Analysis and Design of Framed Columns Under Minor Axis Bending

T. KANCHANALAI AND LE-WU LU

Columns in a building frame are often subjected to combined axial load and bending moment as a result of the frame action in resisting applied loads. A major concern in the design of framed columns is the effect of instability, which may reduce significantly the strength of the column or entire structure. There are two types of instability failure to which careful consideration must be given in design: member instability and overall frame instability. Figure 1a shows a typical load versus lateral deflection relationship of an unbraced frame. The gravity load P acting through the lateral deflection Δ produces a secondary overturning moment, called P-Δ moment in the current literature. This additional moment reduces the strength and stiffness of the structure. Failure occurs when the lateral stiffness becomes so small that it is insufficient to resist any increase of the applied load. This is represented by the peak (instability limit) of the load-deflection curve. The member instability effect results from the axial load acting through the deflection δ occurring within the individual columns (Fig. 1b). It is obvious that if a frame is fully braced against sidesway only member instability effect need be considered in the design of its columns. Experience has shown that in a sway frame, frame instability is considerably more important than member instability, but the latter could lead to premature local failure. It has been reported that member instability may limit the load-carrying capacity of an unbraced frame even if the structure as a whole still has adequate stiffness to resist frame instability.[1]

Much of the previous work on member instability was on columns subjected to combined axial load and major axis bending. Columns bent about the minor axis have received little attention, although there are several beneficial aspects.[2,3] These columns can usually develop their full in-plane strength without the occurrence of lateral-torsional buckling. Also, the shape factor about the minor axis is about 35 percent larger than that about the major axis. The column formulas contained in most design specifications are based essentially on the studies of columns subjected to major axis bending. A discussion of the development of the currently used design formulas can be found in Ref. 3.

As for overall frame instability, the past work was concerned mostly with building frames in which the columns are oriented for major axis bending.[5-7] Although various approaches have been proposed to account for this effect in design,[8-15] specific code provisions are still being developed at this time.

This paper presents a detailed study of the effects of member and frame instability in framed columns under minor axis bending. An important objective is to develop

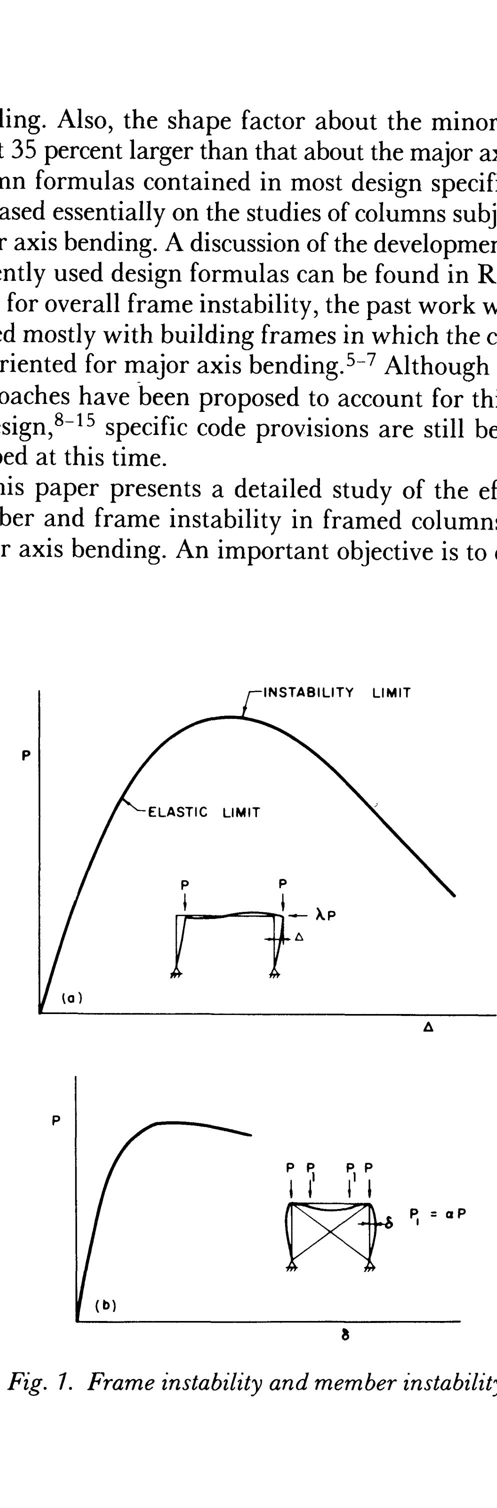

Fig. 1. Frame instability and member instability

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suitable design procedures which will adequately take into account these effects. Specifically, the following are presented in the paper:

1. Ultimate strength analysis of non-sway, pinned-end columns. Numerical solutions for three cases of loading are given.
2. Development of improved interaction formulas for non-sway columns.
3. Analytical study of the behavior of non-sway columns with end restraints.
4. Ultimate strength solutions of two sway frames subjected to combined gravity and lateral loads.
5. A proposed procedure for the design of columns in sway frames. The procedure makes use of a new set of column interaction formulas incorporating the concept of direct moment amplification.

The ultimate strength solutions presented are obtained for wide-flange columns made of A36 steel (yield stress F_y = 36 ksi). The solutions consider the effect of cooling residual stresses. The magnitude and distribution of the residual stresses are the same as those assumed in the previous studies on beam-columns bent about the major axis. Examples are given to illustrate the application of the new design formulas and procedures.

CURRENT DESIGN PROCEDURES

In the allowable-stress method of design, a first-order elastic analysis is performed at the working load, neglecting any effect of instability, and the resulting bending moment and axial force distribution is then used to proportion the members. The formulas used in designing the columns are:

$$\frac{f_a}{F_a} + \frac{C_m f_b}{\left(1 - \frac{f_a}{F'_e}\right) F_b} \leq 1.0 \quad (1)$$

$$\frac{f_a}{0.6\,F_y} + \frac{f_b}{F_b} \leq 1.0 \quad (2)$$

(See Part 1 of the AISC Specification for notation.) The first formula checks the column against possible failure by instability and the second insures that no excessive yielding occurs at the ends of the column. The problem of frame instability was not considered initially in the development of these formulas. They are based on the approximate ultimate strength interaction equations for beam-columns subjected to end moments about the major axis. The equations are:

$$\frac{P}{P_{cr}} + \frac{C_m M_o}{\left(1 - \frac{P}{P_e}\right) M_m} = 1.0 \quad (3)$$

$$\frac{P}{P_y} + \frac{M_o}{1.18\,M_p} = 1.0 \quad ; \quad M_o \leq M_p \quad (4)$$

in which P_{cr} represents the critical buckling load of the column and may be determined approximately from the basic column curve recommended by the Structural Stability Research Council (SSRC):

$$P_{cr} = P_y \left[1 - 0.5\left(\frac{KL/r}{C_c}\right)^2\right] \quad \text{for } \frac{KL}{r} < C_c \quad (5a)$$

$$P_{cr} = P_e = \frac{\pi^2 EI}{(KL)^2} \quad \text{for } \frac{KL}{r} \geq C_c \quad (5b)$$

For columns subjected to minor axis bending, M_m in Eq. (3) is equal to M_p, the full plastic moment about the same axis. The applicability of Eqs. (3) and (4) to columns bent about the minor axis has not been fully established, even for the case of symmetrical bending.

In Eqs. (1) and (3), the expressions $1/(1 - f_a/F'_e)$ and $1/(1 - P/P_e)$ are called the amplification factors and have the effect of amplifying the computed bending stress f_b or the moment M_o. The factor C_m is to adjust for the shape of the moment diagram. For non-sway columns, the AISC Specification gives:

$$C_m = 0.6 + 0.4\beta \quad (6)$$

in which β is the end moment ratio ($\beta = +1.0$ for the case of symmetrical single curvature bending and $\beta = -1.0$ for antisymmetrical double curvature bending). A limiting value of $C_m = 0.4$ is specified, which was established from studies on lateral-torsional buckling of columns under major axis bending.[16] Since lateral-torsional buckling is not a problem in the case of minor axis bending, this restriction on C_m could probably be removed.

Equations (1) and (2) are in use in the design of columns in both sway and non-sway frames. To account for the restraining effect offered by the adjacent members and the overall frame action, the actual length of the column is modified by an effective length factor K. The values of F_a and F'_e in Eq. (1) are then calculated using the effective column length, which is smaller than the actual length for columns in a non-sway frame and is greater than the actual length in a sway frame. The use of an effective length greater than the actual length in sway frame design is to recognize, in an indirect way, the effect of frame instability. An additional provision is to use a C_m value of 0.85, which is likely to be greater than that required by Eq. (6). The reason for this is that double curvature bending (negative values of β) often prevails in framed columns, especially when the frame is also subjected to lateral load. It is apparent that both measures may result in an increase in the sizes of the columns but not the girders. On the other hand, if the design is governed by Eq. (2), then nowhere is the effect of frame instability taken into account. This may lead to unsafe designs.

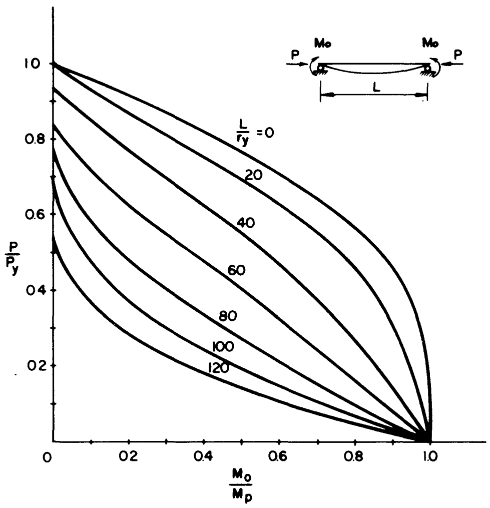

Fig. 2. Ultimate strength of columns subjected to equal end moments

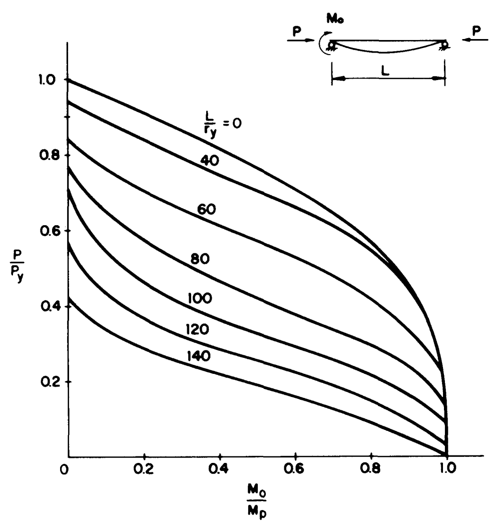

Fig. 3. Ultimate strength of columns subjected to moment at one end

Several studies have recently been made to examine the adequacy of the current design procedure. It has been reported that the use of K-factors greater than 1.0 and C_m = 0.85 in column design increases the strength of sway frames only slightly, and this increase is likely to be less than the reduction caused by the P-Δ moment.[6,7] If the P-Δ moment is large in comparison with the lateral load moment, then the present approach could produce designs with a load factor less than 1.30 (this is the load factor specified in Part 2 of the AISC Specification for the case of combined loading). No similar study has been carried out on frames with weak-axis column orientation.

The effective length factor is usually determined for each individual column using the alignment chart for the "sway permitted case". K values as large as 3 or 4 are not uncommon, and they may differ widely for the individual columns in the same story.[8] A more rational approach in which the amplification factors of the individual columns are replaced by a single storywise amplification factor has been suggested.[1] This is known as the "modified effective length" approach, and the required amplification factor is given by $1/(1 - \Sigma f_a/\Sigma F'_e)$ or $1/(1 - \Sigma P/\Sigma P_e)$, in which Σ represents summation over all the columns in the story. F'_e and P_e are based on the effective column length.

Methods which permit the direct inclusion of the P-Δ moment in design calculations have been proposed. In one of the methods, known as the P-Δ method, the secondary moment is determined through a series of successive iterations, starting with the moment and deflection from a first-order analysis.[12,13] The secondary moment thus obtained is then included in proportioning the members. In another method, the second-order moment is calculated by applying an amplification factor to the first-order moment, much like the procedure used to account for member instability. This method will be referred to as the "direct moment amplification" method. In both the modified effective length approach and the direct moment amplification approach, the columns are treated as non-sway columns and their design is governed by Eqs. (1) and (2). The actual column length (K = 1.0) is used in determining F_a and F'_e and C_m is that given by Eq. (6).

NON-SWAY UNRESTRAINED COLUMNS

Ultimate strength solutions of pinned-end columns subjected to three types of applied load have been obtained. Results are presented in the form of interaction curves for beam-columns subjected to (1) equal end moments ($\beta = +1.0$), (2) one end moment ($\beta = 0$), and (3) a concentrated lateral load at mid-span. The results in Figs. 2 and 3 are obtained by numerically integrating the moment-thrust-curvature relationships using the column-deflection-curve approach.[17] The curves in Fig. 4 are adapted from the solutions for columns in a sway frame whose height is equal to half of the column length and with an infinitely stiff girder.[18] Intersections on the vertical axis represent the ultimate strength of the columns subjected to pure axial compression.

A comparison of the column strength shown in Fig. 3 with the strength predicted by Eq. (3) is given in Fig. 5 for three column slenderness ratios.* The agreement is not considered satisfactory. For columns of low slenderness ratio, Eq. (3) is very conservative and may underestimate the moment-carrying capacity by more than 100% in some

* *To make the comparison consistent, the first term in Eq. (3) assumes the value defined by the intersection on the vertical axis of the theoretical curve of Fig. 3.*

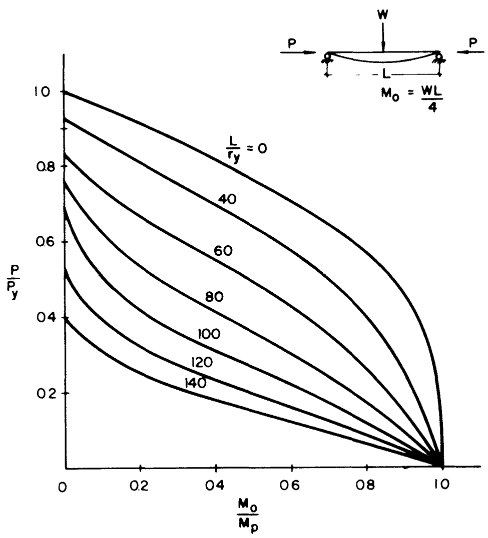

Fig. 4. Ultimate strength of columns subjected to concentrated load at midspan

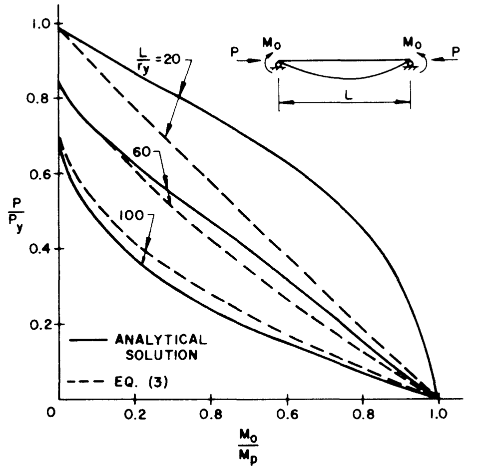

Fig. 5. Comparison of analytically determined column strength with predicted strength by Eq. (3)

cases. For slender columns, on the other hand, Eq. (3) becomes unconservative.* It is recalled that for major axis bending Eq. (3) has been found to give good predictions of column strength.[4]

Since Eq. (3) does not provide good predictions when applied to columns bent about the minor axis, it is highly unlikely that the current design procedure, which is based on this equation and Eq. (4), would yield accurate results. It is also felt that the current procedure can not be signifi-

* *The accuracy of Eq. (3) may be improved by using $C_m = 1 + 0.27\, P/P_e$.*

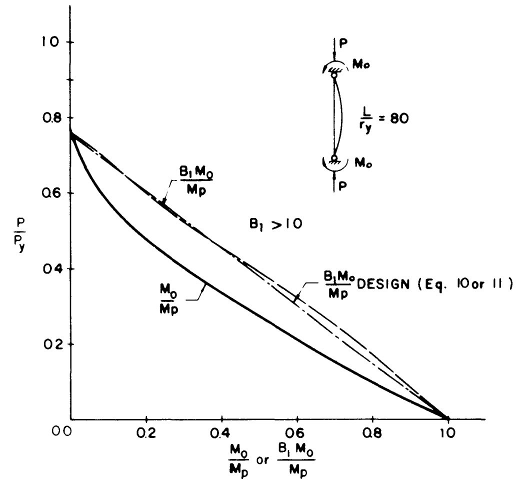

Fig. 6. Second-order moment in column subjected to equal end moments

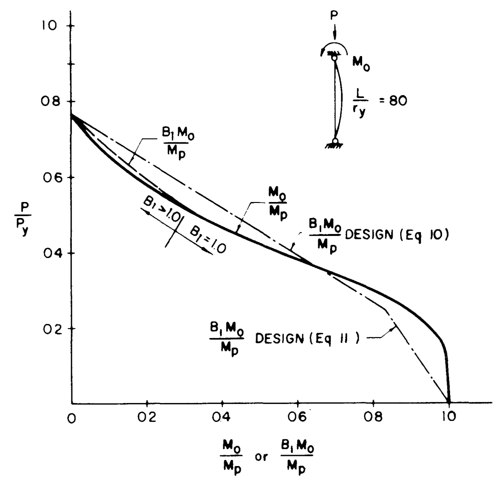

Fig. 7. Second-order moment in column subjected to moment at one end

cantly improved by merely improving Eq. (3).* A different design procedure is therefore developed. In this procedure, column strength is determined by two new interaction formulas, which retain all the important features of Eq. (3). Included in these formulas is an amplification factor B_1, whose value should always be greater than or at least equal to 1.0:

$$B_1 = \frac{C_m}{1 - \dfrac{P}{P_e}} \geq 1.0 \qquad (7)$$

* *A possible way to improve Eq. (3) is described in the Appendix.*

New coefficients are introduced into the formulas to allow for a more accurate evaluation of the effect of moment amplification.

The ultimate strength solutions given in Figs. 2 and 3 are used in "reverse" to develop the new interaction formulas. For each column, B_1 values are first calculated at various levels of axial load. These values are then multiplied by the M_o values given by the interaction curves. Figure 6 shows the resulting B_1M_o versus P/P_y relationship for a column with $L/r_y = 80$ and subjected to symmetrical bending. Another plot is given in Fig. 7 for the same column, but having only one end moment. In the latter case, B_1M_o is equal to M_o (that is, $B_1 = 1.0$) for P/P_y between 0 and 0.5. Another aspect to be noted in the calculation of B_1 is that inelastic column action is considered in determining the parameter P_e. The basic SSRC column curve (Eq. 5a) adopted in this study implies that columns buckle inelastically when the critical load is between $0.5P_y$ and P_y. The buckling load may be determined by replacing elastic modulus E by the tangent modulus E_t given by

$$E_t = E \cdot 4 \frac{P}{P_y}\left(1 - \frac{P}{P_y}\right) \tag{8}$$

or, nondimensionally,

$$\frac{E_t}{E} = \tau = 4 \frac{P}{P_y}\left(1 - \frac{P}{P_y}\right) \tag{9}$$

Equation (8) or (9) is used in computing P_e when $P/P_y >$ 0.5.

The relationship between P and B_1M_o has been found to be approximately linear for $M/M_p < 5/6$ for all slenderness ratios included in this study. Based on this observation, the following set of bilinear equations is proposed for predicting the load-carrying capacity:

$$\frac{P}{P_{cr}} + m_1 \frac{B_1M_o}{M_p} = 1.0 \tag{10}$$

and

$$\frac{P}{P_{cr}} + n_1 \frac{B_1M_o}{M_p} = n_1 \tag{11}$$

in which

$$n_1 = 6 - 5\, m_1 \tag{12}$$

The coefficient m_1, which defines the slope of the P versus B_1M_o plot, can be determined graphically using the available analytical results. The m_1 values thus determined are plotted as a function of L/r_y in Fig. 8. By curve fitting, the following expression for m_1 is obtained:

$$m_1 = 0.27 + 0.3\,\beta + 0.61\,\lambda \leq 1.0 \tag{13}$$

in which λ is the normalized slenderness ratio defined by

$$\lambda = \frac{1}{\pi}\sqrt{\frac{F_y}{E}}\frac{L}{r_y} \tag{14}$$

The adequacy of the proposed equations may be seen in Figs. 6 and 7, where the predicted moment capacities are compared with the theoretically calculated amplified moment, B_1M_o. Comparisons have also been made for columns bent in double curvature (negative β), and Eqs. (10) and (11) have been found to give good estimates of the ultimate strength.

A similar treatment has also been carried out for beam-columns subjected to a concentrated load at mid-span (Fig. 4). In this case B_1 is given by the approximate expression

$$B_1 = \frac{C_m}{1 - \dfrac{P}{P_e}} = \frac{1 - 0.2\dfrac{P}{P_e}}{1 - \dfrac{P}{P_e}} \tag{15}$$

When plotted, the P versus B_1M_o curves show a similar trend as those given in Fig. 6 for columns subjected to equal end moments, and it is found that the ultimate strength can be closely predicted by Eqs. (10) and (11) with m_1 modified as follows:

$$m_1 = 0.85(0.27 + 0.23\beta + 0.61\lambda) \leq 1.0 \tag{16}$$

A β value of 1.0 is to be used in the above equation.

NON-SWAY RESTRAINED COLUMNS

The response of a column with end restraints is considerably different from that of a pinned-end column. When a bending moment is applied to a joint of a restrained column, it is resisted partly by the column and partly by the restraining member. The exact distribution depends on the rotational stiffnesses of the members. An increase in the axial load reduces the stiffness of the column. This results in an increase in the portion of the moment resisted by the

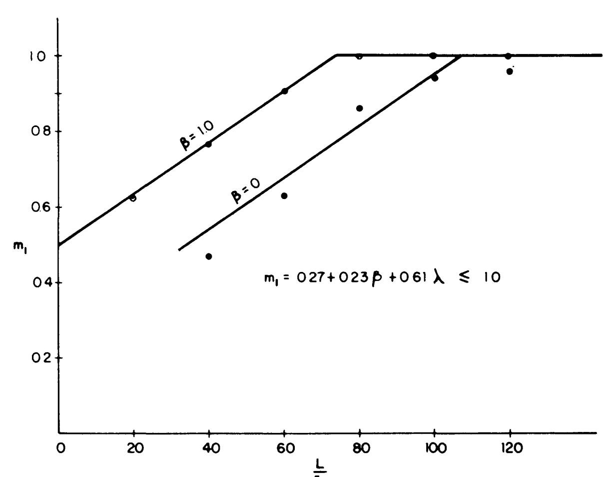

Fig. 8. Determination of coefficient m_1 in Eq. (10)

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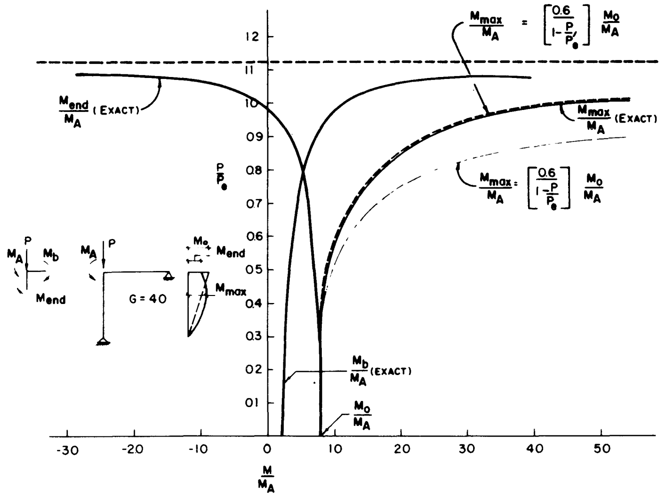

Fig. 9. Elastic second-order analysis of non-sway restrained column

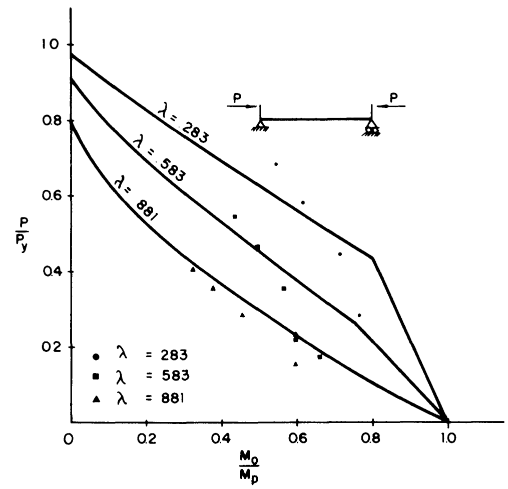

Fig. 10. Comparison of proposed interaction equations for non-sway columns with test results

restraining member. Figure 9 illustrates the behavior of a simple restrained column. The restraint provided by the beam is defined in terms of the G value.

$$G = \frac{EI_c/L_c}{EI_b/L_b} \tag{17}$$

in which I_c, I_b are, respectively, the moments of inertia of the column and the restraining beam, L_c is the height of the column, and L_b is the length of the beam. The joint moment M_A is held constant while the axial load P increases from zero to the critical value (corresponding to the Euler buckling load of the column). Elastic behavior is assumed throughout. It is seen that as P increases the column end moment M_{end} decreases and the beam moment M_b increases. At high levels of P, the direction of M_{end} becomes reversed and M_b is equal to the sum of M_A and M_{end}. The restraining beam must therefore be designed for a larger moment capacity.

Also shown in Fig. 9 is the variation of the maximum moment M_{max} in the column as a function of P. At low levels of P, M_{max} occurs at the column top, and it is equal to M_o, the first-order moment. As P increases, M_{max} gradually moves away from the column top and eventually reaches a value considerably greater than M_o. For a given value of P, M_{max} may be determined by using the ampli-

Table 1. Summary of Johnston and Cheney Tests

Column	L in.	$\frac{L}{r_y}$	F_y ksi	λ	e[a] in.	P kips	M_o kip-in.	$\frac{P}{P_y}$	$\frac{M_o}{M_p}$
C22	12.59	23.7	40.8	.283	0.35	46.6	16.1	0.69	0.55
C23	12.59	23.7	40.8	.283	0.47	38.9	18.3	0.58	0.62
C24	12.59	23.7	40.8	.283	0.71	29.6	21.0	0.44	0.71
C25	12.59	23.7	40.8	.283	1.18	19.1	22.5	0.28	0.76
C28	25.86	48.8	40.8	.583	0.35	36.6	12.8	0.55	0.43
C29	25.86	48.8	40.8	.583	0.47	30.8	14.5	0.46	0.49
C30	25.86	48.8	40.8	.583	0.71	23.5	16.7	0.35	0.56
C31	25.86	48.8	40.8	.583	1.17	14.9	17.4	0.22	0.59
C32	25.86	48.8	40.8	.583	1.65	11.8	19.4	0.18	0.66
C34	39.12	73.8	40.8	.881	0.35	27.2	9.5	0.41	0.32
C35	39.12	73.8	40.8	.881	0.47	23.6	11.1	0.35	0.37
C36	39.12	73.8	40.8	.881	0.71	19.0	13.5	0.28	0.46
C37	39.12	73.8	40.8	.881	1.18	14.9	17.6	0.22	0.60
C38	39.12	73.8	40.8	.881	1.65	10.6	17.5	0.16	0.59

[a] Eccentricity of applied load

fication factor given by Eq. (7). It is interesting to note that very close agreement with the exact solution may be obtained if P_e is replaced by P'_e which is based on the effective length KL_c ($K < 1.0$) of the column. The reason for this is that the restraining beam tends to delay the development of the second-order moment in the columns. Equations (10) and (11) are therefore applicable to restrained columns if P'_e is used in calculating the amplification factor B_1.

COMPARISON WITH TEST RESULTS

Equations (10) and (11) have been checked against previously reported tests on wide-flange columns conducted by Johnston and Cheney at Lehigh University.[19] All the columns had pinned ends and were loaded eccentrically with varying amounts of end eccentricities. The essential properties of the test specimens and the results obtained are summarized in Table 1. In Fig. 10, the test results are compared with the proposed interaction equations. Except in the region of low axial load, the proposed equations give good predictions of the ultimate strength.

DESIGN EXAMPLE 1

Given:
The pinned-end column in Fig. 11 is subjected to an axial load of 80 kips and a minor axis bending moment of 48 kip-in. The ends of the column are braced against sway. Design the column by the allowable-stress method, using the proposed interaction formulas. Use A36 steel.

Solution:
Equations (10) and (11) may be written in terms of the working stresses and the allowable stresses:

$$\frac{f_s}{F_a} + m_1 \frac{B_1 f_b}{F_b} \leq 1.0 \qquad (18)$$

$$\frac{f_a}{F_a} + n_1 \frac{B_1 f_b}{F_b} \leq n_1 \qquad (19)$$

and B_1 in this case is

$$B_1 = \frac{0.6}{1 - \dfrac{f_a}{F'_e}} \geq 1.0$$

Try W6 × 25:

$A = 7.34$ in.^{2}, $S_y = 5.61$ in.^{3}

$r_y = 1.52$ in., $\dfrac{L}{r_y} = 9.47$, $\lambda = 1.06$

From AISC Manual:
$F_a = 13.64$ ksi, $F'_e = 16.65$ ksi

$F_b = 0.75F_y = 27.0$ ksi

$$f_a = \frac{80}{7.34} = 10.90 \text{ ksi}$$

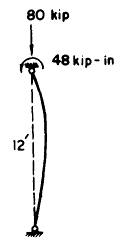

Fig. 11. Design Example 1

$$f_b = \frac{48}{5.61} = 8.56 \text{ ksi}$$

$$B_1 = \frac{0.6}{1 - \dfrac{10.90}{16.65}} = 1.74 > 1.0$$

$$m_1 = 0.27 + (0.61 \times 1.06) = 0.92$$

Check Eq. (18):

$$\frac{10.90}{13.64} + 0.92\left(\frac{1.74 \times 8.56}{27.0}\right) = 1.31 \textbf{ n.g.}$$

Try W8 × 28:

$A = 8.25$ in.^{2}, $S_y = 6.63$ in.^{3}

$r_y = 1.62$ in., $L/r_y = 88.9$, $\lambda = 0.998$

$F_a = 14.33$ ksi, $F'_e = 18.89$ ksi, $F_b = 27.0$ ksi

$$f_a = \frac{80}{8.25} = 9.70 \text{ ksi}$$

$$f_b = \frac{48}{6.63} = 7.24 \text{ ksi}$$

$$B_1 = \frac{0.6}{1 - \dfrac{9.70}{18.89}} = 1.23 > 1.0$$

$$m_1 = 0.27 + (0.61 \times 0.998) = 0.88, \ n_1 = 1.60$$

Check Eq. (18):

$$\frac{9.70}{14.33} + 0.88\left(\frac{1.23 \times 7.24}{27.0}\right) = 0.97 < 1.0 \text{ o.k.}$$

Check Eq. (19):

$$\frac{9.70}{14.33} + 1.60\left(\frac{1.23 \times 7.24}{27.0}\right) = 1.20 < 1.60 \text{ o.k.}$$

Use W8 × 28.

Note that in the above calculations different factors of safety are used for F_a, F'_e, and F_b, as specified in the AISC Specification. It has been found that the use of nonuniform factors of safety leads to conservative results.[18] It is, however, not the intent of this paper to discuss this aspect of the design problem.

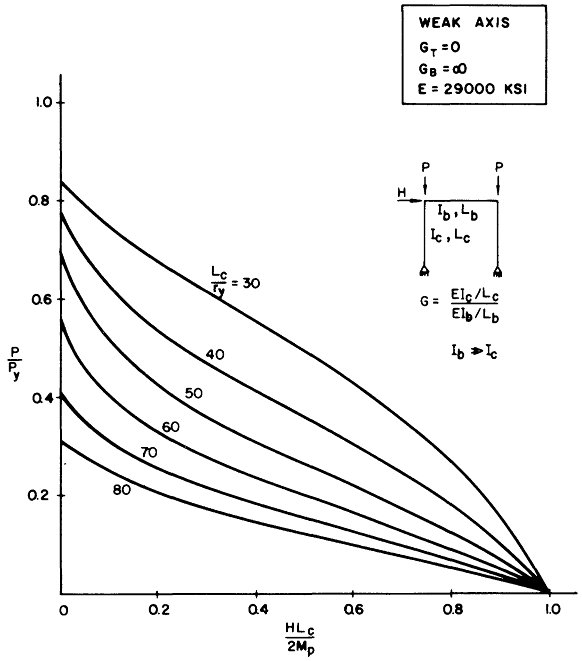

Fig. 12. Ultimate strength of columns in sway frames (both joints rigid)

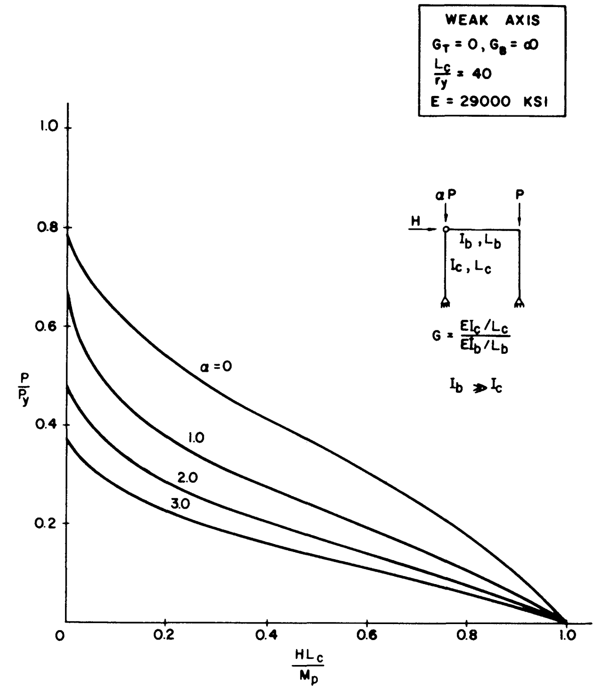

Fig. 13. Ultimate strength of columns is sway frames (one joint hinged)

RESTRAINED COLUMNS IN SWAY FRAMES

Analytical work has recently been carried out to study the strength of restrained columns in laterally unbraced frames. The frames selected were simple portal frames having pinned bases, as shown in Figs. 12 and 13. The frame in Fig. 12 is symmetrical and its stiffness to resist lateral load (or sidesway buckling) is provided by both columns. On the other hand, the frame in Fig. 13 has only one column that resists the lateral load and the P-Δ moment. The column with hinged top resists only vertical load.

The solutions given in Figs. 12 and 13 are obtained by following an approach developed in Ref. 20, and the details can be found in Ref. 18. Each curve defines, for a given structure and loading condition, the relationship between the axial load and the first-order column end moment when failure due to frame instability occurs. The analytical results will be used to develop a proposed design procedure for columns in sway frames.

Before discussing the design procedure, it is useful to examine first the behavior of a sway column and compare it with that of a non-sway column. Shown in Fig. 14 are the axial load versus end moment relationships of a restrained sway column (case b) and an unrestrained non-sway column (case a). Both columns have a slenderness ratio of 40, and, for the sway column, the stiffness of the restrained beam is assumed to be infinite ($G = 0$). The curve for case (a) is taken directly from Fig. 3. For case (b), two curves are shown: the dashed curve gives the first-order moment at the top of the column and the solid curve shows the second-order moment which includes the contribution of the P-Δ moment. Both curves are for the ultimate load condition. The end moment M_{end} of the sway column is considerably lower than the moment M_o of the non-sway column, except when the axial load is low. This suggests that the interaction equations developed for non-sway

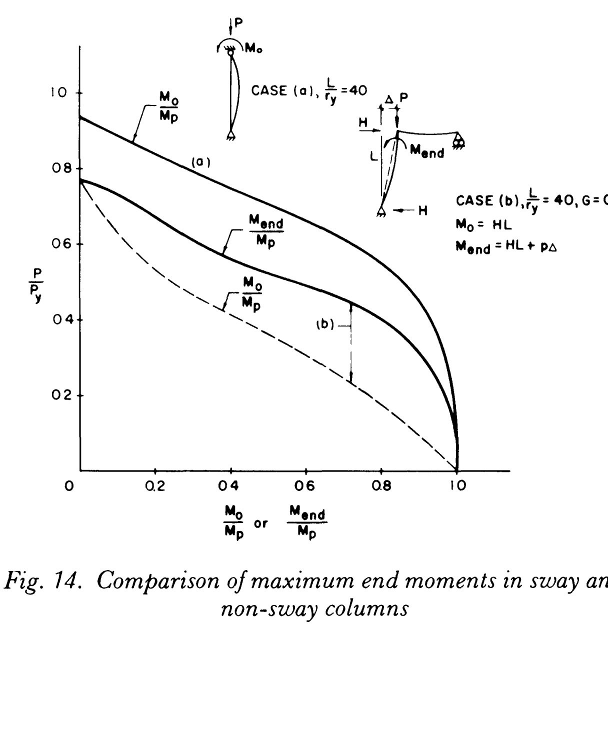

Fig. 14. Comparison of maximum end moments in sway and non-sway columns

columns are not directly applicable to sway columns. Some modifications are necessary.

One of the important considerations in the design of sway columns is the effect of frame instability. Several approaches to account for this effect have been proposed, and a brief description of these approaches is given in the section "Current Design Procedures". Two of these, the "modified effective length" approach and the "direct moment amplification" approach, apply an amplification factor (designated as B_2) to the first-order moment. The amplification factors used in these methods are:

$$B_2 = \frac{1}{1 - \dfrac{\sum P}{\sum P'_e}} \tag{20}$$

in the modified effective length approach,[1] and

$$B_2 = \frac{1}{1 - \dfrac{\sum P\Delta}{\sum HL}} \tag{21}$$

in the direct moment amplification approach.[9,10] In Eq. (21), ΣP and ΣH are, respectively, the total (cumulative) gravity and lateral loads in a story and Δ is the first-order story sway (or drift). A comparison of the B_2 values given by Eqs. (20) and (21) and the theoretically computed amplification factors (ratio of the second-order moment to the first-order moment) is given in Fig. 15. Equation (20) gives good predictions of the amplified moment, although for the case $\alpha = 1.0$ the equation is slightly conservative. The B_2 value given by Eq. (21) is generally too low, particularly at high axial loads. A better approximation for B_2 is

$$B_2 = \frac{1}{1 - 1.2\dfrac{\sum P\Delta}{\sum HL}} \tag{22}$$

which, as shown in Fig. 15, agrees very closely with the theoretical results. In Eq. (22), when the column axial load exceeds $0.5P_y$, Δ is to be calculated using the E_t (or τ) value given by Eq. (8) or (9).

It is the writers' opinion that the direct moment amplification approach would give more consistent and rational results than would the modified effective length approach, especially for frames carrying heavy gravity loads. Also, in the direct moment amplification approach, the quantities that enter into the calculation of B_2 are those which more truly characterize the problem of frame instability. A view similar to this has been expressed recently for reinforced concrete frame design.[21] Because of these and other observations reported in Refs. 6, 7, and 12, the formulas proposed in this paper for sway columns will be based on the direct moment amplification concept.

Each curve in Figs. 12 and 13 gives the relationship between the gravity load P and the maximum first-order moment at the column top, M_o. Multiplying M_o by the factor B_2 according to Eq. (22) gives the amplified moment

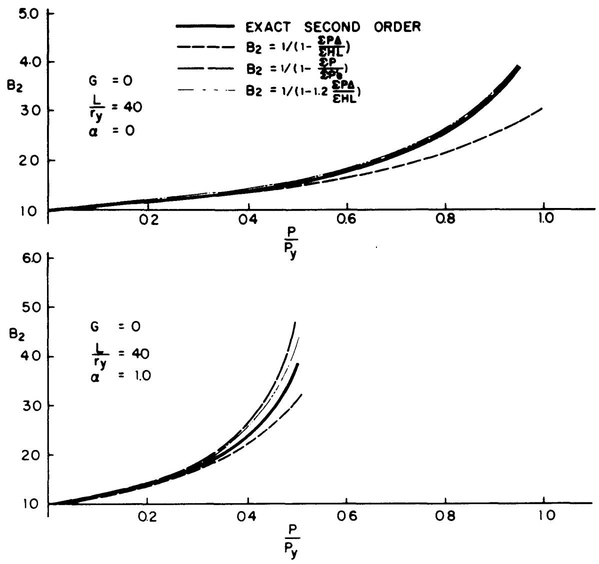

Fig. 15. Methods to account for effect of frame instability

at the column top. Figure 16 shows two B_2M_o curves (dashed) for the frame illustrated in Fig. 13. The curve for $\alpha = 0$ resembles closely the M_{end}-P curve in Fig. 14. The two curves should coincide if the exact B_2 values were used to construct the curve in Fig. 16.

The above development suggests that a possible way to include both the member instability and the frame instability effects in column design is to use the amplified moment $B_1B_2M_o$. However, for the frames included in this study, the effect of member instability has not been found to appreciably affect the strength of the columns. This is because the effect of frame instability tends to "override" the effect of member instability, as illustrated in Fig. 14.

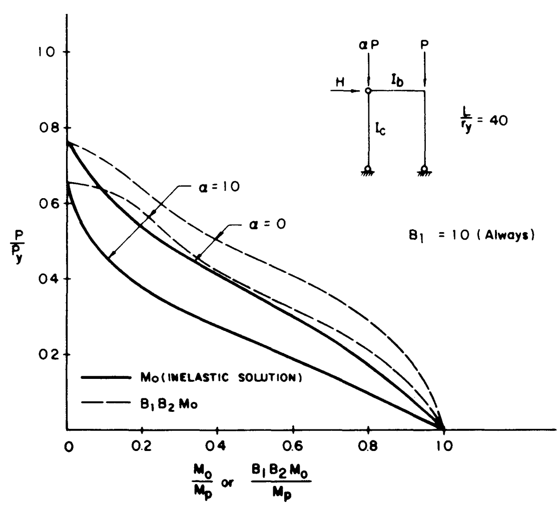

Fig. 16. Second-order moment in column in sway frame

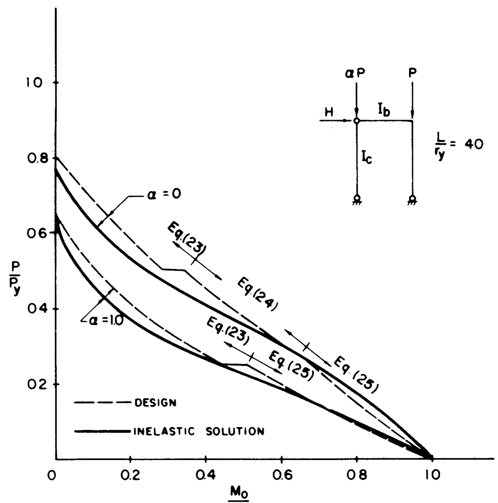

Fig. 17. Comparison of proposed interaction equations for sway columns with analytical solution

A B_1 value of 1.0 is therefore adopted in the proposed column formulas.

All the available ultimate strength solutions have been carefully analyzed, and the following empirical equations are found to adequately represent the column strength:

When $\frac{\sum P\Delta}{\sum HL} > \frac{1}{3}$:

$$\frac{P}{P_{cr}} + \frac{B_2 M_o}{M_p} = 1.0 \tag{23}$$

When $\frac{\sum P\Delta}{\sum HL} \leq \frac{1}{3}$:

$$\frac{P}{P_{cr}} + m_2 \frac{B_2 M_o}{M_p} = 1.0 \tag{24}$$

and

$$\frac{P}{P_{cr}} + n_2 \frac{B_2 M_o}{M_p} = n_2 \tag{25}$$

in which

$$m_2 = 0.85 \tag{26}$$

and

$$n_2 = 6 - 5m_2 \tag{27}$$

The P_{cr} in Eqs. (23), (24), and (25) is based on the actual length of the column ($K = 1.0$). Examples of comparing the proposed interaction equations with the analytical solutions are shown in Fig. 17. The proposed equations predict reasonably well the ultimate strength of the frame.

It is important to point out that in the direct moment amplification approach the amplified moment $B_2 M_0$ is also to be used in the design of beams. This may require larger beam sizes.

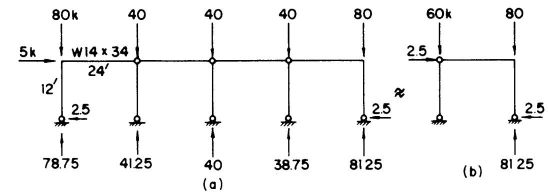

Fig. 18. Design Example 2

DESIGN EXAMPLE 2

Given:
Design the columns of the frame in Fig. 18 by the allowable-stress method for the gravity and lateral loads shown. The frame is permitted to sway in its own plane, but is adequately braced in the perpendicular direction. The W14X34 beams are oriented for major axis bending and all the columns for minor axis bending. Use A36 steel for the columns.

Solution:
In the allowable-stress format, Eqs. (23), (24), and (25) become:

When $\gamma \frac{\sum P\Delta}{\sum HL} > \frac{1}{3}$:

$$\frac{f_a}{F_a} + \frac{B_2 f_b}{F_b} \leq 1.0 \tag{28}$$

When $\gamma \frac{\sum P\Delta}{\sum HL} \leq \frac{1}{3}$:

$$\frac{f_a}{F_a} + m_2 \frac{B_2 f_b}{F_b} \leq 1.0 \tag{29}$$

and

$$\frac{f_a}{F_a} + n_2 \frac{B_2 f_b}{F_b} \leq n_2 \tag{30}$$

in which γ is the factor of safety or the load factor and can be taken as 1.67, and

$$B_2 = \frac{1}{1 - 1.20\gamma \frac{\sum P\Delta}{\sum HL}} \tag{31}$$

The need to incorporate the γ factor in the calculation is explained in Ref. 12.

Because of symmetry, it is possible to simplify the frame of Fig. 18a to that of Fig. 18b. Also, each of the exterior columns is assumed to resist half of the applied lateral load, that is, 2.5 kips. A first-order analysis gives a column top moment of 30 kip-ft and an axial load (vertical reaction) of 81.25 kips. For combined gravity and lateral loads, the AISC Specification permits a 33% increase in the allowable stress. This can be conveniently handled by using 75% of the working values in the calculations.

Interior columns: These columns receive lateral support from the exterior columns and their design requirement is that they should not buckle as a pinned end column. The actual column length is therefore used. The design load of the columns is

$$P = 0.75 \times 41.25 = 30.94 \text{ kips}$$

Try W4X13:

$A = 3.83 \text{ in.}^2, \quad r_y = 1.00 \text{ in.}, \quad L/r_y = 144$

$F_a = 7.20 \text{ ksi}$

$$f_a = \frac{30.94}{3.83} = 8.08 \text{ ksi} > F_a \ \textbf{n.g.}$$

Try W5X16:

$A = 4.68 \text{ in.}^2, \quad r_y = 1.27 \text{ in.}, \quad L/r_y = 113$

$F_a = 11.26 \text{ ksi}$

$$f_a = \frac{30.94}{4.68} = 6.61 \text{ ksi} < F_a \ \textbf{o.k.}$$

Exterior columns: The design loads are (Fig. 18b):

$$P = 0.75 \times 81.25 = 60.94 \text{ kips}$$

$$M_o = 0.75 \times 30 \times 12 = 270 \text{ kip-in.}$$

Try W12X65:

$A = 19.1 \text{ in.}^2, \quad I_y = 174 \text{ in.}^4, \quad S_y = 29.1 \text{ in.}^3$

$r_y = 3.02 \text{ in.} \quad L/r_y = 47.7$

$F_a = 18.55 \text{ ksi}, \quad F_b = 27.0 \text{ ksi}$

$$f_a = \frac{60.94}{19.1} = 3.19 \text{ ksi}$$

$$f_b = \frac{270}{29.1} = 9.28 \text{ ksi}$$

For the frame of Fig. 18b, the ratio $\Delta/\Sigma H$ (flexibility) is given by:

$$\frac{\Delta}{\sum H} = (G+1)\frac{L_c{}^3}{3EI_c}$$

Therefore,

$$\frac{\sum P\Delta}{\Delta HL_c} = \frac{\pi^2}{3}(G+1)\frac{\sum P}{P_e}$$

in which $G = (I_c/L_c)/(I_b/L_b)$ and $P_e = \pi^2 EI_c/L_c{}^2$. The moment of inertia of the W14X34 section is 340 in.^{4} The following G and P_e values are obtained:

$$G = \frac{174/144}{340/288} = 1.02$$

$$P_e = \frac{\pi^2 \times 29{,}000 \times 174}{(144)^2} = 2402 \text{ ksi}$$

The total gravity load acting on the frame is:

$$\sum P = 0.75\ (80 + 60) = 105 \text{ kips}$$

Substitution of G, ΣP, and P_e gives:

$$\frac{\sum P\Delta}{\sum HL_c} = \frac{\pi^2}{3}(1.02 + 1)\frac{105}{2402} = 0.291$$

and

$$\gamma\frac{\sum P\Delta}{\sum HL_c} = 1.67 \times 0.291$$

$$= 0.486 > \frac{1}{3} \rightarrow \text{check Eq. (28)}$$

The required B_2 factor is given by Eq. (31):

$$B_2 = \frac{1}{1 - (1.2 \times 0.486)} = 2.40$$

Equation (28):

$$\frac{3.19}{18.55} + \frac{2.40 \times 9.28}{27.0} = 0.993 \ \textbf{o.k.}$$

The column designs are now complete. Use W5X16 for the interior columns and W12X65 for the exterior columns.

In an actual design, the frame must also be checked for the gravity load alone case. To apply the proposed design procedure to this case, a small fictitious lateral load, say equal to 0.5% of the gravity load, may be assumed for the stability check. In this example, the gravity loading condition controls the design of the interior columns, but the W5X16 section is still adequate. The combined loading condition controls the exterior columns.

SUMMARY

This paper deals with the analysis and design of framed columns subjected to minor axis bending. Both sway and non-sway columns are included in the study. A review is presented of the current design procedures, which are based largely on the previous studies on columns bent about the major axis. For non-sway columns, the interaction formulas given in the AISC Specification have been found to give results which do not agree well with the theoretical solutions. For sway columns, specific design provisions need to be developed to account for the effect of frame instability.

Ultimate strength solutions for non-sway columns have been obtained for three loading cases and numerical results are presented in the form of interaction curves. Based on these curves, a new set of column design formulas, Eqs. (10) and (11), has been developed and its application is illustrated in Design Example 1. The formulas are applicable to columns subjected to symmetrical and nonsymmetrical end moments and to lateral load. The strength predicted by these formulas compares favorably with the available test results.

The elastic behavior of a simple restrained column has been studied in detail. It is shown that the column may "shed" its entire resisting moment when axial load exceeds a certain value.

Ultimate strength solutions of two unbraced frames subjected to combined gravity and lateral loads have been presented and new design formulas for sway columns, based on the direct moment amplification approach, are proposed. These formulas have essentially the same appearance as the formulas for non-sway columns, except that they use a different set of amplification factors and empirical coefficient. Design Example 2 illustrates the application of the new formulas.

Although the formulas and design procedures presented in this paper are for columns subjected to minor axis bending, the basic concepts and approaches adopted in their development appear to be applicable also to the case of major axis bending. A follow-up paper will present a more complete discussion of the design requirements and provisions for columns in both braced and unbraced frames.

ACKNOWLEDGMENTS

This study is part of an investigation sponsored at Lehigh University by the Committee of Structural Steel Producers and the Committee of Steel Plate Producers of the American Iron and Steel Institute. William C. Hansell was project supervisor representing AISI.

The theoretical solutions given in Figs. 9, 12, 13, and 15 were first presented in the senior writer's Ph.D. dissertation submitted to the University of Texas at Austin in 1977. This dissertation was prepared under the supervision of Joseph A. Yura. The interaction curves shown in Figs. 2 and 3 were obtained by Francois Cheong-Siat-Moy, using a computer program prepared by Lee C. Lim.

The writers gratefully acknowledge the extensive discussion they had with some members of Task Committee 10 of the AISC Specification Advisory Committee, especially William J. LeMessurier and Ira Hooper.

NOMENCLATURE

The symbols used in this paper are defined in the AISC Specification, except the following:

B_1 = amplification factor accounting for member instability effect
B_2 = amplification factor accounting for frame instability effect
E_t = tangent modulus
H = lateral load
M_o = first-order moment
m_1 = empirical coefficient in interaction formulas for non-sway column
m_2 = emprical coefficient in interaction formulas for sway column
n_1 = $6 - 5m_1$
n_2 = $6 - 5m_2$
α = proportionality constant for vertical load
β = end moment ratio
γ = factor of safety or load factor (1.67)
Δ = first-order story sway
δ = deflection of column
λ = normalized slenderness ratio
τ = E_t/E

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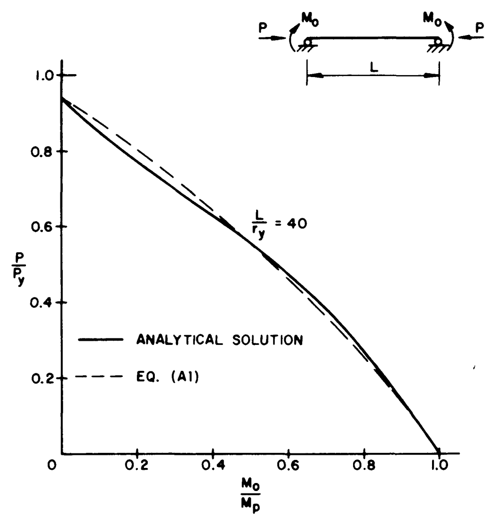

Fig. A1. Comparison of analytical solution with Eq. (A1)—equal end moment case

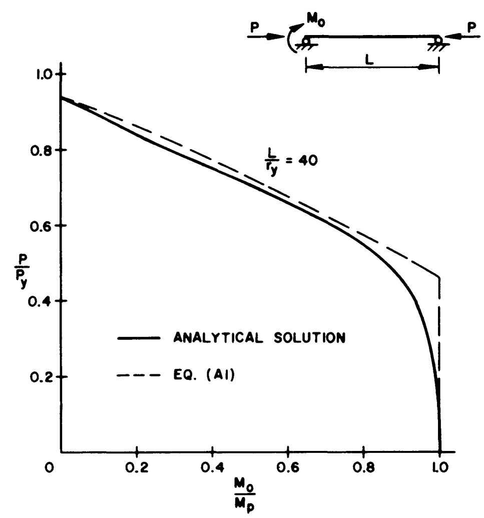

Fig. A2. Comparison of analytical solution with Eq. (A1)—one end moment case

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APPENDIX

A POSSIBLE MODIFICATION TO THE PRESENT COLUMN FORMULA

The following equation has been proposed in Ref. 22 as an improvement of Eq. (3) for non-sway columns:

$$\frac{P}{P_{cr}} + \frac{C_m}{1 - \mu \dfrac{P}{P_y}} \frac{M_o}{M_p} = 1.0 \qquad \text{(A1)}$$

in which μ is an empirical coefficient depending on λ. For H or I section members subjected to minor axis bending, μ is given by:

$$\mu = 2.47\,\lambda - 1.47 \quad \text{for } \lambda \leq 1.0 \qquad \text{(A2)}$$

$$\mu = \lambda^2 \quad \text{for } \lambda > 1.0 \qquad \text{(A3)}$$

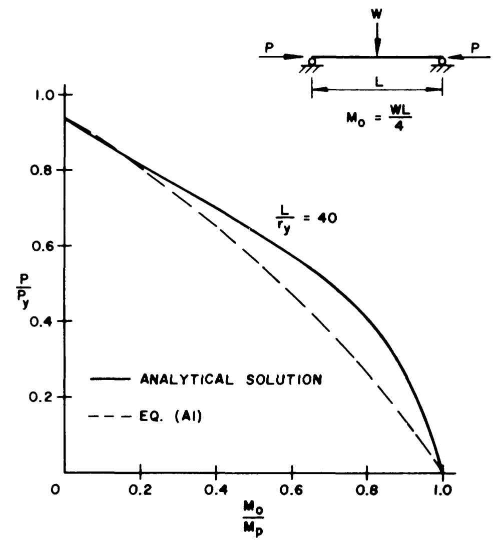

Fig. A3. Comparison of analytical solution with Eq. (A1)—lateral load case

In Eq. (A2), μ takes on values between -1.47 and 1.00. For $\lambda > 1.0$ the amplification factor in Eq. (A1) becomes $C_m/(1 - P/P_e)$, which is the same as that in Eq. (3).

Figures A1, A2, and A3 show comparisons between Eq. (A1) and the analytical solutions presented in Figs. 2, 3 and 4 for a column with $L/r_y = 40$. The equation is quite accurate for the case of equal end moments, but becomes conservative for lateral loading. Also, as shown in Fig. A2, the equation gives unconservative results when the axial load is low. A cut-off at $M_o/M_p = 1.0$ should be specified.

Discussion

Analysis and Design of Framed Columns Under Minor Axis Bending

Paper presented by T. KANCHANALAI AND LE-WU LU
(2nd Quarter, 1979)

Discussion by **D. A. Nethercot**

The writer was interested to read of this investigation dealing specifically with the behaviour of beam-columns bent about the minor axis, since he has also been concerned with certain aspects of this problem in the context of the revision of codes of practice in the U.K.

Results for the two cases of Figs. 2 and 3 have previously been obtained by Young,[23] who used them as part of the basis for his own critical length design approach for in-plane failure of beam-columns.[24] Figures 19 and 20 show the author's results to be in reasonable agreement with this alternative solution. The differences between the two sets of results are greatest for columns subjected to small end moments. The authors do not state whether the effects of initial curvature were included in their analysis and it seems likely that this, together with differences in the cross-sectional shape and the assumed pattern of residual stresses, could well be responsible, since such factors would be expected to become increasingly significant as the column approaches the pure axial load case. Young also considered five different sections in minor-axis bending; examination of the scatter in his results (admittedly made somewhat difficult due to the novel form of plot adopted) naturally raises the question of how representative are the results of Figs. 2 through 4, when viewed in the context of the complete range of possible sections.

Although the authors use their own results for the $M_o = 0$ case to construct Fig. 5, they then base the development of their new interaction approach on axial strengths calculated using the SSRC Basic Column Formula. As indicated in Figs. 19 and 20, the two approaches do not yield identical results. The writer's experience[25,26] suggests that changes in the axial strength portion of an interaction equation can produce surprisingly large differences in the predictions of that interaction formula over its whole range of application. As an example of this, a change from the variant of the ECCS maximum strength column curves proposed in the new U.K. Draft Steelwork Code[27] to the SSRC Basic Column Curve when used with the well-known SSRC interaction formula[28] caused the mean value of (predicted load)/(test load) for the 24 tests reported by Mason, Fisher and Winter[29] to change from 1.152 to 1.016. When an ultimate load version of the simple linear formula of BS.449 was used, the corresponding values became 1.046 and 0.876. In view of the possible change from the SSRC Basic Column Formula to a set of maximum strength column curves, have the authors considered how the accuracy of their interaction formulas [Eqs. (10) and (11)] would be affected, and whether any modifications would be necessary?

In the writer's study,[25] it was also found that while (for the nonsway case) it was probably necessary to retain the effect of the amplification factor for minor-axis bending, for practical purposes this effect could be approximated without much loss in accuracy, with the result that evaluation of the interaction formula was simplified. In view of the authors' comments later in the paper, where they go on to consider the effects of sway (last paragraph on p. 37) it

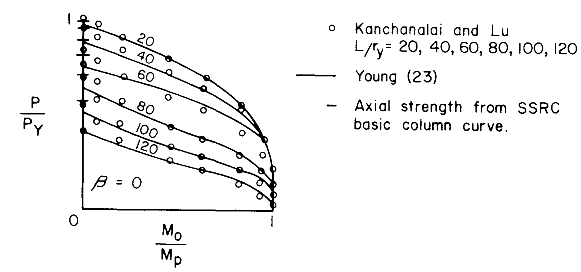

Fig. 19. Comparison of ultimate strength results for columns subjected to one end moment

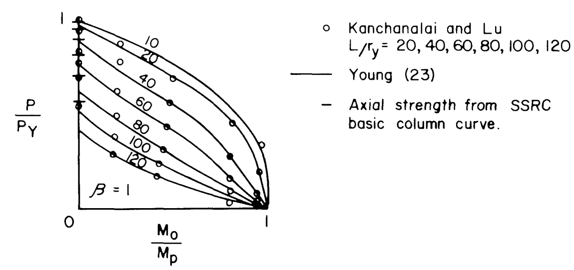

Fig. 20. Comparison of ultimate strength results for columns subjected to equal end moments

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would be of interest to know whether the possibility of simplifying the expressions for B_1 by means of some approximation for P_e had been considered. It should be noted, however, that this point is also affected by the presence or otherwise of the effects of initial curvature in the interaction formula. The importance of the amplification term was found to be greatly reduced in the type of expression used in the U.K.,[27] as well as by ECCS,[30] over the SSRC interaction equation.

It would have been of interest to see a somewhat more detailed comparison of the authors' interaction formula with the test data of Johnston and Cheney. For the full set of 28 tests (the 10 tests at constant load eccentricity and variable slenderness are omitted from Table 1) the writer obtained a mean (test load)/(predicted load) of 1.088 using the SSRC interaction and Basic Column formulas. It is difficult to decide whether the comparison presented by the authors in Fig. 10 is superior. This is particularly true for the most stocky ($\lambda = 0.283$) columns for which the SSRC formula (because it can never become convex as required by Fig. 5) consistently underpredicts (but less so as the amount of bending is increased) and for which Fig. 10 suggests rather erratic predictions for the new method. Perhaps the authors can also indicate why the additional 10 tests were not used in their comparison?

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