Yield Line Analysis of Column Webs with Welded Beam Connections

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In welded construction the connections, for the most part, are easily designed and fabricated. However, one of the more difficult connections occurs when a beam must be welded to a column web. This is so for two reasons: (1) analyzing the column web's capacity is arduous and (2) making the actual connection in accordance with the design is no simple task because of the space restrictions imposed on the welder by the column flanges. This paper will deal only with the first consideration—analyzing the column web strength—using Yield Line Analysis.

The Yield Line approach has been utilized previously by Abolitz and Warner[1] for brackets welded to column webs, and by Blodgett[2] for welding rolled sections to box columns. The assumptions used here differ somewhat from those of the above authors. The final result is a series of curves covering a very high percentage of restrained beam to column web connections involving hot rolled shapes. The curves relate beam moments to column web thickness for a series of beam sizes and for W14 columns with yield strengths of 36 and 50 ksi. For special cases not covered by these curves, but for which the yield line pattern assumed is valid, direct solutions may be developed by use of the formulas used to construct these curves.

ASSUMPTIONS

1. Nominal sizes are assumed for beam groupings.
2. The assumed yield pattern is as shown in Fig. 1, and all lines are stressed to F_y of the column.
3. The length of lines ④ plus ⑤ is assumed to equal the length of line ③, and their angles are assumed to be the same.
4. The web surface enclosed by lines ① and ④ remains plane.
5. The effect of the web angles (designed for shear only) is negligible.

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YIELD LINE ANALYSIS

The external work done in bending the beam end must equal the internal work required to form the yield lines.

$$\text{External work} = M\theta \qquad (1)$$

$$\text{Internal work} = \Sigma m_i\theta_i \qquad (2)$$

where

M = Beam end moment
θ = Beam end rotation
θ_i = Yield line hinge rotation for line **i**
m_i = Plastic moment along line **i**
= $F_y Z_i$ where Z_i is the plastic modulus along **i**
= $F_y(w_i t^2/4)$ where w is the length of line **i**

Therefore,

$$M\theta = \frac{F_y t^2}{4}\,\Sigma w_i\theta_i \qquad (3)$$

Considering line ① (see Fig. 1), the internal work becomes $F_y t^2/4$ times its angle of rotation θ_1 times the length of line ① times 2 (since line ① occurs at each flange). Substituting, this becomes $(F_y t^2/4) \times (\theta + \theta d/12t) \times b \times 2$, or $F_y\theta[(bt^2/2) + (bdt/24)]$. Similarly, lines ② thru ⑤ can be calculated.

INTERNAL WORK SUMMARY

Line ①: $$F_y\theta\left(\frac{bt^2}{2} + \frac{bdt}{24}\right) \qquad (4)$$

Line ②: $$F_y\theta\left(\frac{bdt}{24} + \frac{adt}{12}\right) \qquad (5)$$

Lines ③, ④, ⑤: $$F_y\theta\left(\frac{d^2t}{2a} + \frac{6dt^3}{a}\right) \qquad (6)$$

Summing the above and equating to external work will yield:

$$M = F_y\left[\frac{(a+b)dt}{12} + \frac{(ab+d^2)t^2}{2a} + \frac{6dt^3}{a}\right] \qquad (7)$$

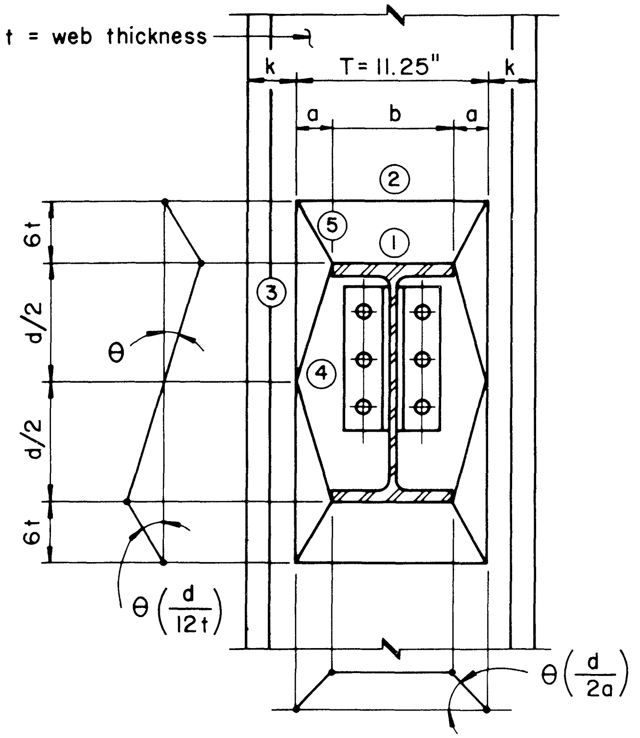

Fig. 1. Assumed yield pattern

Note that this formula is based on the assumption that ultimate strength is determined by yielding in a bending mode along the assumed yield line pattern. As a approaches zero, the formula would seem to indicate that the capacity of the column web to resist moment approaches infinity, which of course is not possible. The correct conclusion, however, is that as a approaches zero, the mode changes from bending to combined bending and shear, and finally to primarily localized shear in the column web at the beam flange tips. Localized shear yielding induces strain hardening under small distortions; therefore, calculated shear stresses in this region equal to shear yield are considered acceptable.

Once a column size and beam grouping has been selected, a, b, and d are known. For example, for group W16×58 through W16×78 (nominal size = 16 x 8½):

$$b = 8.5$$

$$a = (11.25 - 8.5)/2 = 1.375$$

$$d = 16$$

Equation (7) becomes

$$M = F_y[13.167t + 97.341t^2 + 69.82t^3] \qquad (8)$$

F_y is constant; thus, a relationship has been established between the column web thickness t and the beam end moment, and the curve for the 16 x 8½ group can easily be generated.

DESIGN CURVES

Chart I, II, and III were developed using the above method for beams with depths of 10 in. to 24 in. and flange widths of up to 9 in. By using a dual ordinate, both F_y = 36 ksi and F_y = 50 ksi columns can be analyzed. In using these curves, the following factors should be kept in mind:

1. Curves are based on plastic yielding of the column web. In elastic design, the beam moments must be adjusted upward to account for this. In the design example, step a2, load factor = 1.7 was used.
2. For beams on opposite sides of the web, they must be of the same nominal depth; however, their flange widths may vary and interpolation between their respective nominal curves is acceptable.
3. As a is decreased, the shear at the column web fillets becomes large. Although the author feels that this is not critical for the beam sizes covered, this shear check has been performed in the design example in steps b1 and b2.

DESIGN EXAMPLE

Given: Column: W14×184 (t_w = 0.84)
Beams: W18×55 each side
All A36 material
$M_{grav.}$ = 100 kip-ft
M_{wind} = 90 kip-ft

Check: (a) column web strength
(b) shear at column fillets

Solution:

(a) Column web strength:

1. Determine the critical moment acting on the web. Figure 2 shows moment directions. The worst possible condition would be where M_{GL} is minimum and M_{GR} is a maximum.

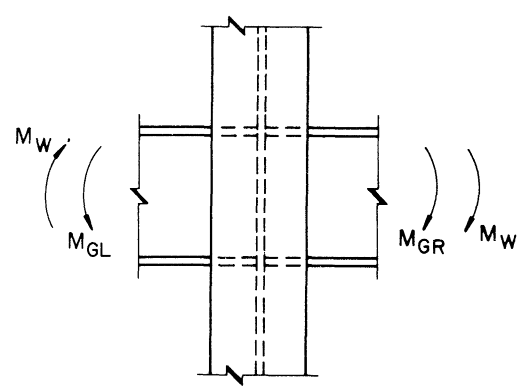

Figure 2

Say $M_{GL} = M_G/2 = 50$ kip-ft and $M_{GR} = 100$ kip-ft.

Total moment $M = 2(90) + (50)$
$= 230$ kip-ft $= 2760$ kip-in.

2. Adjust moment for elastic design and for the $\frac{1}{3}$ stress increase for wind:

$M = (1.7)(2760)(0.75) = 3520$ kip-in.

3. Determine minimum web from Fig. 6 (18 x $7\frac{1}{2}$ beam; $F_y = 36$ column)

$$t = 0.79 < 0.84 \text{ in. } \textbf{o.k.}$$

(b) Shear at column fillets:

1. Determine force to be distributed from beam flange to the fillet. The stress distribution across the flange is not a constant, but its maximum is F_y. Assume the force (F) to consider is as shown in Fig. 3.

$$F = (36 \text{ ksi})(0.63 \times 1) = 22.67 \text{ kips}$$

2. Determine shear along $2a + t_f$ caused by F:

$$f_v = \frac{22.67}{0.84(2 \times 1.875 + 0.63)} \quad \textbf{o.k.}$$
$$= 6.16 < 20.8 \text{ ksi}$$

where 20.8 ksi is the shear yield, $F_y/\sqrt{3}$.

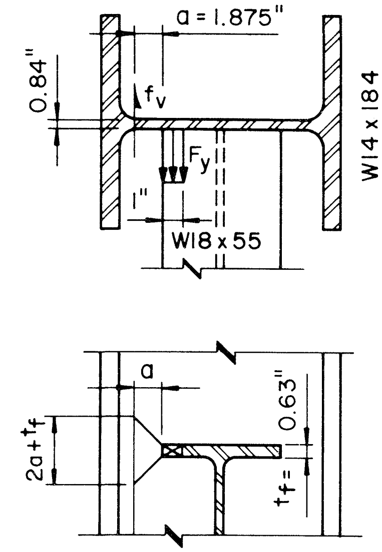

Figure 3

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1. *Abolitz, A. L. and M. E. Warner* Bending Under Seated Connections *AISC Engineering Journal, Vol. 2, No. 1, January 1965.*
2. *Blodgett, O. W.* Design of Welded Structures *James F. Lincoln Arc Welding Foundation, Cleveland, Ohio p. 3.6-6.*

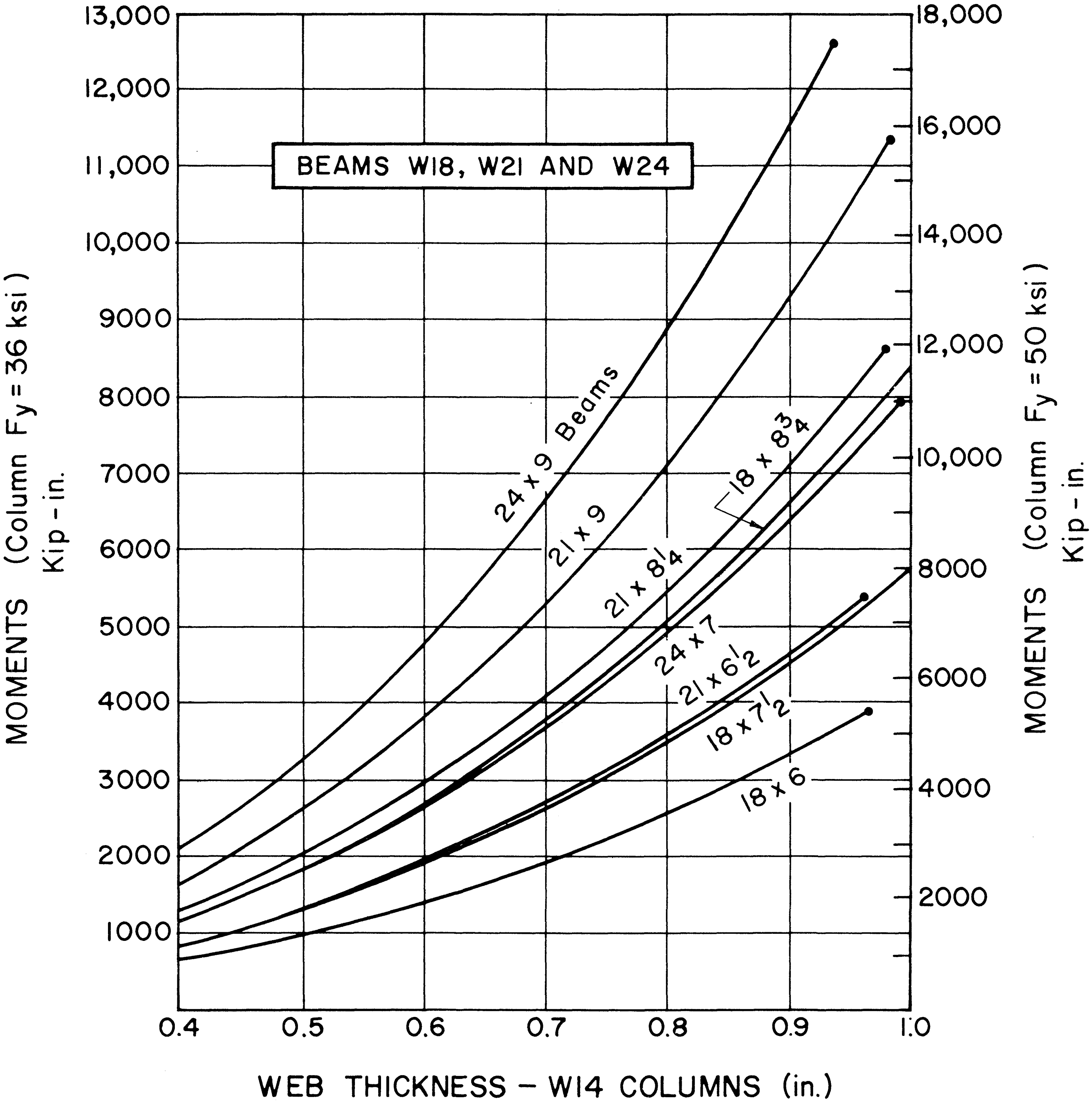

Chart I. W18, W21, and W24 beams

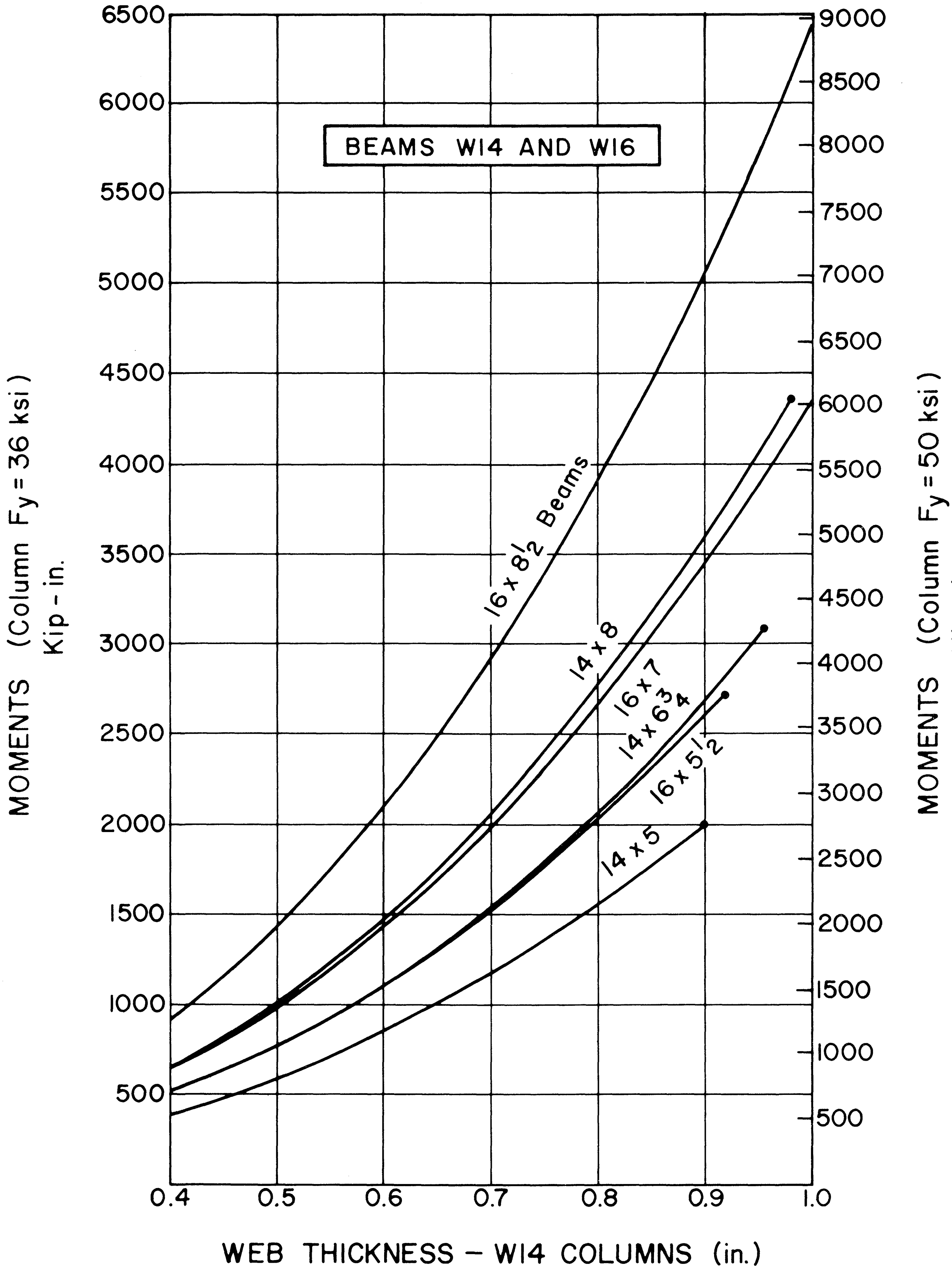

Chart II. W14 and W16 beams

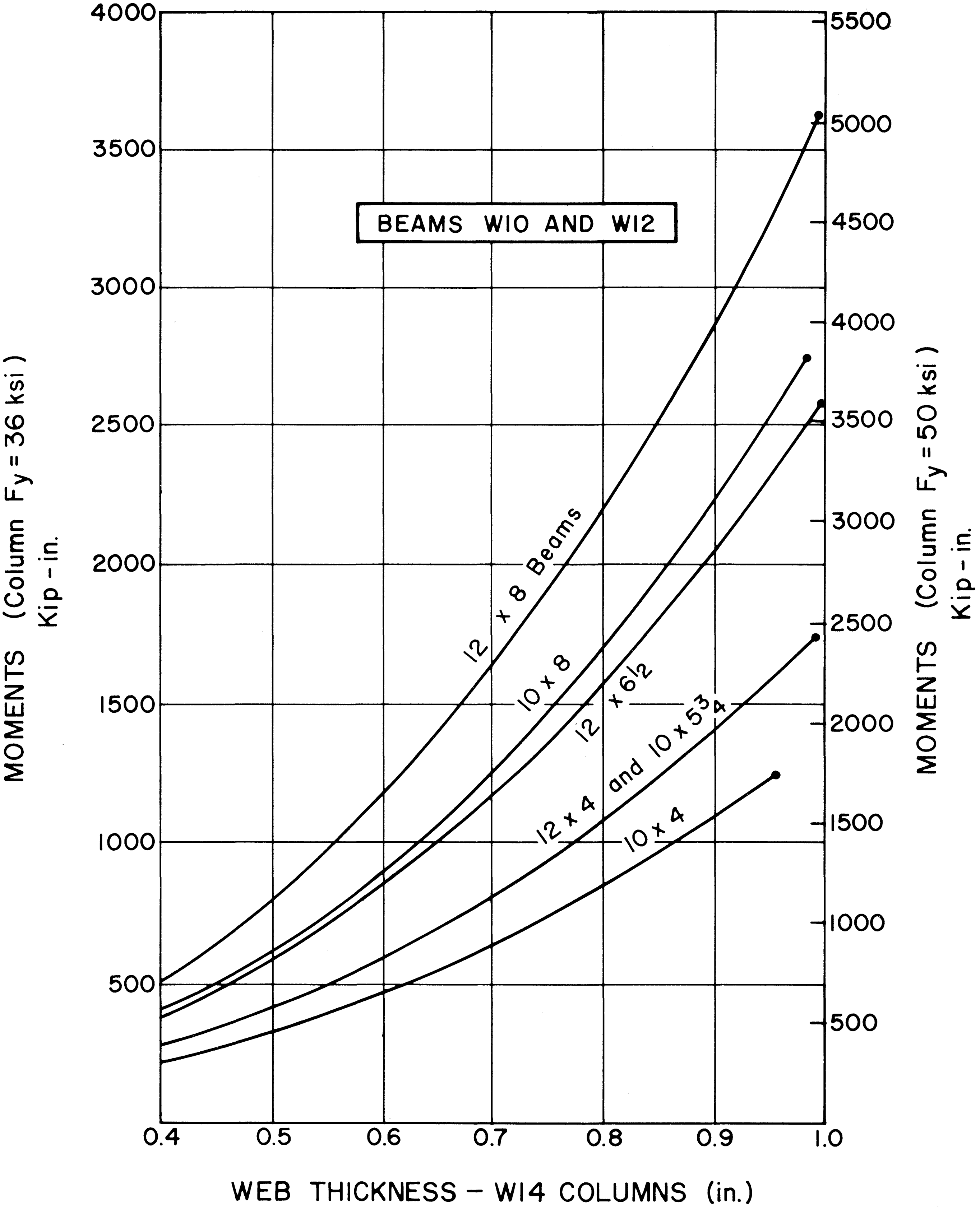

Chart III. W10 and W12 beams

Discussion

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Paper presented by FRANK W. STOCKWELL, JR.
(1st Quarter, 1974 issue)

Discussion by **Richard H. Kapp**

THE CONCERN of the author for an analytical approach to the problem of column (or beam) web adequacy is very welcome. However, the assumptions made do not necessarily lead to the critical yield line. Assumption 3 and the selection of $6t$ (for the distance above and below the beam flange affected by the rotation) yield a different beam end moment M for a given thickness of column web than the writer's approach. The writer's analysis of this problem is similar to that in "Yield Line Analysis of a Web Connection in Direct Tension" by the writer (AISC Engineering Journal, 2nd Quarter, 1974).

Assumption 3 is not made, and the lengths of lines ④ and ⑤ and their angles of rotation are calculated. The distance $6t$ is treated as a variable e (since the yield pattern does not depend on the thickness of the web, but on the geometry of the connected parts). The resulting moment equation varies with respect to e.

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$$M = \frac{F_y t^2 d}{2} [(2a + b)/e + 2e/a + (ba + d^2 + 2a^2)/da] \quad (1)$$

Taking the first derivative of Eq. (1) with respect to e, equating to zero, and solving for e yields the particular e which causes the critical M (the second derivative yields a positive quantity, thus M is minimum for e).

$$e = \sqrt{(2a + b)a/2} \quad (2)$$

Substituting Eq. (2) into Eq. (1) yields the equation for the critical moment M_{cr}.

$$M_{cr} = \frac{F_y t^2 d}{2} [2\sqrt{(4a + 2b)/a} + (2a + b)/d + d/a] \quad (3)$$

Using the author's example with $M = 3520$ kip-in., a web of 0.80 in. is required and the selected column is adequate. The e which gives the yield line shape can be calculated from Eq. (2) and is 3.25 in., which is less than the author's assumed $6t = 5.04$ in. However, the assumptions made by the writer do not much affect the use of the author's graphs when they are compared to graphs which could be made from these equations.

If the assumption is made that $2a + b = T$, as the author has done, then Eqs. (2) and (3) can be written as follows:

$$e = \sqrt{Ta/2} \quad (2a)$$

$$M_{cr} = \frac{F_y t^2 d}{2} [2\sqrt{2T/a} + T/d + d/a] \quad (3a)$$