Yield Line Analysis of Bolted Hanging Connections

THOMAS S. DRANGER

The beam-to-girder connection made by fastening the top flange of a beam to the bottom flange of a transverse girder with bolts acting in tension is an example of a bolted hanging connection. Connections of this type are not often used for the support of primary members. When their use is unavoidable, their allowable load is estimated by a rough analysis and they are detailed very conservatively. It is desirable to rationalize a simple analysis, practical for general office use, so that the designer may be at ease with an economical solution when the use of bolted hanging connections is required.

The end plate moment connection for rigid beam-to-column joints has some problems associated with thin column flanges that are not necessarily resolved by providing web stiffeners, particularly when invoking Sect. 1.15.5 of the AISC Specification,[1] since that section addresses itself to welded, not bolted, connections.[3] Analysis of the column flange at bolts acting in tension can be the same as for bolted hanging connections.

This paper gives one method for analyzing bolted hanging connections and considers the influence of the following variables:

1. Superimposed Stresses: The effect of longitudinal stresses due to axial load or bending.
2. Bolt Spacing: The effects of end distance, pitch, and gage.
3. Prying Action: The increase in fastener tension due to "clamping" the connected material at the bolt location.

Energy methods applied to yield line analysis were found to organize the variables in simple expressions suitable for routine office use. This method seeks a solution by equating the work required to (virtually) accomplish the plastic deformation of the connected material (called the internal work) to the work (virtually) performed by moving the load through the distance compatible with the deformation of the connected material (called the external work).[4,5] The deformation of the connected material is assumed to occur by bending on idealized yield lines and a pattern of yield lines which will permit movement of the load is called a mechanism. The result of the analysis is an upper bound on the magnitude of load supported. Thus, while small modifications of a mechanism have negligible effects on the computed capacity, it is imperative not to overlook the general form of the mechanism that gives the least capacity. A "correct" solution must be statically admissible. Five mechanisms are investigated in this paper.

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NOMENCLATURE

a = the lesser of e or $2t$, in.
b = $\frac{1}{2}(G - t_w - \frac{1}{8})$
= one-sixteenth-inch less than the distance between the gage line and the near face of the web, in.
b_f = flange width, in.
d = nominal bolt diameter, in.
e = $\frac{1}{2}(b_f - G)$
= the distance between the gage line and the edge of the flange, in.
F = $P + Q$
= total tension per bolt, kips
F_b = superimposed longitudinal flange stress, ksi
F_y = yield stress of the connected material, ksi
G = gage, in.
m = $F_y t^2/4$
= local plastic moment capacity per inch of yield line for an otherwise unstressed flange, kip-in./in.
P = applied load per bolt, kips
Q = prying force per bolt, kips
r = $(F_y - F_b)/F_y$
= ratio of plastic moment capacity reduced for superimposed stresses to the plastic moment capacity of otherwise unstressed material
S = pitch, in.
t = thickness of the connected material, usually the flange, in.
t_w = web thickness, in.
w = length of beam tributary to a bolt, in.
x = a variable dimension as figured, in.
θ = a vector component of rotation, rad.

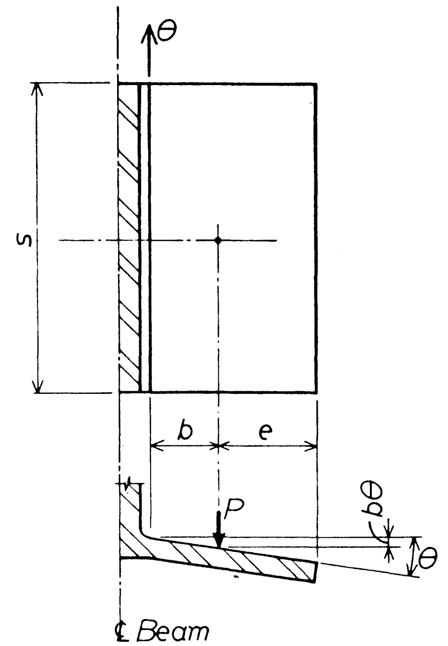

Figure 1

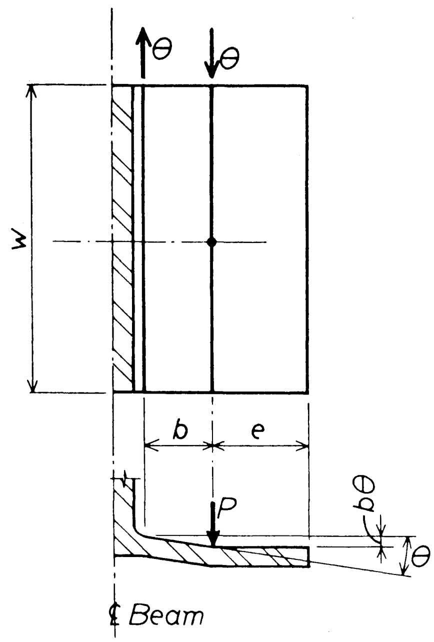

Figure 2

FLANGE CAPACITY

Five mechanisms are shown in Figs. 1 through 5. The mechanisms in Figs. 1 and 2 involve the entire length of the member tributary to the bolt. In Figs. 2, 4, and 5 the connected material must act to clamp a portion of the flange in a plane parallel to the original surface of the undeformed flange. In Figs. 1 and 3 the material is not clamped.

In analyzing the flange, each mechanism compatible with the service conditions is evaluated to determine the lowest load P which will cause failure. The formulas required to determine load P for each mechanism are derived as follows:

Figure 1—The mechanism consists of a single yield line along the fillet at the web. The plastic moment capacity of this yield line is mS. Then,

$$\text{Internal work} = mS\theta$$

To rotate through the small angle θ ($\theta = \tan\theta$), the load P moves a distance $b\theta$, so that

$$\text{External work} = Pb\theta$$

Equating the internal and external work,

$$\text{Internal work} = \text{External work}$$

or

$$mS\theta = Pb\theta$$

Then,

$$P = mS/b$$

and since

$$m = F_y t^2/4$$

$$P = F_y t^2 S/4b \quad (1)$$

Figure 2—The internal work is the sum of the work on two yield lines: one along the fillet at the web and one on the gage line. Following the procedure used for Fig. 1:

$$\text{Internal work} = 2mw\theta$$

$$\text{External work} = Pb\theta$$

$$Pb\theta = 2mw\theta$$

$$P = 2mw/b$$

$$P = F_y t^2 w/2b \quad (2)$$

Figure 3—The internal work is the sum of the work on three yield lines. It is convenient to choose θ as the component of rotation parallel to the axis of the member. The transverse component of rotation is inversely related to the variable x. The unit plastic moment capacity in the direction of θ is m, and the transverse unit plastic moment capacity is rm. The work on yield line (iii) is equal to the work on yield line (i).

The work on yield line (i) is:

$$m(x/2)\theta + rm(b + e)[2\theta(b + e)/x]$$

The work on yield line (ii) is:

$$2rm(b + e)[2\theta(b + e)/x]$$

Taking the sum of the work on the three yield lines and combining like terms:

$$\text{Internal work} = mx\theta + 8rm(b + e)^2\,\theta/x$$

Recognizing that P will be a function of the variable x:

$$\text{External work} = P_x b\theta$$

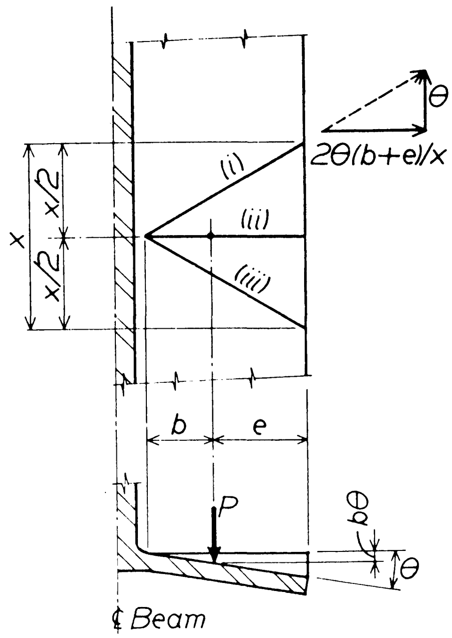

Figure 3

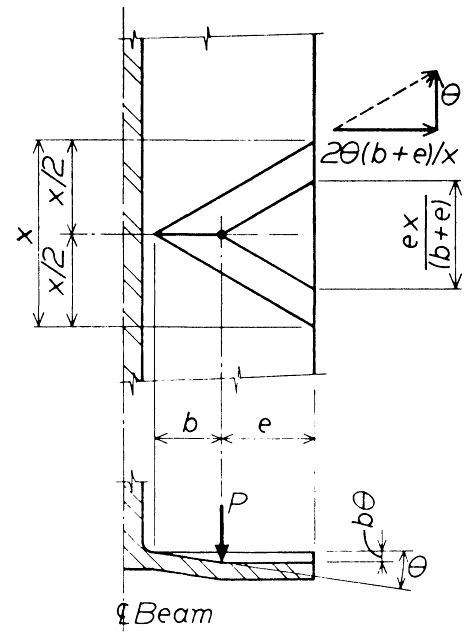

Figure 4

Equating internal and external work and rearranging terms:

$$P_x = m[x + 8r(b + e)^2/x]/b$$

P_x is minimum when P'_x is zero. Taking the derivative:

$$P'_x = m[1 - 8r(b + e)^2/x^2]/b = 0$$

Solving for x:

$$x = (8r)^{1/2}(b + e)$$

Find load P by substituting this value of x into P_x:

$$P = m[(8r)^{1/2}(b + e) + (8r)^{1/2}(b + e)]/b$$
$$= 4m(2r)^{1/2}(1 + e/b)$$

Substituting the value of m:

$$P = F_y t^2(2r)^{1/2}(1 + e/b) \qquad (3)$$

Figure 4—This mechanism is analyzed by the same procedure used for Fig. 3.

$$\text{Internal work} = mx\theta + m[e/(e + b)]x\theta + 4rm(e + b)2\theta(e + b)/x$$
$$= m\theta[(2e + b)/(e + b)]x + 8rm(e + b)^2\theta/x$$

$$\text{External work} = P_x b\theta$$

$$P_x = m[(2e + b)x/(e + b) + 8r(e + b)^2/x]/b$$

$$P'_x = m[(2e + b)/(e + b) - 8r(e + b)^2/x^2]/b = 0$$

$$x = [8r(e + b)^3/(2e + b)]^{1/2}$$

$$P = m[(2e + b)^{1/2}(8r)^{1/2}(e + b)^{1/2} + (8r)^{1/2}(e + b)^{1/2}(2e + b)^{1/2}]/b$$
$$= 4m[(2r)^{1/2}(1 + 2e/b)(1 + e/b)]^{1/2}$$

$$P = F_y t^2[2r(1 + 2e/b)(1 + e/b)]^{1/2} \qquad (4)$$

Figure 5—This mechanism is analyzed by the same procedure used for Fig. 3.

$$\text{Internal work} = 2mx\theta + 4rm(e + b)(2\theta b/x)$$
$$= 2mx\theta + 8rm(e + b)\theta b/x$$

$$\text{External work} = P_x b\theta$$

$$P_x = m[2x/b + 8r(e + b)/x]$$

$$P'_x = m[2/b - 8r(e + b)/x^2] = 0$$

$$x = 2[rb(e + b)]^{1/2}$$

$$P = m\left\{4[r(1 + e/b)]^{1/2} + 4[r(1 + e/b)]^{1/2}\right\}$$
$$= 8m[r(1 + e/b)]^{1/2}$$

$$P = 2F_y t^2 [r(1 + e/b)]^{1/2} \qquad (5)$$

Figure 6 shows the relationship between the values of load P (as $P/F_y t^2 r^{1/2}$) found for the mechanisms shown in Figs. 3, 4, and 5 for various values of e/b. When $e = 0$, the mechanisms of Figs. 3 and 4 are identical and, as expected, Fig. 6 indicates like values for the load P. For values

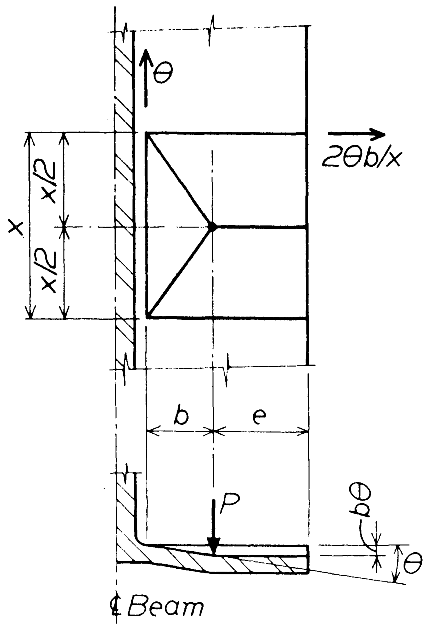

Figure 5

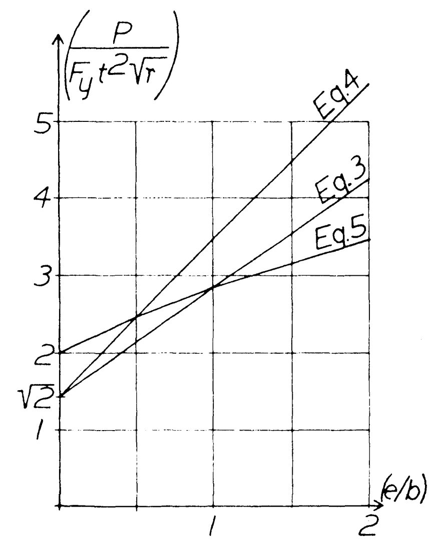

Figure 6

of $e/b > 1$, the mechanism of Fig. 5 governs, even if the material is not clamped. Because of this and because the loads found for the mechanisms of Figs. 3 and 4 differ by less than 15%, there is little to be saved in debating whether or not, in doubtful cases, conditions exist to clamp the connected material. It is a conservative expedient to limit investigation to Figs. 3 and 5 when the tributary length per bolt is large. If the tributary length per bolt is small, the mechanisms in Figs. 1 and 2 should also be investigated.

The average shear stress f_v in the flange between the bolt and the web is:

$$f_v = 2m/\tfrac{1}{2}bt = F_y t/b$$

Since the shear must not exceed $0.55F_y$, then:

$$0.55F_y \geq F_y t/b$$

$$b \geq 1.8t$$

Thus, dimension b should be taken as not less than approximately $2t$.*

In an analysis (beyond the scope of this paper) of plates with distant edges, an absolute upper bound on the load P indicates that, in applying Eq. (5), the value of e/b should not exceed 3.

FASTENER TENSION

After determining the flange capacity, the capacity of the fastener in tension is checked. A theoretical prying force is statically determinate from the mechanism found to govern the flange capacity. However, the real prying force is highly indeterminate, and the empirical formulas such as those developed at the University of Illinois[2] seem more appropriate:

For A325 Bolts:

$$Q/P = \frac{100bd^2 - 18wt^2}{70ad^2 + 21wt^2} \tag{6}$$

For A490 Bolts:

$$Q/P = \frac{100bd^2 - 14wt^2}{62ad^2 + 21wt^2} \tag{7}$$

These formulas can be applied in a straightforward manner for Fig. 2. For the other cases, a value for w can be found by equating the load P found for Fig. 2 to the load P found for the governing mechanism, and solving for w.

Thus, for Fig. 1:

$$F_y t^2 w/2b = F_y t^2 S/4b$$

Then,

$$w = S/2 \tag{1a}$$

Similarly, for Fig. 3:

$$w = 2(2r)^{1/2}(b + e) \tag{3a}$$

For Fig. 4:

$$w = 2[2r(b + e)(b + 2e)]^{1/2} \tag{4a}$$

For Fig. 5:

$$w = 4b[r(1 + e/b)]^{1/2} \tag{5a}$$

* *Unless the designer especially investigates the effects of shear stresses.*

EXAMPLES

Example 1—A W14×30 beam at a stair opening has a pair of hanger rods for the support of the stair landing. The ends of the rods are threaded and fastened with nuts through holes in the bottom flange of the beam at the usual gage line. What factored load can each hanger safely support if the factored bending stress at this location is 25 ksi?

Given:

$F_b = 25$ ksi	$t = 0.383$ in.
$G = 3.5$ in.	$t_w = 0.270$ in.
$b_f = 6.733$ in.	$F_y = 36$ ksi

Solution:

$$r = (36 - 25)/36 = 0.306$$

$$b = \tfrac{1}{2}(3.5 - 0.27 - 0.125) = 1.552 \text{ in.}$$

$$e = \tfrac{1}{2}(6.733 - 3.5) = 1.615 \text{ in.}$$

$$e/b = 1.615/1.552 = 1.04$$

The tributary length of beam per hanger is large. Therefore, from Fig. 6, Eq. (5) governs.

$$P = 2(36)(0.383)^2[0.306(2.04)]^{1/2}$$
$$= 8.34 \text{ kips per hanger}$$

Example 2—A W10×21 beam is suspended by four A325 bolts from the bottom chord of a truss to support the ceiling of a canopy. The truss is concealed by a masonry wall supported on the extended end of the W10 beam (see Fig. 7). The factored reaction is 29 kips. The factored bending stress in the beam is 21 ksi. Check the capacity of the connection for usual bolt gages.

Given:

$F_b = 21$ ksi	$b_f = 5.75$ in.
$S = 5.5$ in.	$t = 0.34$ in.
$G = 2.75$ in.	$t_w = 0.24$ in.
$d = 0.75$ in.	$F_y = 36$ ksi

Solution:

$$r = (36 - 21)/36 = 0.417$$

$$b = \tfrac{1}{2}(2.75 - 0.24 - 0.125) = 1.192 \text{ in.}$$

$$e = \tfrac{1}{2}(5.75 - 2.75) = 1.50 \text{ in.}$$

$$e/b = 1.50/1.192 = 1.258$$

$$a = 2(0.34) = 0.68 \text{ in.}$$

From Fig. 6, the capacity of the W10 flange is governed by Eq. 5.

$$P = 2(36)(0.34)^2[0.417(2.258)]^{1/2}$$
$$= 8.07 \text{ kips per bolt}$$

The allowable capacity is limited by the strength of the flange to 4(8.07) = 32.3 kips.

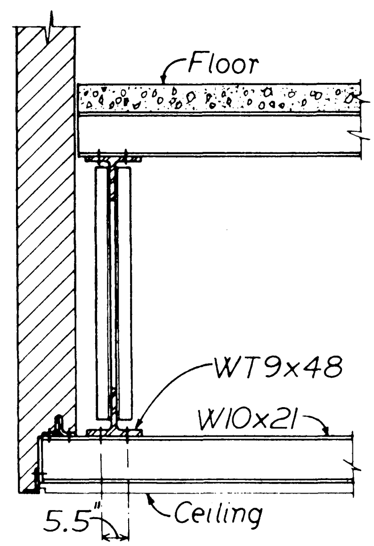

Figure 7

The fastener tension is checked for the direct load plus the prying force. Applying Eq. (5a):

$$w = 4(1.192)[0.417(2.258)]^{1/2}$$
$$= 4.627 \text{ in.} < S = 5.5 \text{ in.}$$

From Eq. (6):

$$Q/P = \frac{100(1.192)(0.75)^2 - 18(4.627)(0.34)^2}{70(0.68)(0.75)^2 + 21(4.627)(0.34)^2}$$
$$= 1.512$$

$$F = (1.512 + 1)P = 2.512\,P$$

But P is the 29 kip reaction divided by four bolts, or 7.25 kips per bolt. Then,

$$F = 2.512(7.25) = 18.21 \text{ kips per bolt}$$

The allowable factored tension load of 30 kips per bolt is greater than the 18.21 kip/bolt.

To complete the analysis, the bottom chord flange capacity is checked by a similar procedure:

$$e/b = 3.125/2.432 = 1.285$$

$$P = 2(36)(0.831)^2[r(2.285)]^{1/2} = 7.25 \text{ kips}$$

Evidently, r must not be less than 0.009305. The connection is adequate if the factored chord stress does not exceed

$$F_b = (1 - 0.009305)(36) = 35.6 \text{ ksi (governs)}$$

Finally, since the bolts are spaced close together, check Eq. (2):

$$P = 36(0.831)^2(2.75)/[2(2.432)] = 14.06 \text{ kips/bolt}$$

The connection is adequate if the longitudinal stress in the bottom chord of the truss does not exceed 35.6 ksi.

SUMMARY

The bolted hanging connection can be more useful to the designer if a practical design procedure is available. Yield line analysis provides a solution, with due regard for superimposed longitudinal member stresses, for the most commonly encountered cases. Limitations are placed on the edge distance and on the proximity of bolt to web.

The effect of prying force on fasteners is determined by accepted empirical formulas. Fasteners in long members are evaluated by analogy to the conventional (short member) case.

REFERENCES

1. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings *American Institute of Steel Construction, New York, 1969.*
2. Behavior of Bolts in Tee-Connections Subject to Prying Action *Structural Research Series No. 353, U. of Illinois, Sept. 1969.*
3. *Graham, J. D., A. N. Sherbourne, R. N. Khabbaz, and C. D. Jansen* Welded Interior Beam-to-Column Connections *American Institute of Steel Construction, New York, 1959.*
4. *Bresler, B., T. Y. Lin, and J. B. Scalzi* Design of Steel Structures, 2nd Ed., John Wiley & Sons, New York, 1968.
5. *Ferguson, P. M.* Reinforced Concrete Fundamentals *Second Edition, John Wiley & Sons, Inc., New York, 1965.*

Electroslag Welding Process

Recent events involving several bridges have drawn attention to and caused initiation of extensive investigations which are still underway on welds produced by the electroslag process. AISC is keeping abreast of the research in progress in the United States and abroad; however, until more complete authoritative information is available, it is not prudent to comment in detail on preliminary unpublished indications of the previously unknown types of discontinuities which have been found in several instances by the current investigations. At the present time, neither the prevalence of these previously unsuspected discontinuities, nor their real significance in tension members, is known.

Because data to evaluate the brittle fraction properties of electroslag welds are lacking, intensive research on the subject is underway. In addition to current investigations in the United States, seventeen companies in Europe are jointly supporting work by the European Research Institute for Welding at three laboratories in England, West Germany and Belgium. This work will produce the largest collection of data on electroslag welds ever assembled. Studies are also underway to develop improved inspection procedures.

It is planned that a more definitive paper on the state-of-the-art and current studies, prepared by a knowledgeable author, will be published in the *AISC Engineering Journal* in the near future.

Discussion

Yield Line Analysis of Bolted Hanging Connections

Paper presented by THOMAS S. DRANGER (3rd Quarter, 1977)

Discussion by **Henry W. Dougherty**

The analysis presented by Mr. Dranger is very interesting. As the author states, this type of connection is not used very often for primary members. In such cases, rather than going through the outlined steps that were presented, why not use stiffener plates between flanges at the upper beam connection, thus minimizing (or eliminating) rotation of the lower flange and allowing the suspended load to be transmitted directly into the upper beam web? In the case of the lower beams, as indicated in Fig. 7, again stiffener plates, or bolts long enough to extend to holes in the lower flange, would solve the problem with no doubt as to transmittal directly into the beam web.

Henry W. Dougherty is President, Dougherty Engineering, Inc., Memphis, Tenn.

* Steel Designers' Manual *U.S. Steel Corporation, May, 1974, pp. 169 and 175.*