Ponding of Two-Way Roof Systems

FRANK J. MARINO

SEVERAL PAPERS HAVE been published[1, 2] analyzing roof beams subject to ponding. However, the scope of these papers has been limited to the one-directional action of beams. That is, the flexural members are considered supported by unyielding knife edges with no consideration given to surface deflection transverse to the beam span under study. The effect of interaction between members in a roof framing system can be considerable and should not be neglected. The present AISC Specification[3] is cognizant of the ponding problem. Chinn[1] points out that the Specification provision is arbitrary in nature and could be overly conservative. It is interesting to note here that in all the cases of collapse attributed to the ponding phenomenon that the author has reviewed, the members involved did violate the present Specification provision. However, the provision may actually be unconservative for very large spans.

The purpose of this paper is to analyze the ponding of a roof system, accounting for the interaction of members and to develop a design aid suitable for office use. A restriction imposed on the analysis is that the structural system must consist essentially of two-way framing (i.e., main girders or primary members and secondary sub-members spanning perpendicular to the girders) with the deck contributing negligible deflection to the system. However, if in the absence of other sub-members the deck spans a substantial distance between main members, it should be treated as the secondary system.

ANALYSIS

Ponding may be defined as a situation caused by the flexible nature of a structural assembly, created when a flat roof retains rain water that causes deflection of the roof system, which in turn increases its volumetric capacity. This process is iterative in nature and continues until convergence, which is termed the *equilibrium position;* or, if the system is divergent, until collapse occurs.

Frank J. Marino is Assistant Research Engineer, American Institute of Steel Construction, New York, N. Y.

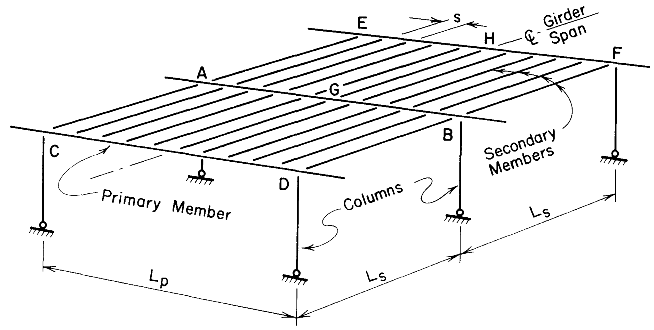


Figure 1

In the case of simply-supported members, the deflection due to dead and live loads on the structure, or accidental negative camber, can initiate ponding. In the case of continuous members having identical stiffness in adjacent spans, ponding action can be initiated by small accidental differences in the levels of these spans before loading. In such circumstances the higher level spans unload, causing accelerated deflections in the adjacent lower spans, and the ponding effect will be similar to that which occurs in a simple span. In either case, it is evident that, to prevent collapse, the equilibrium position must be reached before the maximum flexural stresses in the member reach yield point.

This phenomenon is by no means unique to steel construction. However, due to steel's high strength-to-weight ratio, as compared with other building materials, the problem may be more acute. It is further accented by the introduction of high strength steels and the popularity of plastic design. Both these factors tend to produce designs of shallower depths and therefore more flexible systems.

Figure 1 shows the system under investigation. The primary member under discussion is interior girder **AB.** The secondary member considered is the beam **GH**, which frames into the girder at its mid-span. This, of course, is the critical secondary member. Figure 2 shows the deflected position of members **AB** and **GH** at the equilibrium position previously defined. Note that the supporting primary members at both ends are assumed to be proportional in stiffness and loading so as to have the same deflection Δ. Figure 3 illustrates the loads imposed on each member by the ponded rain. In each case, the deflected elastic curve for the members is assumed as a half sine wave. The ponding loads on the primary member, due to deflection of both the primary and secondary members, are assumed to vary as the ordinates of a half sine wave. The reactions of secondary member on the primary member are assumed continuously distributed rather than as discreet concentrated loads. The ponding loads on the secondary member, however, take the shape of a sine curve due to the deflection of the secondary member plus a uniform load due to the deflection of the primary member.

Considering the primary member, the following observations can be drawn from the load diagram:

$$W_p = \frac{2}{\pi}\gamma L_s(\Delta_o + \Delta_w)L_p$$

and

$$W_s = \frac{(2)^2}{(\pi)^2}\gamma L_s(\delta_w - \delta_{1w})L_p + \frac{2}{\pi}\gamma L_s(\delta_o + \delta_{1w})L_p$$

The bending moment at mid-span of the primary member is:

$$M = \frac{\gamma L_s L_p^2(\Delta_o + \Delta_w)}{\pi^2} + \frac{2\gamma L_s L_p^2(\delta_w - \delta_{1w})}{\pi^3} + \frac{2\gamma L_s L_p^2(\delta_o + \delta_{1w})}{8\pi}$$

The mid-span deflection due to the ponding rain can be calculated by the conjugate beam method as:

$$\Delta_w = \frac{\gamma L_s L_p^4}{\pi^4 EI_p}\left[(\Delta_o + \Delta_w) + \frac{2}{\pi}(\delta_w - \delta_{1w}) + \frac{\pi}{4}(\delta_o + \delta_{1w})\right]$$

and letting

$$\frac{\gamma L_s L_p^4}{\pi^4 EI_p} = C_p$$

$$\Delta_w = C_p\Delta_o + C_p\Delta_w + \frac{2}{\pi}C_p\delta_w - \frac{2}{\pi}C_p\delta_{1w} + \frac{\pi}{4}C_p\delta_o + \frac{\pi}{4}C_p\delta_{1w}$$

Solving for Δ_w:

$$\Delta_w = \alpha_p\left(\Delta_o + \frac{\pi}{4}\delta_o\right) + \alpha_p\left(\frac{2}{\pi}\delta_w + \frac{\pi}{4}\delta_{1w} - \frac{2}{\pi}\delta_{1w}\right) \quad (1)$$

where

$$\alpha_p = \frac{C_p}{1 - C_p}$$

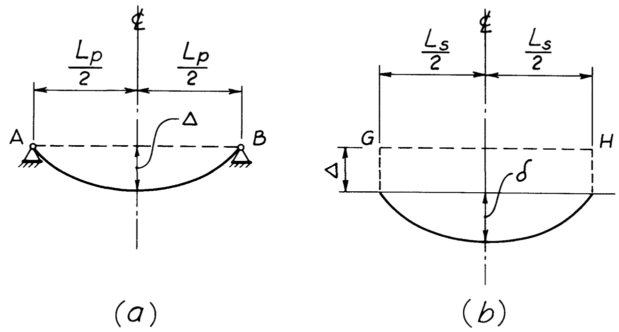


Figure 2

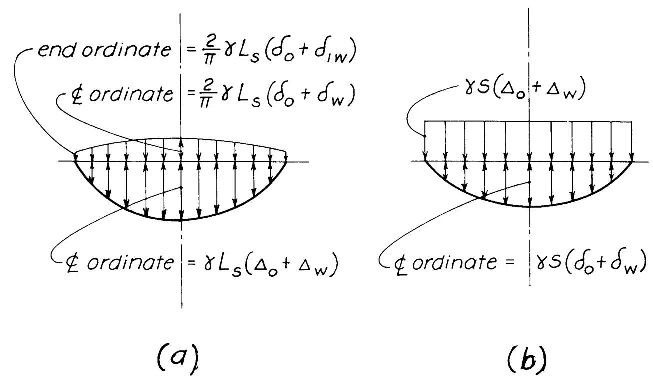


Figure 3

By similar deduction the mid-span deflection of the critical secondary member due to ponded rain can be expressed as:

$$\delta_w = \alpha_s\left(\delta_o + \frac{\pi^2}{8}\Delta_o\right) + \frac{\pi^2}{8}\alpha_s\Delta_w \quad (2)$$

where

$$\alpha_s = \frac{C_s}{1 - C_s}$$

Also,

$$\delta_{lw} = \alpha_s\delta_o \quad (2a)$$

By combining Equations (1), (2) and (2a),

$$\Delta_w = \frac{\alpha_p\left(\Delta_o + \frac{\pi}{4}\delta_o\right) + \frac{\pi}{4}\alpha_p\alpha_s(\delta_o + \Delta_o)}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \quad (3)$$

and

$$\delta_w = \frac{\alpha_s\left(\delta_o + \frac{\pi^2}{8}\Delta_o\right) + \frac{\pi^2}{8}\alpha_p\alpha_s\left(\frac{\pi}{4}\delta_o + \Delta_o + \frac{\pi}{4}\alpha_s\delta_o - \frac{2}{\pi}\alpha_s\delta_o\right)}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \quad (4)$$

It is evident that as the quantity $(\pi/4\alpha_p\alpha_s)$ approaches unity (or $\alpha_p\alpha_s$ approaches $4/\pi$) the ponding deflections Δ_w and δ_w will approach infinity. Therefore one could conclude that if the parameter $(\alpha_p\alpha_s)$ were less than $4/\pi$, Δ_w and δ_w would have a finite limit, termed the equilibrium position. However, another factor must be considered. The analysis so far has assumed elasticity on the part of the structural members. If stresses in any member of the system were to exceed the elastic limit before reaching the point of theoretical equilibrium, a runaway condition could ensue with respect to ponding. Therefore, in addition to complying with the stipulation concerning the parameter $(\alpha_p\alpha_s)$, the design should also ensure that the maximum flexural stress of any member be maintained below the yield point of the material.

In order to develop a criterion that is suitable for design office use, the following substitutions were made:

$$\rho = \frac{\delta_o}{\Delta_o} = \frac{C_s}{C_p} \tag{5}$$

This is self-evident from the observation that both the deflection and the flexibility constant for a beam are directly proportional to L^4/EI.

Substituting Equation (5) into Equations (3) and (4) yields:

$$\Delta_w = \frac{\alpha_p\Delta_o\left[1 + \frac{\pi}{4}\alpha_s + \frac{\pi}{4}\rho(1 + \alpha_s)\right]}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \tag{6}$$

and

$$\delta_w = \frac{\alpha_s\delta_o\left[1 + \frac{\pi^3}{32}\alpha_p + \frac{\pi^2}{8\rho}(1 + \alpha_p) + 0.185\alpha_s\alpha_p\right]}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \tag{6a}$$

Noting that deflection is proportional to stress,

$$\frac{f_w}{f_o} = \frac{\Delta_w}{\Delta_o}$$

and placing as a limitation on the stress induced by ponding,

$$f_w \leq \frac{F_y}{\text{F.S.}} - f_o$$

then

$$\Delta_w \leq \left[\frac{\frac{F_y}{\text{F.S.}} - f_o}{f_o}\right]\Delta_o = \left[\frac{1}{\text{F.S.}}\frac{F_y}{f_o} - 1\right]\Delta_o$$

A factor of safety (F.S.) of 1.25 against yielding is suggested for design office use.

By substituting into Equation (6):

$$\left[\frac{1}{\text{F.S.}}\frac{F_y}{f_o} - 1\right]_p \geq \frac{\alpha_p\left[1 + \frac{\pi}{4}\alpha_s + \frac{\pi}{4}\rho(1 + \alpha_s)\right]}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \tag{7}$$

Similarly for the secondary member:

$$\left[\frac{1}{\text{F.S.}}\frac{F_y}{f_o} - 1\right]_s \geq \frac{\alpha_s\left[1 + \frac{\pi^3}{32}\alpha_p + \frac{\pi^2}{8\rho}(1 + \alpha_p) + 0.185\alpha_s\alpha_p\right]}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \tag{8}$$

Equations (7) and (8) afford a relatively easy method for checking the ponding stability of a two-way roof system.

Figures 4 and 5 are design aids which plot the relationship between the parameters C_p and U_p, and C_s and U_s, for Equations (7) and (8), respectively. The term U represents the left side of these equations. To use these charts, tentatively select member sizes, as usual, on the basis of the design loading. Then from the known characteristics, compute the values of U_p, U_s, C_p, and C_s. To check the primary member, enter Fig. 4 at the left with the value of U_p. Proceed to the right to the intersection with the curve representing the flexibility constant of the secondary member (C_s). Descend to the abscissa and read the maximum flexibility constant of the primary member to satisfy Equation (7). If the actual C_p is larger than this value, it indicates that the system is potentially unstable and the design should be revised. A similar procedure can be used, employing Fig. 5, to check the secondary member.

As a further simplification, the parameters C_p and C_s can be computed from the following expressions in which γ, π and E have been replaced by their numerical equivalent:

$$C_p = \frac{l_s l_p^4}{32 \times 10^4 I_p}$$

$$C_s = \frac{s l_s^4}{32 \times 10^4 I_s}$$

where l_s, l_p and s are in feet and I_s and I_p are in in.^{4}

It is important to note that the span involved in this analysis is the distance between *support points* (i.e., column spacing) and *not* between splice points. The flexibility limitation obtained by this analysis should be applied to simple and continuous spans alike. This is because unequal deflections in adjacent continuous spans can result in a greater accumulation in one

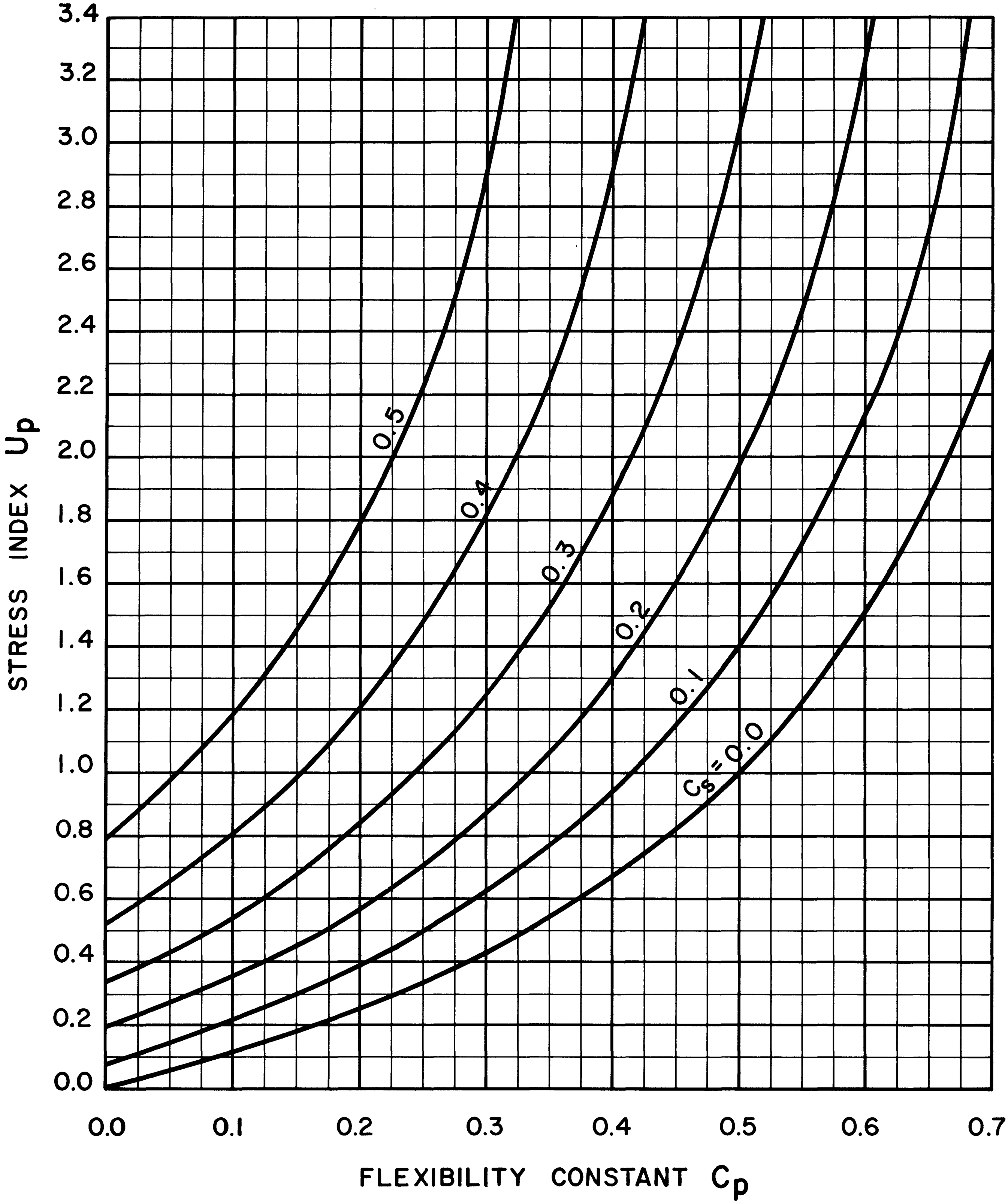
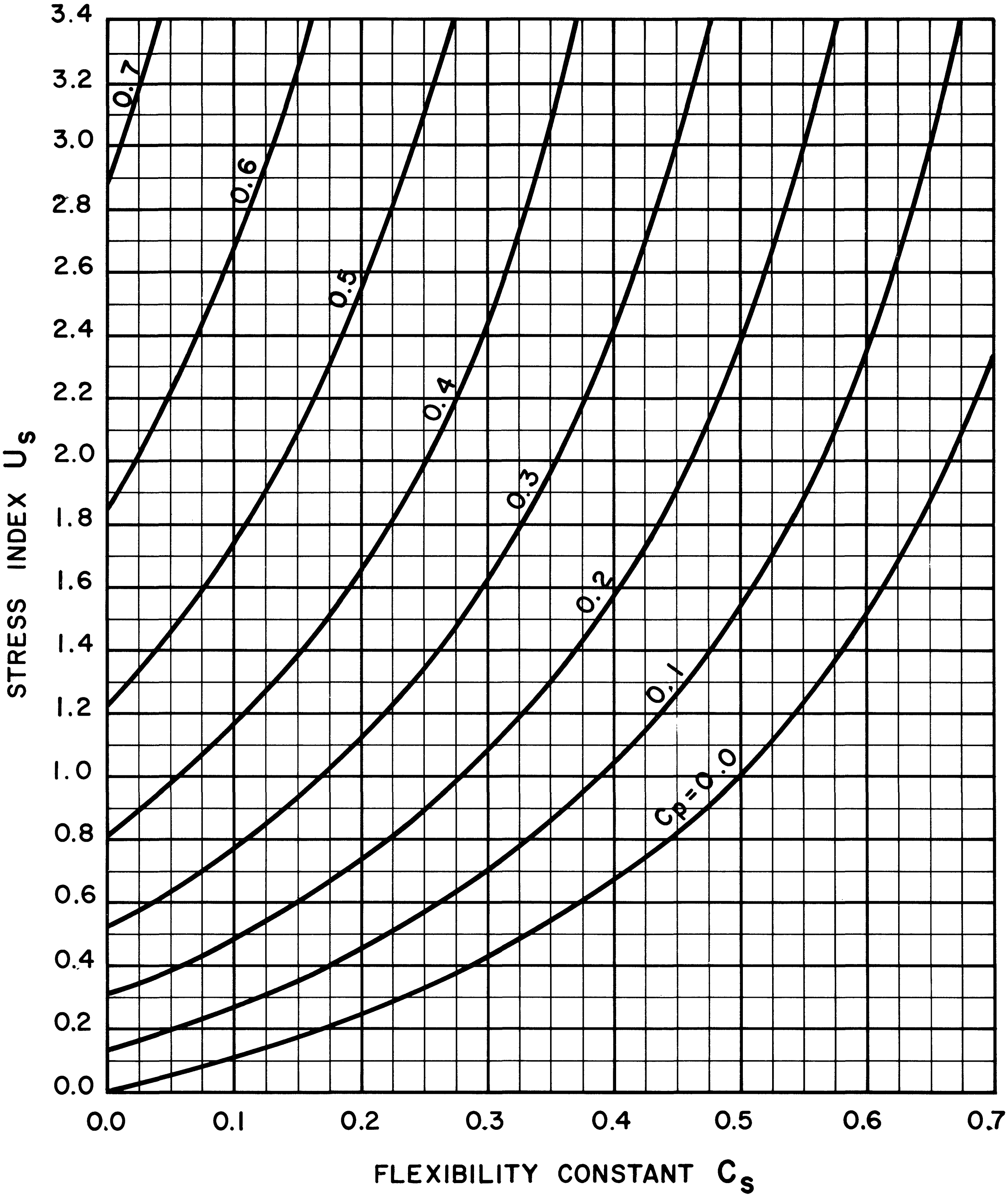

STRESS INDEX U_p
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
0.4
0.3
0.2
0.1
C_s=0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
FLEXIBILITY CONSTANT C_p

Figure 4



3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
STRESS INDEX U_S
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
FLEXIBILITY CONSTANT C_S
0.7
0.6
0.5
0.4
0.3
0.2
0.1
C_P=0.0

Figure 5

span which, in turn, tends to unload the adjacent spans. This reduces the restraint at the ends of the ponded span, increases the deflection due to ponding in that span, and ultimately causes the continuous beam to act as though it were simply supported.

Another consideration to bear in mind relates to the calculation of U_p and U_s. In these terms, the value of f_o is the stress associated with all dead and live loads which are likely to be on the roof at the time that ponding commences. This would include any anticipated water load due to reservoir action of curbs and similar architectural features.

DISCUSSION AND CONCLUSIONS

The most desirable method to preclude the effect of ponding is to provide sufficient slope to the roof surface along with adequate drainage facilities to prevent the accumulation of rain water in the first instance. For the slope to be sufficient, the upward pitch provided to a roof surface must exceed the downward slope of the beam's elastic curve at or near the support point, caused by all gravity loads. Experience as well as theoretical considerations indicate that a pitch of ⅛-in. per ft will suffice for this purpose under normal conditions of *free* drainage. However, the hydraulics of roof drainage is actually a very complex problem which requires careful study and is not included in the scope of this paper. In many cases, it is not feasible to drain a roof area without incurring the risk of some accumulation.

This analysis presented has been tested against several cases of roof collapse attributed to ponding. In each case the instability would have been predicated by a significant margin.

REFERENCES

1. *Chinn, James* Failure of Simply-Supported Flat Roofs by Ponding of Rain, *AISC Engineering Journal, April, 1965.*
2. *Haussler, R. W.* Roof Deflection Caused by Rainwater Pools, *Civil Engineering, October, 1962.*
3. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings, *American Institute of Steel Construction, New York, N. Y., April 17, 1963.*

NOMENCLATURE

C_p Flexibility constant of primary member $= \dfrac{\gamma L_s{}^4 L_p}{\pi^4 E I_p}$

C_s Flexibility constant of secondary member $= \dfrac{\gamma S L^4{}_s}{\pi^4 E I_s}$

E Modulus of elasticity (psi)

F.S. Factor of safety

F_y Yield point of member considered (psi)

I_p Moment of inertia, primary member (in.^{4})

I_s Moment of inertia, secondary member (in.^{4})

L_p Span of primary member (in.)

L_s Span of secondary member (in.)

M_p Bending moment in primary member, at mid-span (lb-in.)

S Spacing of secondary members (in.)

U_p Stress index of primary member $= \left[\dfrac{1}{\text{F.S.}}\dfrac{F_y}{f_o} - 1\right]$

U_s Stress index of secondary member $= \left[\dfrac{1}{\text{F.S.}}\dfrac{F_y}{f_o} - 1\right]$

W_p Total water load, on primary member, due to volumetric configuration of primary members (lb)

W_s Total water load, on primary member, due to volumetric configuration of secondary members (lb)

d_p Depth of primary member (in.)

d_s Depth of secondary member (in.)

f_o Extreme fiber flexural stress in a member at onset of ponding (psi)

f_p Extreme fiber flexural stress in primary member (psi)

f_s Extreme fiber flexural stress in secondary member (psi)

f_w Extreme fiber flexural stress in a member due to ponding (psi)

l_p Span of primary member (ft)

l_s Span of secondary member (ft)

s Spacing of secondary members (ft)

α_p Flexibility parameter of primary member $= \dfrac{C_p}{1 - C_p}$

α_s Flexibility parameter of secondary member $= \dfrac{C_s}{1 - C_s}$

γ Unit weight of water (lb/in.^{3})

Δ_o Deflection in primary member at onset of ponding (in.)

Δ_w Deflection in primary member due to ponding effect (in.)

δ_o Deflection in secondary member at onset of ponding (in.)

δ_w Deflection in secondary member at center line of primary member due to ponding effect (in.)

δ_{1w} Deflection in secondary member at end of primary member due to ponding effect (in.)

ρ Initial deflection ratio $= \delta_o/\Delta_o$

APPENDIX

Example 1—An industrial building has been designed with 50 ft-0 in. x 38 ft-0 in. bays. The structural members of the flat roof have been proportioned by conventional analysis. Check the design for ponding.

Given: Girders (50 ft-0 in. span): 21WF55

$I_p = 1140.7$ in.^{4}

$f_b = 23.0$ ksi

Secondary members

(38 ft-0 in. span): 20H7 open web joist

$I_s \cong 160$ in.^{4}

$f_b = 28.5$ ksi

Joist spacing: 6 ft-3 in. o.c.

Live load: 20 psf

Dead load: 15 psf

Solution:

Assume that one-quarter of L.L. is on roof at outset of ponding.

$$f_o \text{ (girder)} = 23 \times \left(\frac{15 + 5}{35}\right) = 13.2 \text{ ksi}$$

$$f_o \text{ (joist)} = 28.5 \times \left(\frac{15 + 5}{35}\right) = 16.3 \text{ ksi}$$

$$U_p = \left(\frac{1}{1.25} \times \frac{36}{13.2}\right) - 1 = 1.18$$

$$U_s = \left(\frac{1}{1.25} \times \frac{50}{16.3}\right) - 1 = 1.45$$

$$C_p = \frac{38 \times 50^4}{32 \times 10^4 \times 1140.7} = 0.65$$

$$C_s = \frac{6.25 \times 38^4}{32 \times 10^4 \times 160} = 0.26$$

(a) Check girder:

From Fig. 4, with $U_p = 1.18$ and $C_s = 0.26$:

Allowable $C_p = 0.32 < 0.65$ **N.G.**

(b) Check joist:

From Fig. 5, with $U_s = 1.45$ and $C_p = 0.65$:

Allowable $C_s < 0 < 0.31$ **N.G.**

Therefore, neither the girders nor joists are suitable on the basis of ponding analysis, even though both are adequately proportioned on the basis of static load strength. Note that neither the girder nor the joist would have met the requirements of AISC Specification Sect. 1.13, which requires a minimum depth of 24 in. for both primary and secondary members in this case.

Example 2—Redesign the roof system of Example 1 to be adequate for ponding.

Solution:

Try: 24WF68 girder: $I_p = 1814.5$ in.^{4}

$f_b = 16.4$ ksi

24J8 joist: $I_s \cong 270$ in.^{4}

$f_b = 19.2$ ksi

$$f_o \text{ (girder)} = 16.4 \times \frac{20}{35} = 9.4 \text{ ksi}$$

$$f_o \text{ (joist)} = 19.2 \times \frac{20}{35} = 11.0 \text{ ksi}$$

$$U_p = \left(\frac{1}{1.25} \times \frac{36}{9.4}\right) - 1 = 2.10$$

$$U_s = \left(\frac{1}{1.25} \times \frac{36}{11.0}\right) - 1 = 1.60$$

$$C_p = 0.65 \times \frac{1140.7}{1814.5} = 0.41$$

$$C_s = 0.26 \times \frac{160}{270} = 0.15$$

(a) Check girder:

From Fig. 4, with $U_p = 2.10$ and $C_s = 0.15$:

Allowable $C_p = 0.55 > 0.43$ **O.K.**

(b) Check joist:

From Fig. 5, with $U_s = 1.60$ and $C_p = 0.41$:

Allowable $C_s = 0.18 > 0.15$ **O.K.**

Use 24WF68 girders and 24J8 joists.

Example 3—A flat roof with bay spacings of 34 ft-0 in. x 24 ft-0 in. has been preliminarily proportioned with 14B26 girders (34 ft-0 in. span) and 12H4 joists (24 ft-0 in. span) on 6 ft-0 in. centers. Check the design for ponding.

Given: 14B26: $I_p = 242.6$ in.^{4}

$f_b = 21.0$ ksi

12H4: $I_s \cong 35$ in.^{4}

$f_b = 30.0$ ksi

Solution:

Assume that one-fifth L.L. is on roof at onset of ponding.

$$f_o \text{ (girder)} = 21.0 \times \frac{19}{35} = 11.4 \text{ ksi}$$

$$f_o \text{ (joist)} = 30.0 \times \frac{19}{35} = 16.0 \text{ ksi}$$

$$U_p = \left(\frac{1}{1.25} \times \frac{36}{11.4}\right) - 1 = 1.50$$

$$U_s = \left(\frac{1}{1.25} \times \frac{50}{16.3}\right) - 1 = 1.50$$

$$C_p = \frac{24 \times 34^4}{32 \times 10^4 \times 242.6} = 0.41$$

$$C_s = \frac{6 \times 24^4}{32 \times 10^4 \times 35} = 0.18$$

(a) Check girder:

From Fig. 4, allowable $C_p = 0.45 > 0.41$ **O.K.**

(b) Check joist:

From Fig. 5, allowable $C_s = 0.18 \geq 0.18$ **O.K.**

Preliminary members are O.K. for ponding.

Note that although these members are adequate for ponding according to the analytic method in this paper, they could not be used for a design under the AISC Specification. Section 1.13 requires a minimum depth of 14.5 for these members, so that 16-in. deep girders and joists would actually have to be provided.

Example 4—An industrial building consists of a flat roof system supported directly on masonry walls. Open web steel joists, 24LH06, span a 40 ft-0 in. clear opening between walls, and 3-in. deep, 20 ga. steel deck spans 14 ft-0 in. c. to c. of joists. Check the design for ponding.

Given: 24LH06: $I \cong 400$ in.^{4}

$f_b = 28.8$ ksi

3-in. deck: $I = 0.83$ in.^{4}/ft

$f_b = 18.2$ ksi

Live load: 20 psf

Dead load: 10 psf

Solution: In this example it is obvious that the masonry walls act as unyielding supports, the joists act as primary flexural members, and the steel deck acts as the secondary flexural members.

Assume that one-half the live load (10 psf) is on the roof at the onset of ponding.

Check joist:

$$f_0 = 28.8 \times \frac{10+10}{30} = 19.2 \text{ ksi}$$

$$U_p = \left(\frac{1}{1.25} \times \frac{50}{19.2} - 1\right) = 1.08$$

$$C_p = \frac{14 \times 40^4}{32 \times 10^4 \times 400} = 0.28$$

From Fig. 4, allowable $C_p = 0.38 > 0.28$ **O.K.**

Check deck:

$$f_0 = 18.2 \times \frac{10+10}{30} = 12.1 \text{ ksi}$$

$$U_s = \left(\frac{1}{1.25} \times \frac{33}{12.1} - 1\right) = 1.18$$

$$C_s = \frac{1 \times 14^4}{32 \times 10^4 \times 0.83} = 0.14$$

From Fig. 5, allowable $C_s = 0.19 > 0.14$ **O.K.**

Therefore, the design is satisfactory for ponding.

Discussion

Ponding of Two-way Roof Systems

Paper presented by FRANK J. MARINO (July, 1966, Issue)

Discussion by **DONALD A. SAWYER**

Mr. Marino has correctly pointed out that the problem of water ponding on flexible roof systems is more involved than some of the earlier papers implied and that the AISC Specification, Sect. 1.13, may give false information about the susceptibility of a given roof to ponding problems. In order to put the design of flat roofs on a more rational basis, he has derived certain relationships that are offered as possible design office procedures. His methods have been illustrated by several examples.

Up through Equation (4), his derivations are sound and there can be no argument with his use of certain acceptable approximations to arrive at the stability criterion for two-way roofs. The device used was that of assuming an arbitrary initial deflection in both the primary and the secondary beams, and proceeding to a final equilibrium position determined by the flexibility constants of the two types of members. For this part of the derivation, it does not matter how the initial deflections were caused, and Mr. Marino has stated that these may be due, in part, to initial crookedness. However, beginning with Equation (5), the derivation goes astray because the initial crookedness component has been omitted. The situation is similar to that for the case of initially crooked columns. It can be shown that initial crookedness in a column does not reduce its Euler buckling load but it does affect the load at which yielding first commences. For beams subject to ponding loads, the initial crookedness does not reduce the critical stability number but it does affect the equilibrium position and thereby the point at which yielding commences.

The major problem with Mr. Marino's derivation is that when initial crookedness is present, C_s/C_p is not a proper measure of δ_0/Δ_0 and Δ_w/Δ_0 is not a proper measure of f_w/f_0. These deficiencies can be remedied as follows. In general, the initial deflected shape of a beam at the onset of ponding is due to two components. One component is that due to the loads that are on the beam at the onset of ponding. This can be related to the stress f_0. The second component is that due to intentional or accidental crookedness. Its magnitude is independent of the initial stresses and would represent the shape of the beam in the unstressed condition. Assuming, with minor error, that the same constant would apply to both the primary and the secondary beams, their stress-related deflection at the onset of ponding can be expressed as

$$\Delta_0' = C_d f_{0p} \tag{9}$$

and

$$\delta_0' = C_d f_{0s} \tag{9a}$$

Assuming that the initial crookedness is known or can be estimated, its magnitude can be expressed as a multiple of the stress-related deflection. Thus,

$$\Delta_0'' = \beta_p \Delta_0' \tag{10}$$

$$\delta_0'' = \beta_s \delta_0' \tag{10a}$$

in which β_p and β_s are positive when the initial crookedness is such as to cause a sag. Again, with some error it may be assumed that

$$\Delta_w = C_d f_{wp} \tag{11}$$

$$\delta_w = C_d f_{ws} \tag{11a}$$

although the load distribution for the ponded water is somewhat different from the load distribution that exists at the onset of ponding. As Mr. Marino indicated, the deflections due to the loads can be expressed in terms of the flexibility constants also. Thus,

$$\Delta_0' = C_y C_p \tag{12}$$

$$\delta_0' = C_y C_s \tag{12a}$$

From Equations (10) and (12), the total deflections at the onset of ponding are

$$\Delta_0 = (1 + \beta_p) C_y C_p \tag{13}$$

$$\delta_0 = (1 + \beta_s) C_y C_s \tag{13a}$$

Then, from Equations (9), (10) and (11)

$$\frac{\Delta_w}{\Delta_0} = \frac{f_{wp}}{(1 + \beta_p) f_{0p}} \tag{14}$$

$$\frac{\delta_w}{\delta_0} = \frac{f_{ws}}{(1 + \beta_s) f_{0s}} \tag{14a}$$

Donald A. Sawyer is Head Professor, Department of Civil Engineering, Auburn University, Auburn, Ala.

From Equations (13)

$$\frac{\delta_0}{\Delta_0} = \frac{(1 + \beta_s)C_s}{(1 + \beta_p)C_p} \tag{15}$$

When Equations (14) and (15) are substituted in the appropriate places in Mr. Marino's derivation of Equations (7) and (8), the following new relationships are found:

$$\left[\frac{1}{\text{F.S.}} \frac{F_y}{f_{0p}} - 1\right] \geq A_p(1 + \beta_p) + B_p(1 + \beta_s) \tag{7a}$$

$$\left[\frac{1}{\text{F.S.}} \frac{F_y}{f_{0s}} - 1\right] \geq A_s(1 + \beta_p) + B_s(1 + \beta_s) \tag{8a}$$

where,

$$A_p = \frac{\alpha_p\left(1 + \frac{\pi}{4}\alpha_s\right)}{1 - \frac{\pi}{4}\alpha_p\alpha_s}$$

$$B_p = \frac{\frac{C_s}{1 - C_p}\frac{\pi}{4}(1 + \alpha_s)}{1 - \frac{\pi}{4}\alpha_p\alpha_s}$$

$$A_s = \frac{\frac{C_p}{1 - C_s}\frac{\pi}{4}(1 + \alpha_p)}{1 - \frac{\pi}{4}\alpha_p\alpha_s}$$

$$B_s = \frac{\alpha_s\left[1 + \frac{\pi^3}{32}\alpha_p + \left(\frac{\pi^3}{32} - \frac{\pi}{4}\right)\alpha_p\alpha_s\right]}{1 - \frac{\pi}{4}\alpha_p\alpha_s}$$

When β_p and β_s are both zero, the modified equations reduce to Mr. Marino's original Equations (7) and (8). Curves similar to Figs. 4 and 5 may be drawn for the four parameters A_p, B_p, A_s and B_s. A few computations will show that the initial crookedness can have a major influence on the results and that the effect of a given amount of crookedness is greatly different depending on whether the beam is erected with the crookedness adding to or subtracting from the deflection due to loads.

Some expected values of β_p and β_s can be estimated from the rolling tolerances allowed by practice. It is common to allow as much as ⅛-in. of crookedness for each 5 ft of beam length. In order to compare this with the dead load deflection of the example problems, it is convenient to express the deflection of a simple beam with its contributing area loaded with w psf as follows:

$$\Delta = \frac{w}{4.11} C_p \tag{16}$$

$$\delta = \frac{w}{4.11} C_s \tag{16a}$$

For Example 2 of the paper, the dead load is 15 psf and $C_p = 0.41$, so that the dead load deflection of the girders would be 1.50 in. The initial crookedness could be as much as $(0.125)(50/5) = 1.25$ in. Thus, β_p could lie somewhere between the limits of -0.84 and $+0.84$. Similarly, β_s could lie somewhere between -1.73 and $+1.73$. These values could affect the conclusions significantly. Obviously, even greater values of β_p and β_s can occur when intentional camber exists.

Note that when β_p and β_s are both equal to -1.00, Equations (7a) and (8a) would indicate that no allowance need be made for ponding regardless of the magnitudes of the flexibility constants. Of course this would be an ideal case where the roof is mathematically flat under full dead load and the rain could drain instantly over zero-height gravel stops. Clearly, one should not rely on such details to avoid ponding problems. However, considerations of favorable camber may help to explain why some apparently critical roofs have survived.

Because initial crookedness has not been taken into account in the numerical examples, it would be proper to rework them completely. However, it will be assumed that perfectly straight beams were used so that a few other comments will be pertinent. Mr. Marino has demonstrated that a designer may apply the procedure for any arbitrary amount of live load present at the onset of ponding. It was probably his intent to show the ease with which the computations could be made using Figs. 4 and 5. However, the impression is given that the designer should choose a proper fraction of live load to be present at the onset of ponding. But Example 3 shows that the procedure can be quite sensitive to this choice. That is, Example 3 demonstrates that the given roof satisfies the suggested criteria when one-fifth of the L.L. (4 psf) is on the roof at the onset of ponding. However, a check will show that the roof does not satisfy the criteria if one-fourth of the L.L. (5 psf) is on the roof at the onset. Thus, a difference of just 1 psf in that assumption would cause the roof to be either accepted or rejected. In southern regions where the true cause of roof live load is somewhat nebulous, it seems that the most logical source of the initial fraction must come from the rain storage caused by gravel stops or inadequate drainage. Some preliminary work by the writer indicates that much useful information for the selection of a factor somewhat akin to Mr. Marino's live load fraction will come from a study of roof hydraulics. In the meantime it appears that a designer must fall back on judgment to make a selection of the live load fraction if he chooses to apply Equations (7) and (8).

The application of the procedure to Example 1 shows that even an infinitely stiff joist would not be accepted. Clearly this is an extreme requirement. Actually, the example shows that the girder is so flexible that the stress requirements are not met even before the joist deflections are taken into account. Therefore, because this is a problem in the interaction of the two beam systems, it would be possible to make a redesign by changing only the girder rather than by changing both the girder and the joists as was done in Example 2. The correct course of action might be determined by a consideration of the weight required for various alternative designs. For the case of Example 1, it does seem best to modify both the girder and the joists to achieve a less heavy design. A check will show that 22H7 joists and a 24W68 girder would be acceptable according to Figs. 4 and 5 rather than the 24J8 joists chosen in Example 2. This modification is 516 lbs/bay lighter than the design of Example 2.

This writer has a similar paper in press[1] that takes into account several important parameters that were omitted from Mr. Marino's paper. That paper and one or two others due for publication within the next several months should do much to answer some of the remaining questions concerning this interesting roof ponding problem.

ADDITIONAL NOMENCLATURE

C_d Deflection constant relating extreme fiber stress to deflection

C_y Deflection constant relating deflection to the flexibility constant

β_p Parameter relating initial crookedness to load deflection at onset of ponding for primary member

β_s Parameter relating initial crookedness to load deflection at onset of ponding for secondary member

Δ_0' Deflection of primary member due to load existing at the onset of ponding

δ_0' Deflection of secondary member due to load existing at the onset of ponding

Δ_0'' Initial crookedness of the unstressed primary member

δ_0'' Initial crookedness of the unstressed secondary member

1. *Sawyer, Donald A.* Ponding of Rainwater on Flexible Roof Systems, *Journal of the Structural Division, ASCE (scheduled for publication in February, 1967).*

Discussion by **FRANK J. MARINO**

THE CONTENT and scope of Professor Sawyer's discussion indicate his keen understanding of the ponding phenomenon. He correctly points out, as the author was aware, that accidental initial crookedness might effect the equilibrium position of a member and hence the level of the maximum flexural stress. However, several factors led the author to ignore this effect in attempting to formulate a relatively simple analytical procedure, suitable for the design office.

The magnitude of the effect is relatively small. The allowable mill tolerance for camber of steel shapes, given in ASTM A6, is 1/8-in. per 10 ft of length, and not 1/8-in. per 5 ft as stated in Professor Sawyer's discussion. Moreover, the AISC Specification, Sect. 1.19.3, requires that, after erection, any accidental camber due to rolling or fabricating processes must be upward. Thus any accidental crookedness present in roof members should serve to lower the magnitude of ponding stresses. The effect, as previously pointed out, is small. Normal factor of safety requirements would cover the unlikely situation where, by some oversight, a roof member was fabricated and erected with a downward accidental camber. For instance, in the example cited by Professor Sawyer, referring to Example 2 of the author's paper, if the girder of that example had been erroneously erected with a downward camber, the amount of that accidental crookedness could have been, at most, 0.625 in. This would have the effect of increasing the final stress level in the girder by approximately 2 ksi, or about 7 percent.

Professor Sawyer points out that conformance to design criteria will depend on the amount of live load that is assumed on the roof at the onset of ponding. The fact that the amount of live load will affect the magnitude of final stress is self evident. Any structural member, designed against performance criteria, will satisfy, or not satisfy, those criteria depending on the amount of load assumed in the analysis. The determination of live load level for any aspect of design is a decision properly made by the designer for the specific conditions under consideration.

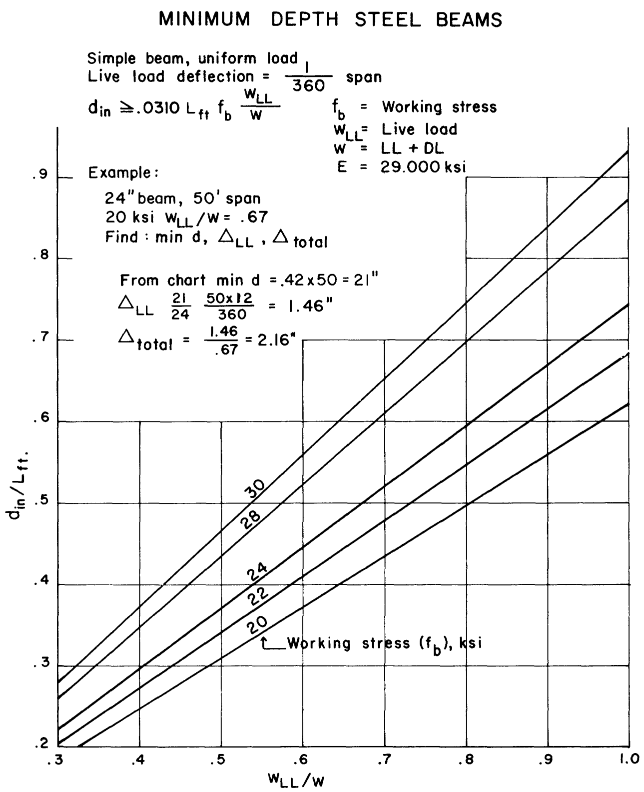
The author must confess that no attempt was made, in formulating the design examples provided with the paper, to reach an optimum or minimum weight design. The intent was merely to illustrate the application of the ponding analysis. Certainly, in attempting to modify an actual design that does not meet the ponding criteria, several avenues are open to the designer. He would normally choose that solution that optimized the design while still conforming to other nonstructural limitations.

Errata

October, 1966 (Vol. 3, No. 4)

Page 168

In Adolph A. Marrone's discussion of the April, 1966, paper by George Froyton, identification of the working stress values was inadvertently omitted from the curves in the chart entitled "Minimum Depth Steel Beams." These values have been added to the chart, and the corrected chart is reproduced below:



July, 1966 (Vol. 3, No. 3)

Page 98

In the Nomenclature of the paper by Frank J. Marino, the terms C_p and C_s should be corrected as follows:

C_p Flexibility constant of primary member $= \dfrac{\gamma L_s{}^4 L_p}{\pi^4 E I_p}$

C_s Flexibility constant of secondary member $= \dfrac{\gamma S L^4{}_s}{\pi^4 E I_s}$

April, 1966 (Vol. 3, No. 2)

Page 73

The equation preceding Equation (11) should be corrected to read:

$$N^2 + N(1 - Kn) - Kn\left(\frac{6m}{1.6} + 1\right) = 0$$