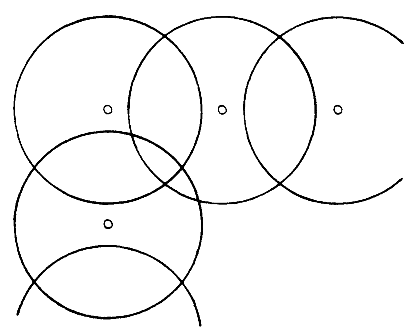

Fig. 2. Stress cones with multiple overlap

Because of the myriad of different anchorage configurations and the ensuing possibility of multiple overlaps, there is no easy panacea for the problem of calculating effective areas. For this reason a "building block" approach has been developed for determining these areas. While parts of this approach are exact in their predictions

M. Lee Marsh is a Structural Engineer with Midwest Technical Inc., Oak Ridge, Tennessee.
Edwin G. Burdette is Professor of Civil Engineering, The University of Tennessee, Knoxville, Tennessee.

of area, the sum total is not exact because some parts of the method are only approximate and lead to relatively small errors (1-3%).

It is always advisable to make a scale sketch of the configurations and their projected areas. This sketch allows the geometry of the problem to be recognized. Once the sketch is made, one or more of the following five area calculations can be employed to estimate the effective area. Simple extensions of any of these calculations can be used for configurations which do not permit evaluation by the five primary methods. Bolt holes are conservatively omitted from the calculations for the sake of simplicity.

The area calculation given in Fig. 3 will accurately evaluate the shaded area. This is a base area from which sub-areas can be subtracted. For instance if the triangular areas at midsides and the square area at the center are deducted from the base area, an accurate calculation of the effective area results.

The given equation for the base area is based on the determination of four individual areas as shown in Fig. 4.

The idea behind this area calculation may be expanded to any configurations of anchorage which are appropriate. Examples of application of the methods follow the explanations of the area calculations.

Method 2 facilitates calculation of the shaded area in Fig. 5. This area is the triangular sub-area referred to in the discussion of Method 1. Its calculation is based on the determination of two individual areas as illustrated in Fig. 6.

Method 3 (Fig.7) allows calculation of the area of a diagonally inscribed square. While the square does not quite conform to the shape of the excluded area, it does provide a way to estimate the area. The error incurred should be minimal and thus not detract greatly from the accuracy of the effective area calculations.

The method is based on finding the area of a square with the length of its sides equal to the diagonal distance d, minus twice the embedment depth l_e (Fig. 7). The square can actually be inscribed between the circles when x is equal to y. For unequal spacing (i.e. $x \neq y$) the square will not fit between the circles. While the equation for this case is not strictly correct, it is a way to calculate an approximate area. This method is an alternative to the use of more complex geometrical relationships.

Method 4 in Fig. 8 provides a means to approximate an interior, excluded area similar to that outlined in Method 3. The principles are much the same in that a slight error is incurred, but it is an error which does not greatly reduce the accuracy of the resulting calculations.

The inscribed square has sides with lengths equal to the out-to-out spacing, x or y, minus twice the embedment depth, l_e (Fig. 8). This square will become a rectangle if the distances x and y are unequal.

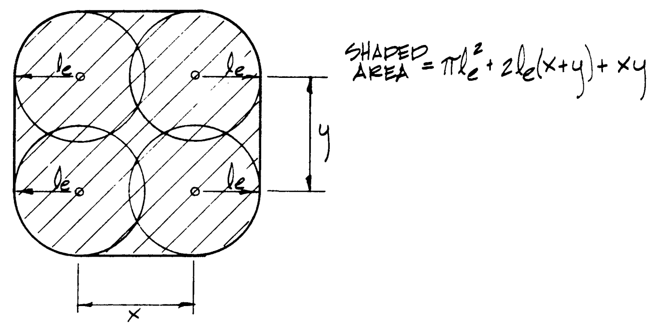

Fig. 3. Method 1 area calculation

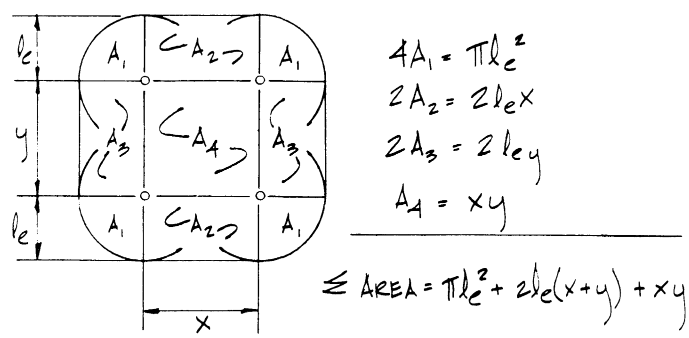

Fig. 4. Individual areas of Method 1

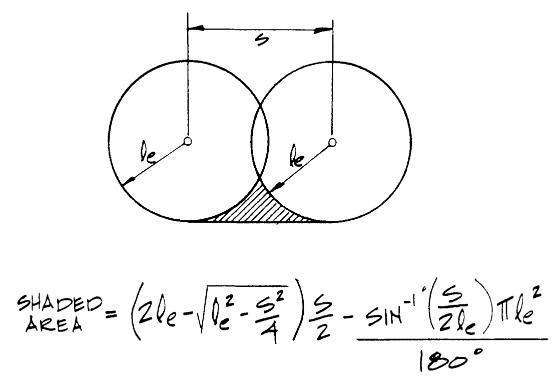

Fig. 5. Method 2 area calculation

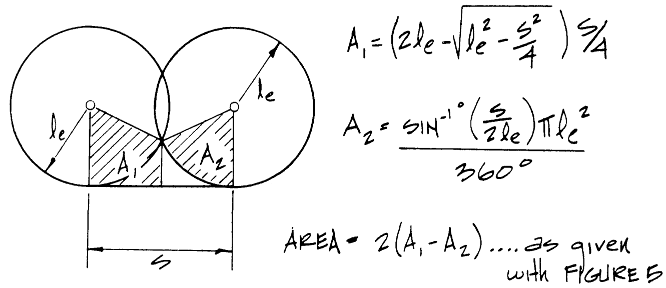

Fig. 6. Individual areas of Method 2

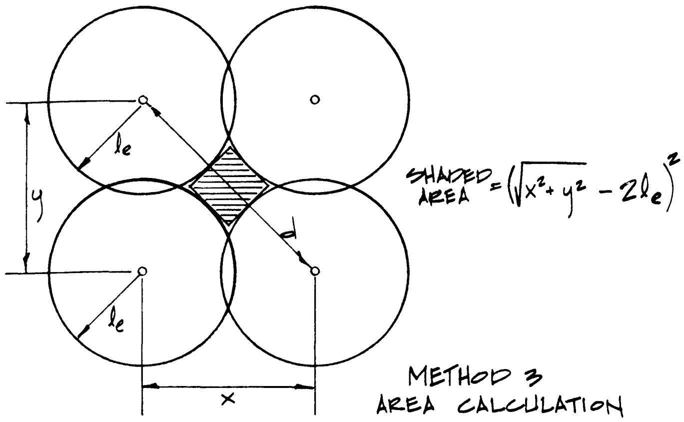

Fig. 7. Method 3 area calculation

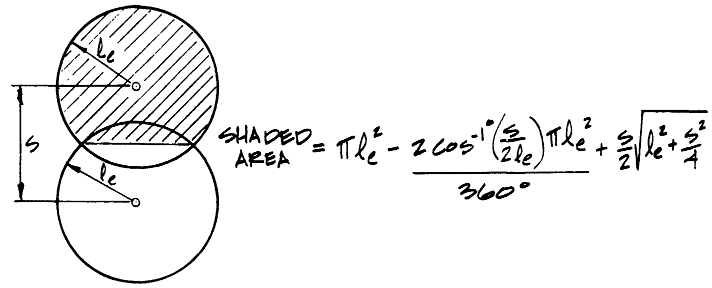

Fig. 9. Method 5 area calculation

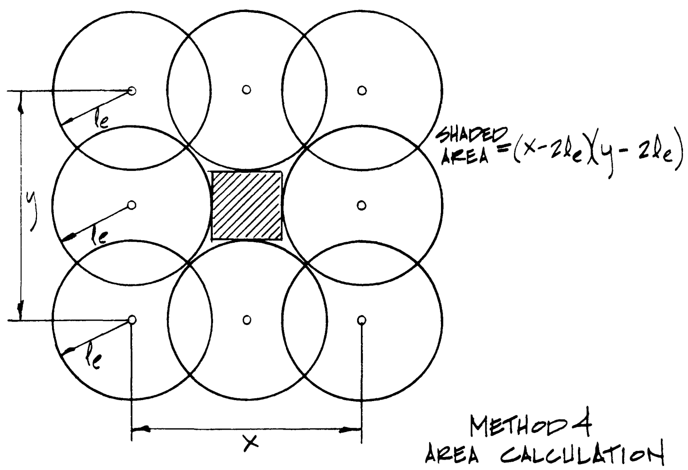

Fig. 8. Method 4 area calculation

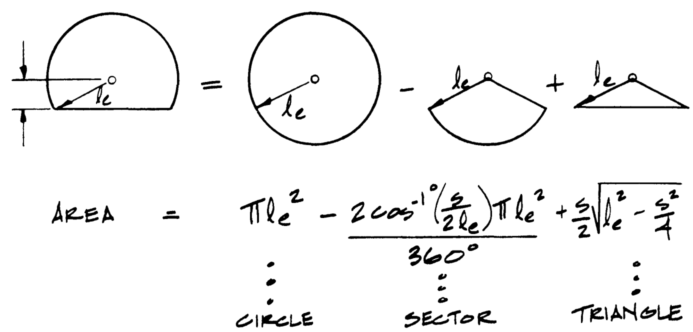

Fig. 10. Individual areas of Method 5

Method 5 allows calculation of the area of a circle which has a segment clipped off, as shown in Fig. 9. The equation given to calculate this area can be readily modified to accommodate more than just one overlap. However, this calculation is possible only if the overlaps are independent of one another, that is, if each overlap only involves two circles and not three.

The area is found by calculating the area of a complete circle, subtracting the area of a sector (2), and then adding back the area of the triangle (2). This procedure is illustrated in Fig. 10.

The area calculations outlined in the five methods presented can be used collectively to estimate the effective projected area of a multi-bolt anchorage. It must be emphasized that some areas will be added while some will be subtracted in order to estimate the effective area.

REFERENCES

1. *Cannon, R. W., D. A. Godfrey and F. L. Moreadith* Guide to the Design of Anchor Bolts and Other Steel Embedments *Concrete International, July, 1981.*
2. *Tuma, J. J.* Engineering Mathematics Handbook *McGraw-Hill Book Company, New York, N.Y., 1970.*

APPENDIX—EXAMPLE CALCULATIONS

Example 1

Calculate the effective area of the anchorage configuration given in Fig. A1.

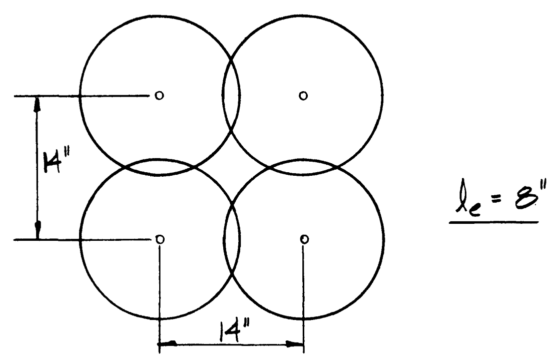

Fig. A1. Example 1: Projected area

Solution: Using Methods 1, 2 and 3:

$$A_1 = \pi l_e^2 + 2l_e (x+y) + xy$$

$$A_1 = \pi 8^2 + 2(8)(14+14) + 14 (14) = 845 \text{ in.}^2$$

$$A_2 = [2l_e - \sqrt{l_e^2-(S^2/4)}] \left(\frac{S}{2}\right) - \left(\frac{\sin^{-1}(S/2l_e)\ \pi l_e^2}{180°}\right)$$

$$A_2 = [2(8)-\sqrt{64-(14^2/4)}] \left(\frac{14}{2}\right) - \left(\frac{\sin^{-1}(S/2l_e)\ \pi l_e^2}{180°}\right)$$

$$A_2 = 16.70 \text{ in.}^2$$

$$A_3 = (\sqrt{x^2+y^2} - 2l_e)^2$$

$$A_3 = [\sqrt{14^2+14^2} - 2(8)]^2$$

$$A_3 = 14.43 \text{ in.}^2$$

Effective Area, $A_{eff} = A_1 - 4A_2 - A_3$

$$A_{eff} = 845 - 4(16.70) - 14.43$$

$$A_{eff} = 764 \text{ in.}^2$$

Check

Using Method 5:

$$A_5 = \pi l_e^2 - \left(\frac{2\cos^{-1}(S/2l_e)\pi l_e^2}{360°}\right)(2) + (2)\left(\frac{S}{2}\right)\sqrt{l_e^2-S^2/4}$$

$$A_5 = \pi 8^2 - \left(\frac{2\cos^{-1}(14/16)\pi 8^2(2)}{360°}\right) + 14\sqrt{8^2-14^2/4}$$

$$A_5 = 190.6 \text{ in.}^2$$

$$A_{eff} = 4(A_5)$$

$$A_{eff} = 4(190.6)$$

$A_{eff} = 762$ in.^{2}—checks earlier results

Example 2

Calculate (estimate) the effective area of the anchorage shown in Fig. A2.

Solution:

$$A_1 = \pi l_e^2 + 2l_e (x+y) + xy$$

$$A_1 = \pi(10)^2 + 2(10)(36+24) + 36(24)$$

$$A_1 = 2{,}378 \text{ in.}^2$$

$$A_2 = [2l_e - \sqrt{l_e^2-(S^2/4)}]\left(\frac{S}{2}\right) - \left(\frac{\sin^{-1}(S/2l_e)\pi l_e^2}{180°}\right)$$

$$A_2 = [2(10)-\sqrt{10^2-(12^2/4)}]\left(\frac{12}{2}\right) - \left(\frac{\sin^{-1}[12/2(10)]\pi 10^2}{180°}\right)$$

$$A_2 = 7.65 \text{ in.}^2$$

$$A_4 = (y-2l_e)(x-2l_e)$$

$$A_4 = [36-2(10)][24-2(10)]$$

$$A_4 = 64 \text{ in.}^2$$

$$A_{eff} = A_1 - 12A_2 - A_4$$

$$A_{eff} = 2{,}378 - 12(7.65) - 64$$

$$A_{eff} = 2{,}222 \text{ in.}^2$$

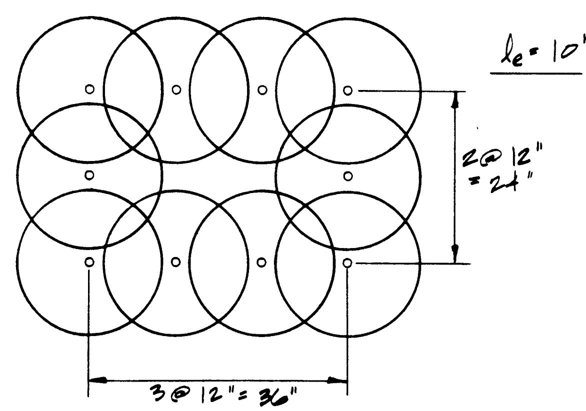

Fig. A2. Example 2: Projected area

ERRATA

Multiple Bolt Anchorages: Method for Determining the Effective Projected Area of Overlapping Stress Cones

Paper by M. LEE MARSH and EDWIN G. BURDETTE (1st Quarter, 1985)

On Page 31, Figure 9, correct the equation for the "shaded area" to read as follows:

$$\text{SHADED AREA} = \pi l_e^2 - \frac{2\cos^{-1}\left(\frac{s}{2l_e}\right)\pi l_e^2}{360\,\text{deg}} + \frac{s}{2}\sqrt{l_e^2 - \frac{s^2}{4}}$$

On page 31, Figure 10, insert missing dimension in far left partial circle, $s/2$.