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Column Design: Past, Present, Future

December 6, 2018

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Stronger.
Steel.

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Course Description

Column Design: Past, Present, Future
December 6, 2018

The historical development of the different approaches for designing metal columns is presented and critiqued. Prior to 1960, emphasis was placed on design methods for an isolated compression member. Since 1960 the focus has been on columns as part of frames, by introducing factors such as effective length factors (K-factors), PΔ, frame stability, plastic design and second-order structural analysis. The development of the current AISC column curve is presented. The single-column curve vs the multiple-column curve controversy is evaluated, and possible future changes to our design approach, due to the current methods of manufacturing rolled steel sections, will be predicted.

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Learning Objectives

- Describe how builders proportioned columns historically, from ancient times through the end of the 18th century.
- List the research advances that took place in the 19th century, which informed the design practices for wrought iron and early steel columns.
- Explain the strategies that engineers have employed to account for the effect of residual stresses in rolled steel shapes, in compression members.
- Identify why current manufacturing processes might cause reformulation of structural steel column design methods in the future.

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Column Design: Past, Present, Future

Joseph A. Yura, PE, PhD
Emeritus Professor in Civil Engineering
University of Texas at Austin

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INTRODUCTION

HISTORICAL REVIEW OF PAST WORK

- Axially Loaded Column Design:
  Ancient $\longrightarrow$1960
- Column Design as Part of a Frame:
  1960 $\longrightarrow$Present

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INTRODUCTION

HISTORICAL REVIEW OF PAST WORK

What is the purpose of a historic review?

- The best way to teach a subject is in the order it evolved
- Helps guide and predict the future approaches

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OUTLINE: (KEY WORDS)

1. ANCIENT – 1650 (Greek temples, calculus)
2. 1650 -1800 (Euler)
3. 1800 -1900 (wrought iron, eccentrically-loaded column)
4. 1900 -1945 (steel frame, AISC)
5. 1945 -1970 (K-factors, plastic design, residual stress)
6. 1970 -1985 (frames, multiple column curves, LRFD)
7. 1985 -2018 (K = 1.0, 2nd order modified analysis)
8. Future

11

OUTLINE: (KEY WORDS)

1. **ANCIENT - 1650 (Greek temples, calculus)**
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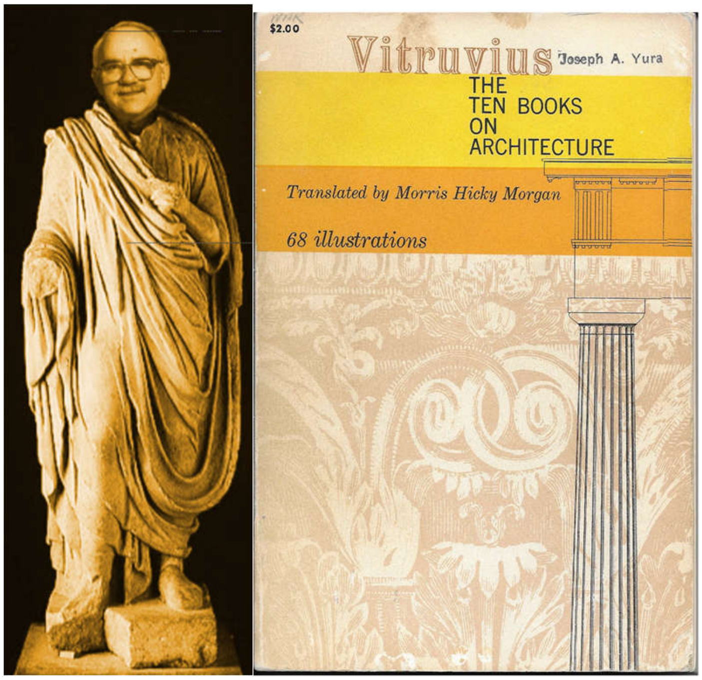
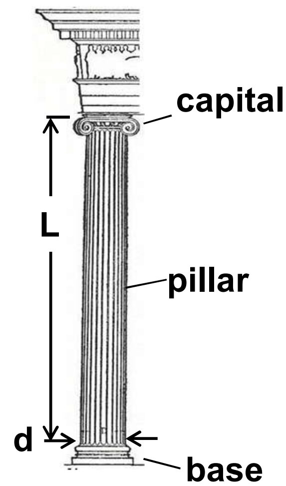
5000 BC: THE FIRST COLUMN FORMULA



$$\frac{L}{d} = \frac{\text{Body Height}}{\text{Foot Imprint}}$$



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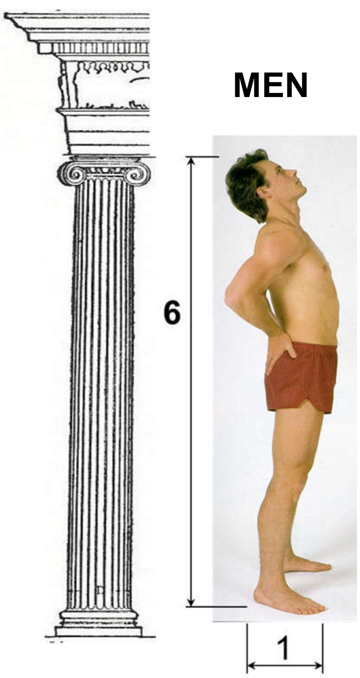
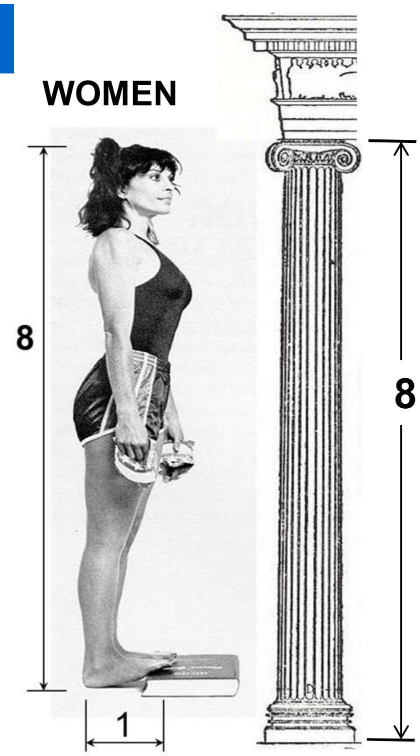
GREEK TEMPLES





L/d ≤ 10 in ACI Code until 1956

8 + base = 10

2005 DATA
Men: 6.7
Women: 7.0

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ANCIENT - 1650

STRUCTURAL MECHANICS, MATERIALS, MATH

- Concept of equilibrium was understood but stress and strain were unknown
- Stone, masonry and wood columns
- **Trigonometry** fully developed by the 10th century. **Differential calculus** developed in 1629

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1650–1800: MECHANICS, MATERIALS, MATH

- Hooke's Law (1660) – $F \propto \Delta L$
  John, James Bernoulli (1691) - $d\theta/ds = 1/r \propto M$
- Mainly stone, masonry and wood columns;
  some cast iron
- Major math advances
- Straight wooden slide rule

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1650–1800: COLUMN DESIGN

- The earliest cast iron columns just followed the proportions of stone columns
- Two Euler solutions

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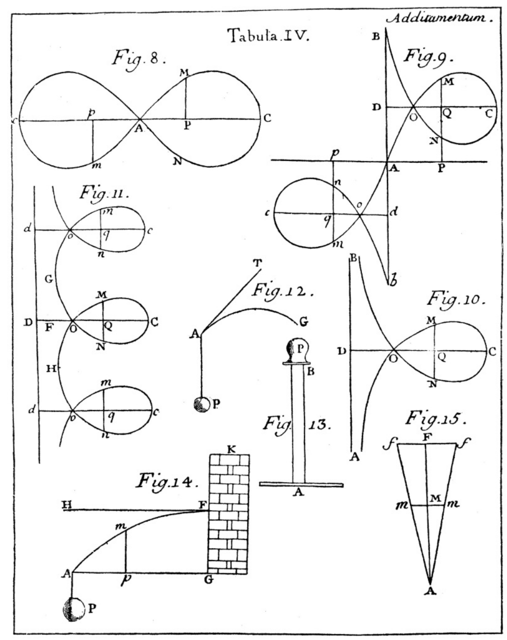
EULER (1744). *Elastic Curves*

Large Deflection Theory

$$\frac{d\theta}{ds} = -\frac{Py}{EI}$$

unknown geometric and elastic material term



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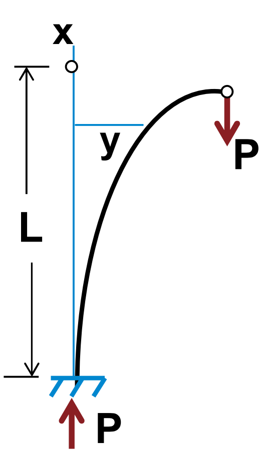
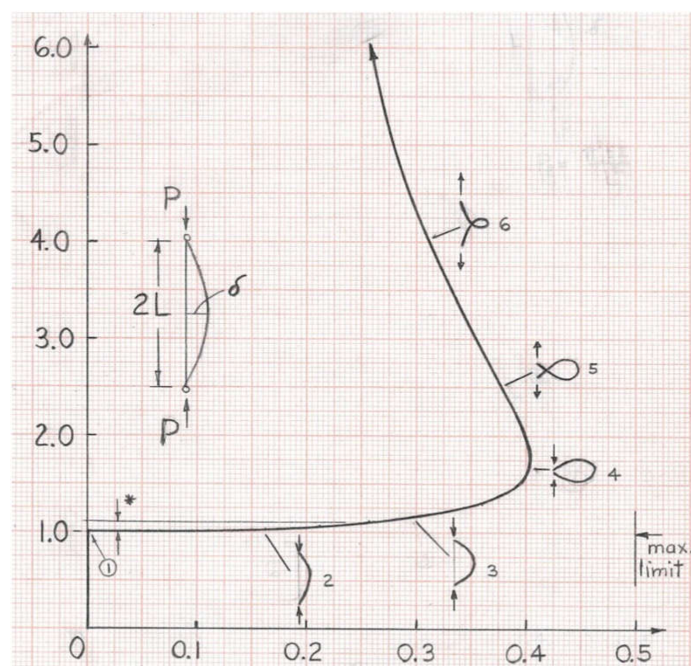
EULER (1744)

$$P_{cr} = \frac{\pi^2 \boldsymbol{EI}}{(2L)^2}$$



$\frac{P}{P_{cr}}$



δ/2L

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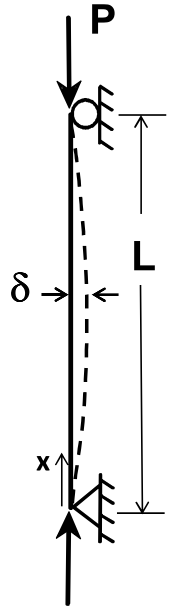
EULER (1757). *On the Strength of Columns*



Small deflection theory:

$$y'' = -\frac{Py}{\boldsymbol{EI}} \xrightarrow{\text{yields}} \boxed{P_E = \frac{\pi^2 \boldsymbol{EI}}{(L)^2}}$$

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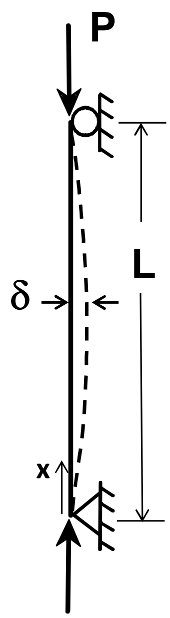
EULER (1757). *On the Strength of Columns*

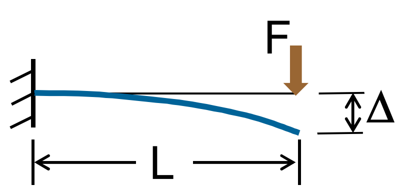


Small deflection theory:

$$y'' = -\frac{Py}{\boldsymbol{EI}} \xrightarrow{\text{yields}} \boxed{P_E = \frac{\pi^2 \boldsymbol{EI}}{(L)^2}}$$

Determine ***EI*** experimentally:

Measure F, Δ, L -



Solve for ***EI*** from

$$\Delta = \frac{FL^3}{3\boldsymbol{EI}}$$

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8. Future

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1800–1880: MECHANICS, MATERIALS, PRACTICE

- **Concepts of stress, strain, E and I established, 1820**
- **Stresses and deflections of simple beams could be calculated**
- **Wrought iron compression members until 1880**
- **Method of joints for truss analysis (Whipple, 1847)**

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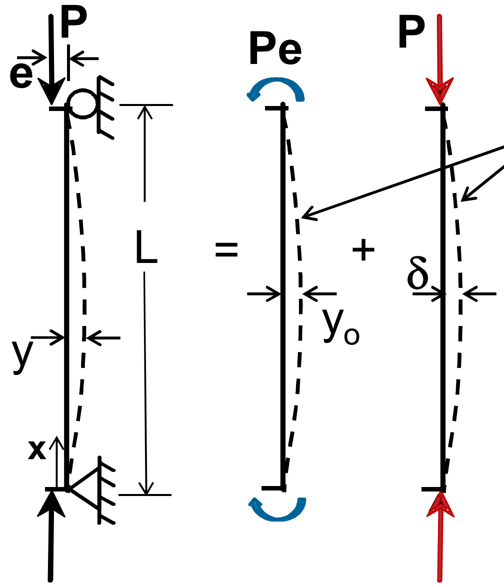
TREDGOLD (1822): FIRST COLUMN DESIGN FORMULA



1st YIELD LIMIT $\boldsymbol{f_a + f_b = F_y}$

Assume same deflected shape (error)

$$M_{max} = P(e + \delta)$$

Rectangular section

$$\boxed{\frac{P}{P_y} = \frac{1}{1 + \frac{6e}{d} + \frac{3}{2}\frac{F_y}{E}\left(\frac{L}{d}\right)^2}}$$

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1800–1880: TEST MACHINES, COLUMN TESTS

- **Test Machines and strain instrumentation came available to provide reliable material properties**
- **Hodgkinson (1840) - 250 column tests**
  Cast iron, wrought iron and wood
  Flat and rounded ends, poorly defined
  Euler not verified : $P_{max} \propto 1/L^{1.7}$
- **One million lb test machine installed in US in 1879**

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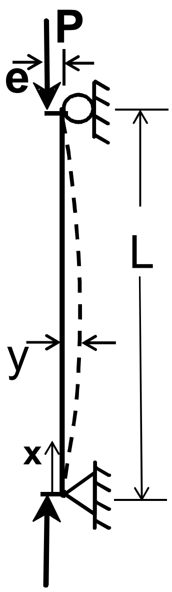
SCHEFFLER (1858): EXACT 2^{ND} ORDER ELASTIC ANALYSIS



Secant Formula

1^{st} YIELD LIMIT $\boldsymbol{f_a + f_b = F_y}$

$$\frac{P}{P_y} = \frac{1}{1 + \frac{ed}{2r^2}\sec\left[\left(\frac{L}{2r}\right)\sqrt{\left(\frac{P}{P_y}\frac{F_y}{E}\right)}\right]}$$

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GORDON-RANKINE COLUMN FORMULA (1845, 1858)

Gordon (1845)
(altered Tredgold)

$$\frac{P}{P_y} = \frac{1}{1 + \cancel{\frac{6e}{d}} + \frac{3}{2}\frac{F_y}{E}\left(\frac{L}{d}\right)^2}$$

fit to Hodgkinson results

a

r

Rankine (1845)

Gordon- Rankine

$$\boxed{\frac{P}{P_y} = \frac{1}{1 + a\left(\frac{L}{r}\right)^2}}$$

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GORDON-RANKINE FORMULA (1845, 1858)

Gordon- Rankine $$\frac{P}{P_y} = \frac{1}{1 + a\left(\frac{L}{r}\right)^2}$$

Wrought Iron Columns: use a FS = 4 for working stress
a= 1/3000 (fixed ends), =4/3000 (pinned ends)

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GORDON-RANKINE- PERRY FORMULAS

Gordon- Rankine $$\frac{P}{P_y} = \frac{1}{1 + a\left(\frac{L}{r}\right)^2}$$

3/2 (Tredgold)

Perry (1889): if $a = {F_y}\big/{(\pi^2 E K^2)}$

$$\frac{P}{P_y} = \frac{1}{1 + \frac{P_y}{P_{cr}}}$$ or $$\frac{1}{P} = \frac{1}{P_y} + \frac{1}{P_{cr}}$$

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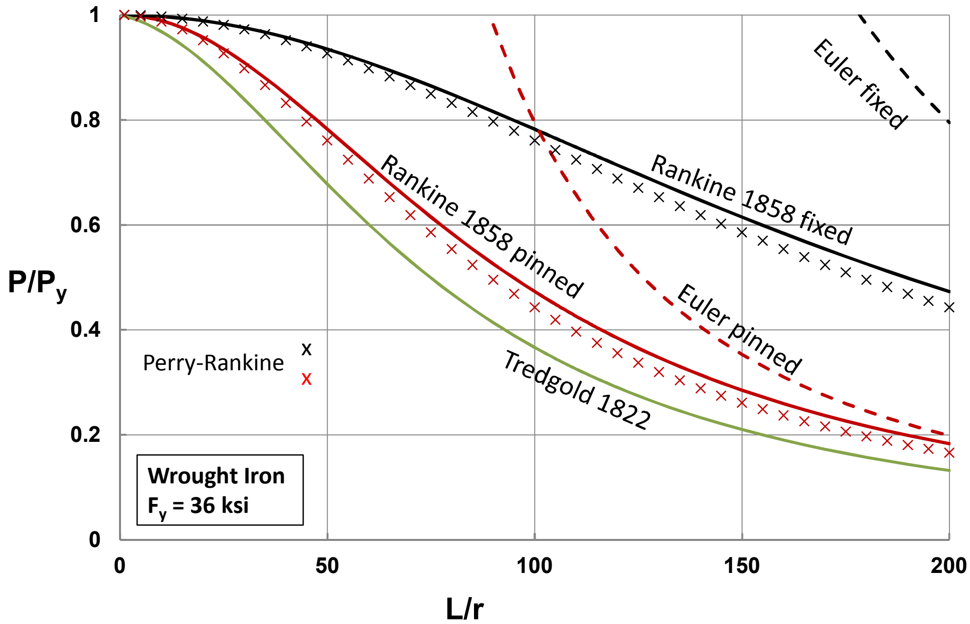
RANKINE COLUMN CURVES



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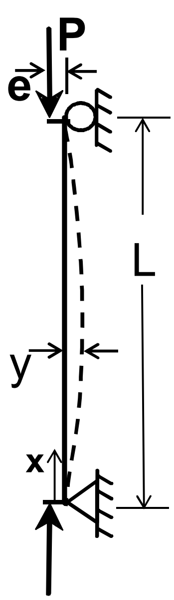
SCHEFFLER (1858): SECANT FORMULA



1st YIELD LIMIT $f_a + f_b = F_y$

$$M_{max} = P(e + y_{max}) = (Pe)\, sec\, \frac{\pi}{2}\sqrt{\frac{P}{P_E}}$$

$$\frac{P}{P_y} = \frac{1}{1 + \frac{ec}{r^2}\sec\left[\left(\frac{L}{2r}\right)\sqrt{\left(\frac{P}{P_y}\frac{F_y}{E}\right)}\right]}$$

Not a practical column formula
no EXCEL Solver available!!

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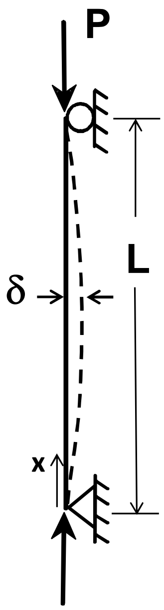
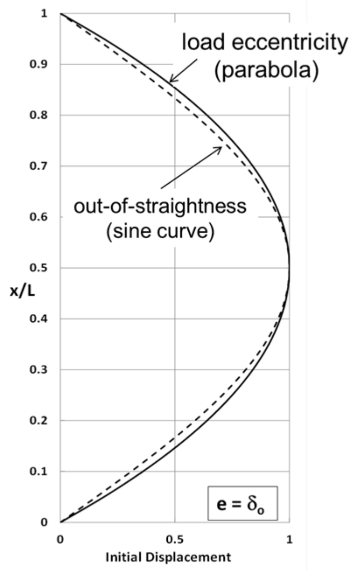
AYRTON-PERRY (1886) EXACT 2ND ORDER ANALYSIS



Initial Out-of-Straightness, δ_o, (sine curve)



$$P\delta = \frac{P\delta_o}{1 - \frac{P}{P_e}}$$

$$f_a + f_b = F_y$$

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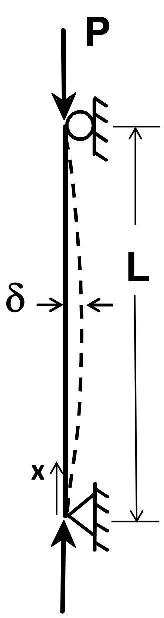
AYRTON-PERRY (1886) COLUMN FORMULA



Initial Out-of-Straightness, δ_o, (sine curve)

$$f_a + f_b = F_y$$

$$\frac{P}{P_y} = \frac{1 + (1+m)\frac{P_e}{P_y}}{2} - \sqrt{\left[\frac{1 + (1+m)\frac{P_e}{P_y}}{2}\right]^2 - \frac{P_e}{P_y}}$$

where $m = \dfrac{\delta_o c}{r^2}$

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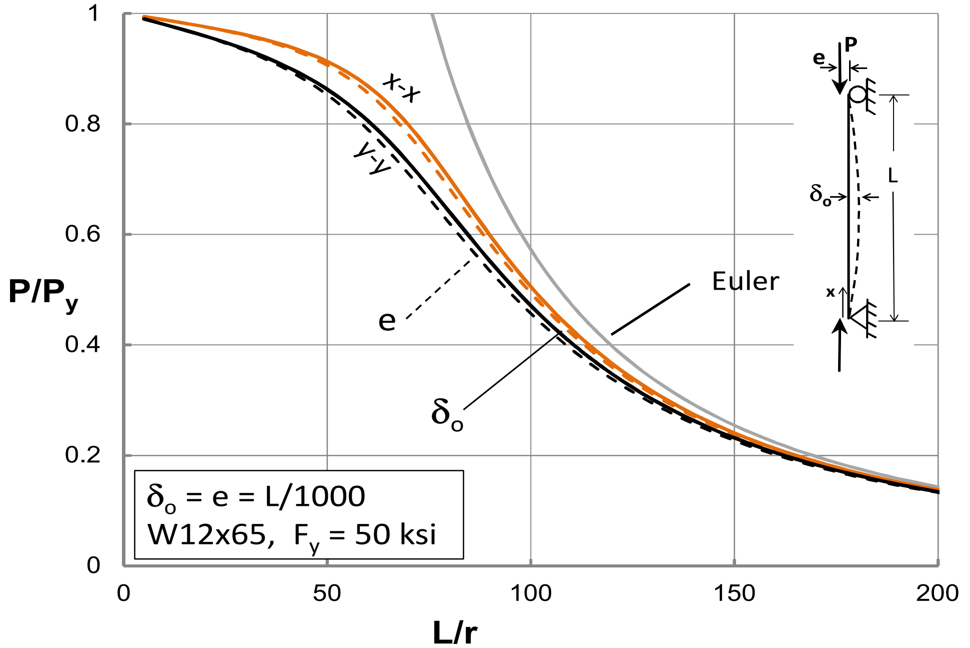
SECANT AND AYRTON-PERRY 1ST YIELD SOLUTIONS



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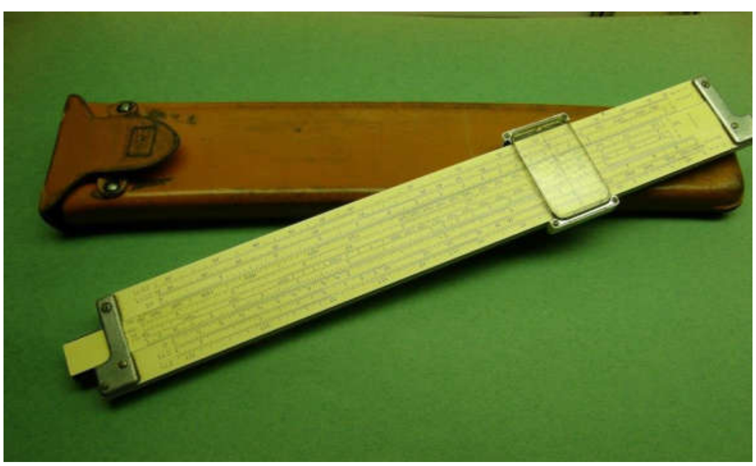
SLIDE RULE

Until 1970



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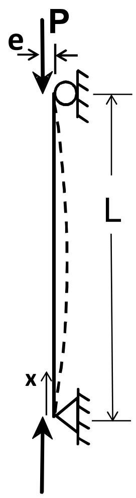
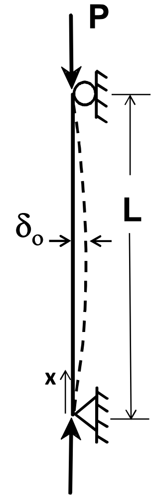
SECANT AND AYRTON-PERRY 1ST YIELD SOLUTIONS



SECANT

- **Both require an input of an imperfection**
- **Both were considered too complicated**
- **Perry could be used to solve the end eccentricity problem by using δ_o = (1.2)e**



PERRY

USE RANKINE

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1880–1900: MECHANICS, MATERIALS, PRACTICE

- **Euler is verified by careful elastic tests with knife - type end fixtures**
- **Engesser extends Euler theory to inelastic range with two inelastic theories**
- **Steel starts to replace wrought iron columns**
- **Railroad bridges failed at the rate of 1/week over a ten-year period**
- **Engineers developed their own design specs and column formulas.**

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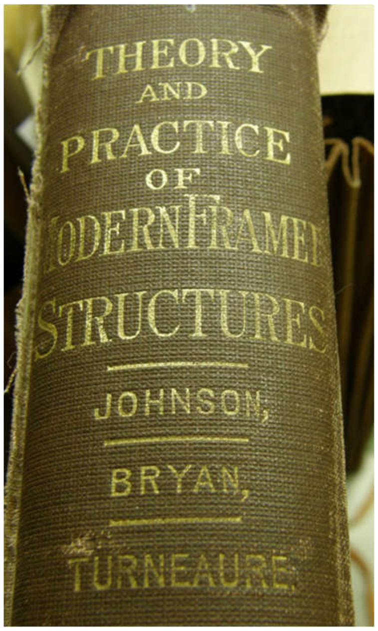
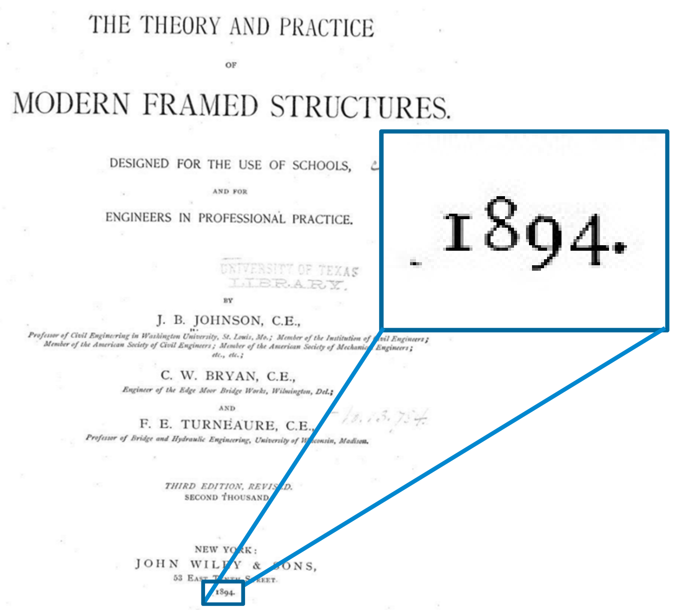
FIRST STEEL DESIGN TEXT





THE THEORY AND PRACTICE

OF

MODERN FRAMED STRUCTURES.

DESIGNED FOR THE USE OF SCHOOLS,

AND FOR

ENGINEERS IN PROFESSIONAL PRACTICE.

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BY

J. B. JOHNSON, C.E.,

Professor of Civil Engineering in Washington University, St. Louis, Mo.; Member of the Institution of Civil Engineers; Member of the American Society of Civil Engineers; Member of the American Society of Mechanical Engineers; etc., etc.;

C. W. BRYAN, C.E.,

Engineer of the Edge Moor Bridge Works, Wilmington, Del.;

AND

F. E. TURNEAURE, C.E.,

Professor of Bridge and Hydraulic Engineering, University of Wisconsin, Madison.

THIRD EDITION, REVISED.

SECOND THOUSAND.

NEW YORK:

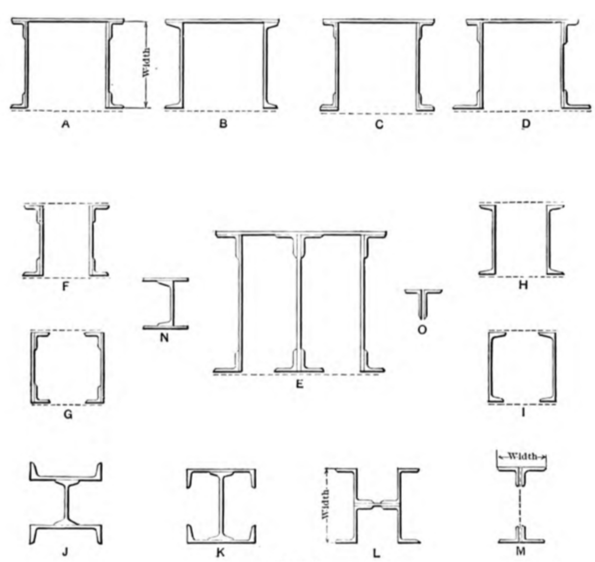
JOHN WILEY & SONS,

53 EAST TENTH STREET.

1894.

1800-1900: TYPICAL TRUSS BRIDGE MEMBERS

Johnson et al (1894)



JOHNSON PARABOLA (1894)

CURVE FIT TO TESTS $\frac{P}{P_y} = 1 - \frac{F_y}{4\pi^2 E}\left(\frac{KL}{r}\right)^2$ for $\frac{P}{P_y} \geq 0.5$;

EULER THEORY $\frac{P}{P_y} = \frac{\pi^2 E}{F_y\left(\frac{KL}{r}\right)^2}$ for $\frac{P}{P_y} < 0.5$

This formulation was adopted in the AISC Spec (1961-1985)

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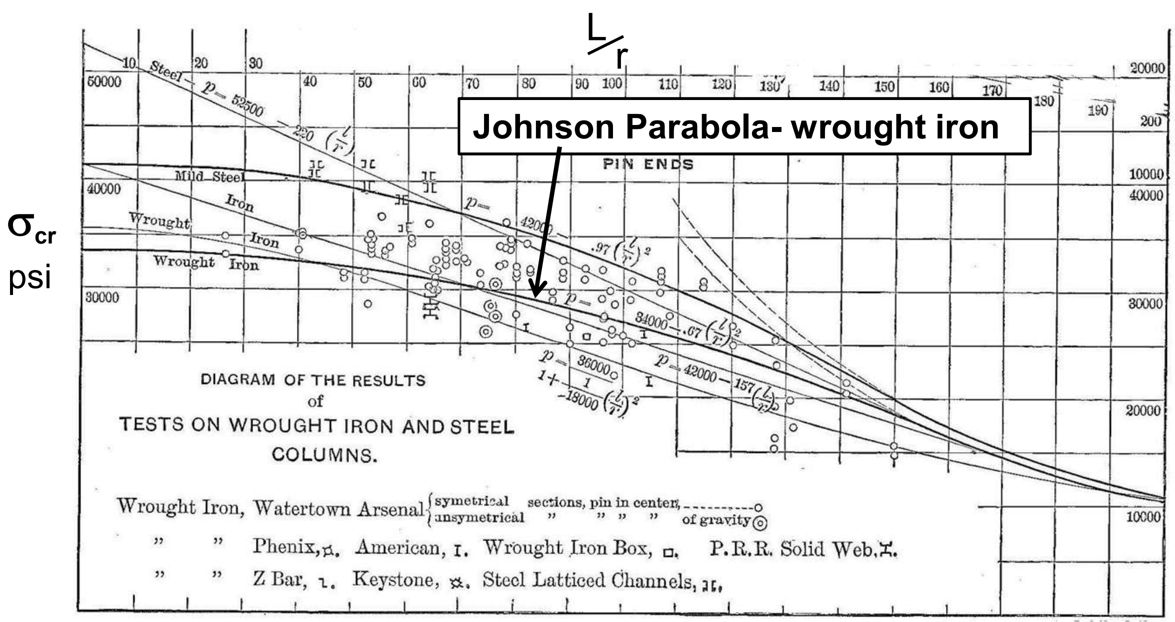
WROUGHT IRON TESTS (1894)



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1800–1900: ENGINEERING EDUCATION

- **First engineering school (Paris) based on science and math, 1795 – our current model**
- **British and US – Classical + "shop" courses**
- **Congress, 1862, established land-grant universities "to promote liberal and practical education"**
- **Engineering schools grew from 4 to 100 by 1900 using the British model of practical courses**

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Assessment Question

[The first **metal** column design formula was a fit to test results. True or False]

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1. ANCIENT – 1650 (Greek temples,)
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8. Future

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1900-1944: STRUCTURAL MECHANICS, MATERIALS

No mention of the word *buckling* in the previous centuries

Google – When did the word *buckling* originate in English

Oxford Dictionary:

Buckling - **a smoked herring**

American Century Dictionary **1909**

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1900-1944: STRUCTURAL MECHANICS, MATERIALS

- BENDING: Slope Deflection, Area-Moment, Moment Distribution
- AXIAL EFFECTS: Stability functions, beam-columns
- Sway frames were a challenge
- Steel (buildings and bridges)-- the W shape arrives

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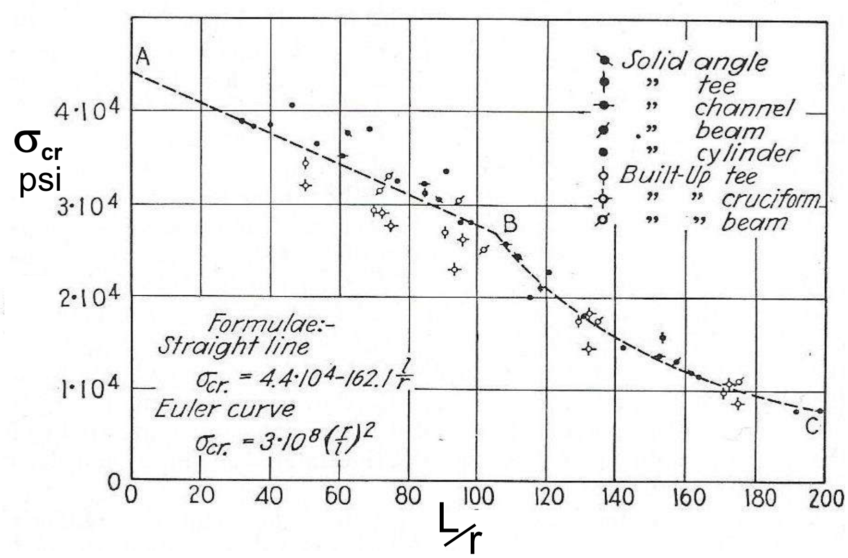
COLUMN DESIGN: TETMAJER STEEL TESTS (1903)

Straight Line Column Formula



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1900-1944: COLUMN DESIGN

BUILDINGS AND BRIDGES

L/r < 120 - elastic Euler formula not applicable

Steel elastic to yield stress

Test scatter due to accidental eccentricities

No perfectly straight members with knife edges

Used eccentricity approach, not buckling

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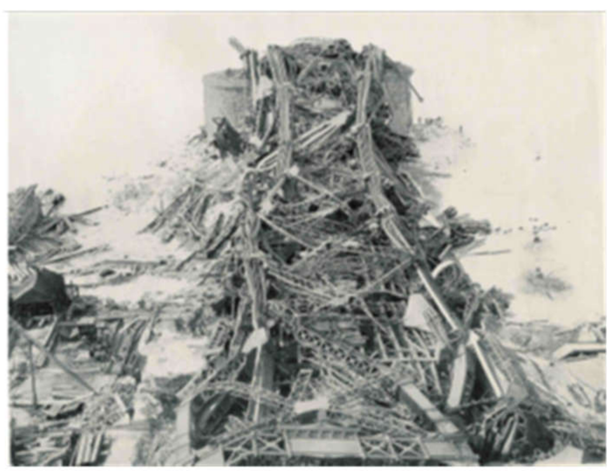
QUEBEC BRIDGE COLLAPSE (1907)



Major Effect: The beginning of consensus steel design specifications

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ASCE COLUMN COMMITTEES 1909-1933

PURPOSE - Determine the strength and safe working values of steel columns

- Planned and executed a large test program
- Two large test machines -10 and 2.3 million lbs
- 320 tests conducted, mostly flat end, some beam-columns
- No material property tests were conducted

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ASCE COLUMN COMMITTEES 1909-1933

PURPOSE - Determine the strength and safe working values of steel columns

CONCLUSION - <u>Secant formula</u> best adapted to various orientations of end eccentricities compared to Ayrton-Perry

A parabolic formula is a close fit for bridges with a tensile working stress = 18,000 psi for L/r < 140

$$\boxed{F_a = 15000 - 0.25\left(\frac{L}{r}\right)^2} \qquad L/r < 140$$

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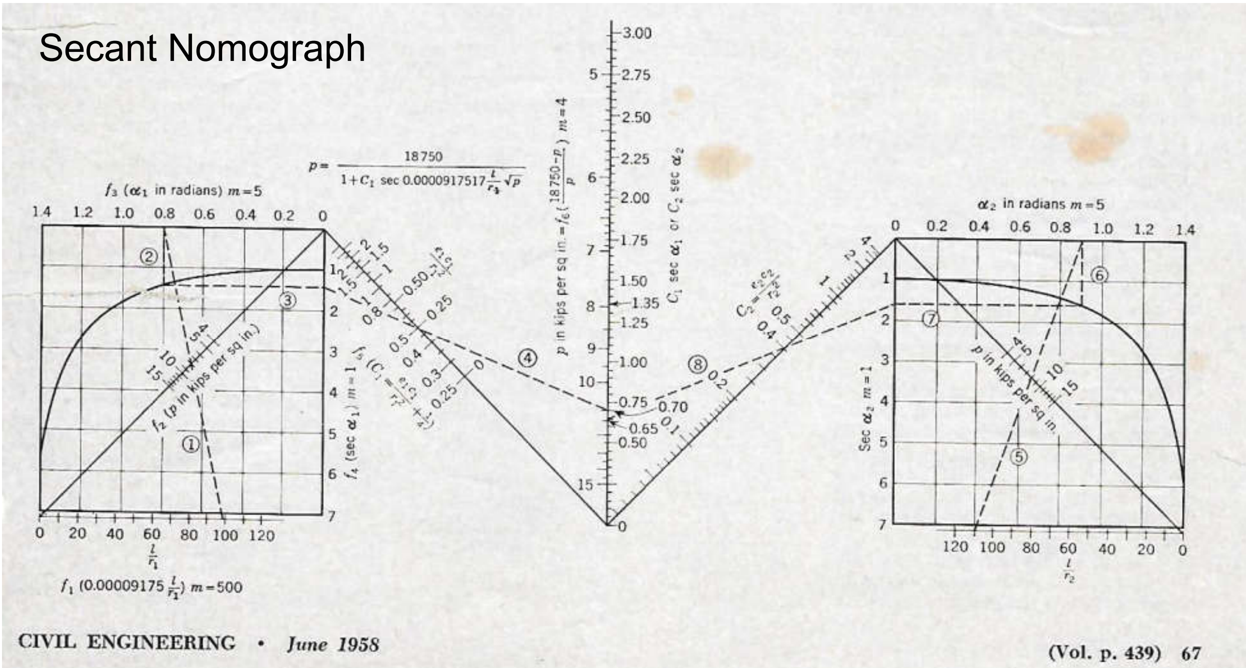
Secant Nomograph



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AISC, est. 1921

1st Spec 1923

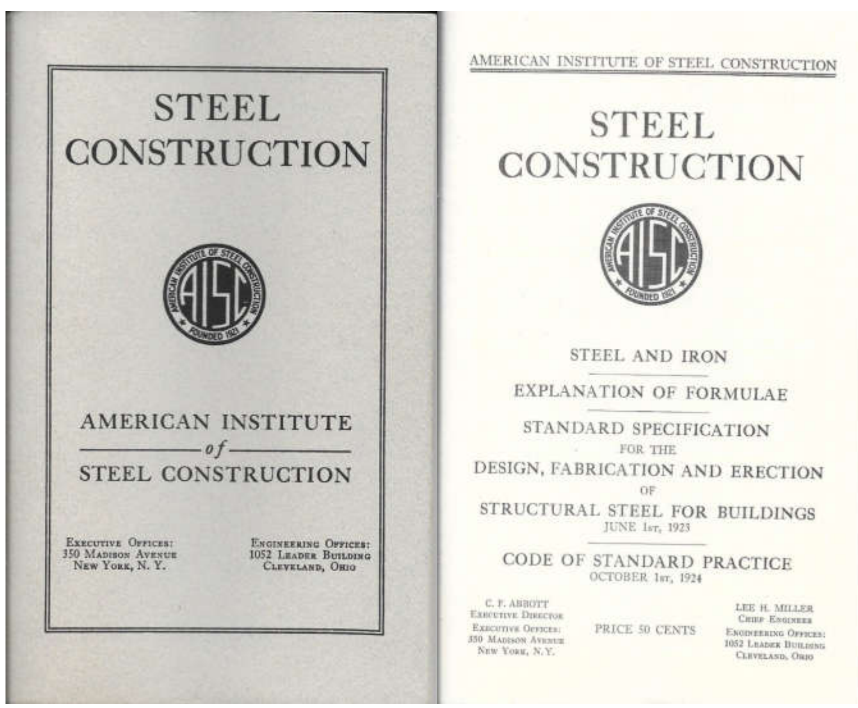
1st Code 1924

1st Manual 1925

78 pages

Commentary

50 cents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STEEL AND IRON

EXPLANATION OF FORMULAE

STANDARD SPECIFICATION FOR THE DESIGN, FABRICATION AND ERECTION OF STRUCTURAL STEEL FOR BUILDINGS JUNE 1st, 1923

CODE OF STANDARD PRACTICE OCTOBER 1st, 1924

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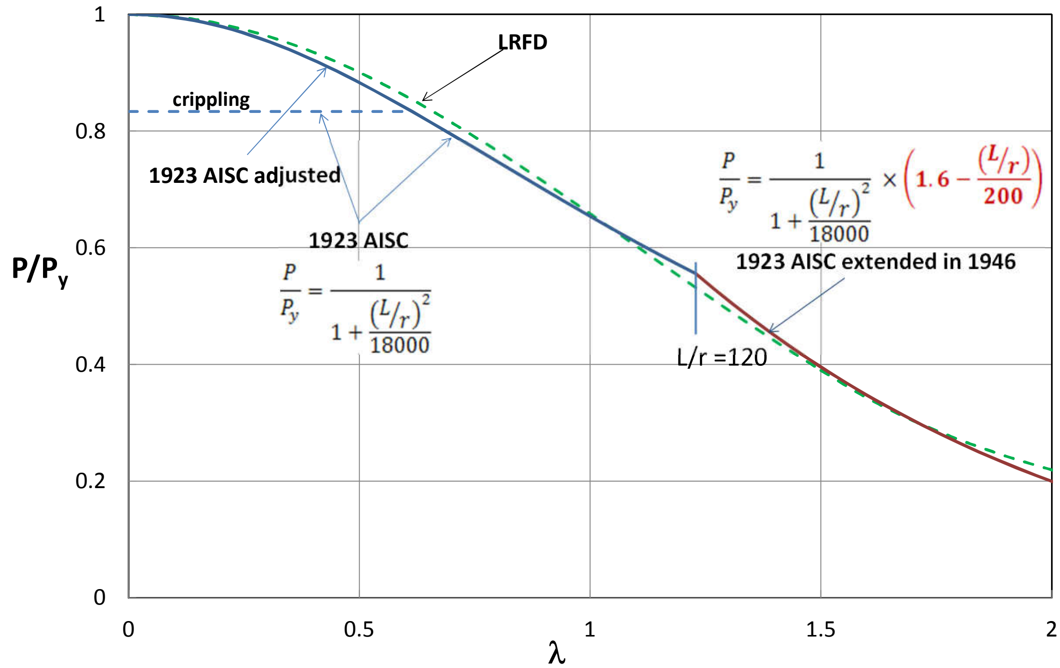
AISC SPECS: 1923 -1936



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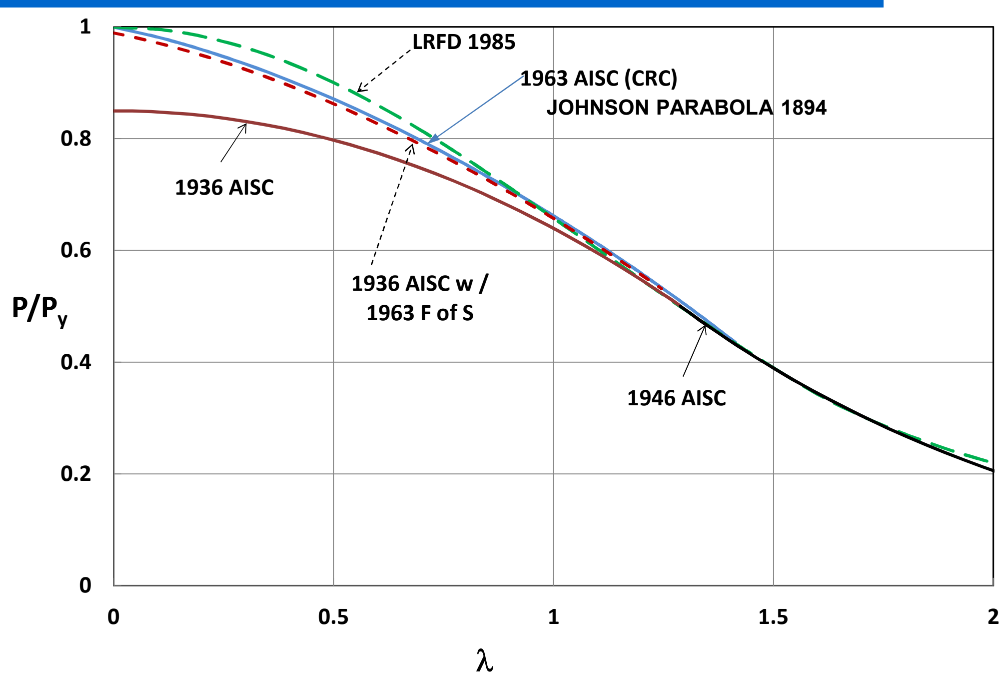
AISC PARABOLIC FORMULAS: 1936 - 1985



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1936 AISC SPEC

Combined Stresses

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$ (Not Derivable)

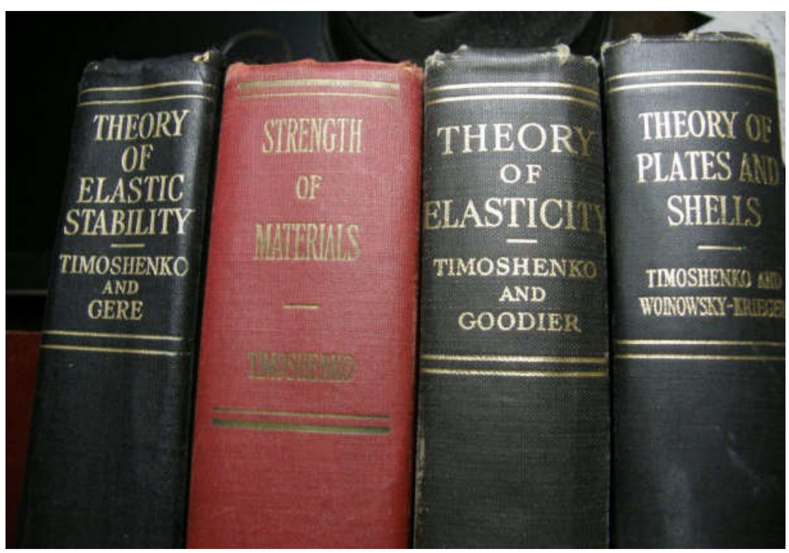
Formerly

$$f_a + f_b \leq F_a$$

57

EDUCATION: S. TIMOSHENKO in US 1922-1972

S. Timoshenko



Universities Morphed to the Math-Science Model

58

OUTLINE: (KEY WORDS)

1. ANCIENT – 1650 (Greek temples,)
2. 1650 -1800 (Euler)
3. 1800 -1900 (eccentrically-loaded column)
4. 1900 -1945 (AISC)
5. 1945 - 1970 (K-factor, residual stress, SSRC)
6. 1970 -1985 (frames, multiple column curves, LRFD)
7. 1985 - 2018 (K = 1.0, 2nd order modified analysis)
8. Future

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1944: STRUCTURAL STABILITY RESEARCH COUNCIL

- **SSRC** Originally called Column Research Council (CRC)
- **Goal**
  Develop Practical Design Procedures Consistent with Accurate Predictions of Structural Strength
- **First Actions**
  Support Bleich's book, *Buckling Strength of Metal Structures*
  Focus on the column as part of a real structure

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SSRC

Tangent modulus buckling theory confirmed by Shanley

Whole cross-section compressive tests (stub columns) show a proportional limit due to residual stresses

Residual stress effects can explain column test scatter

Bleich (chapter 1) recommends buckling theory for basis of design and a parabolic design curve

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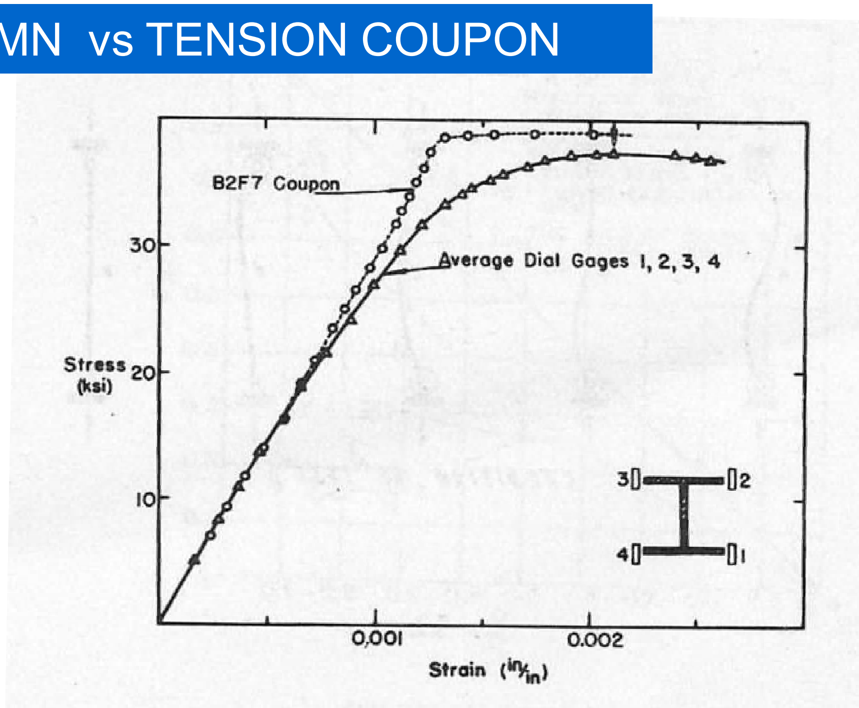
STUB COLUMN vs TENSION COUPON



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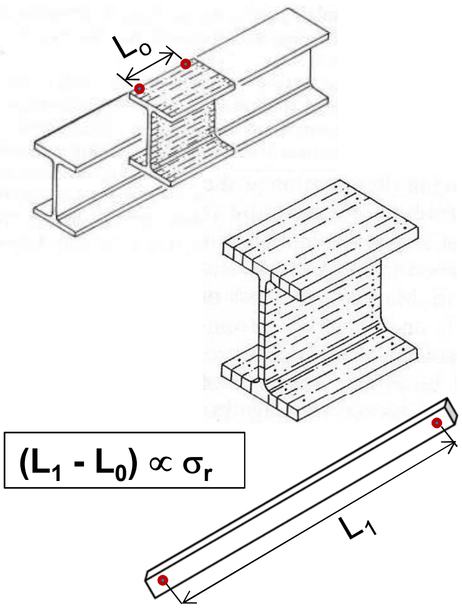
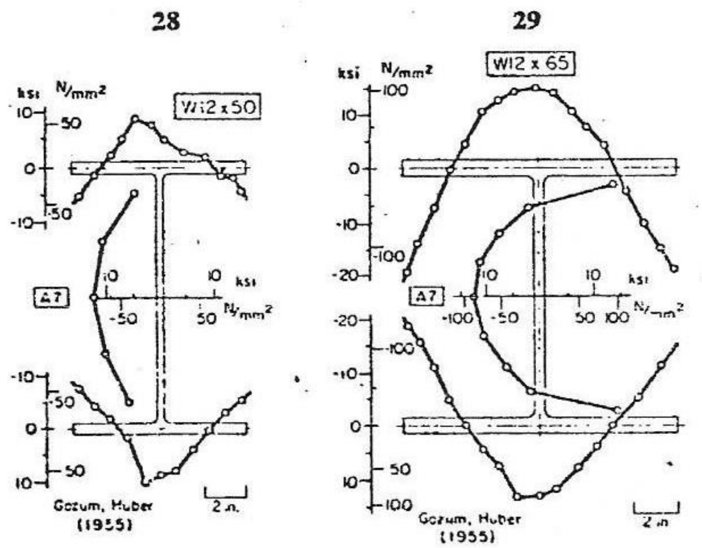
1950-1970:RESIDUAL STRESSES MEASUREMENTS

Tebedge, Tall 1974





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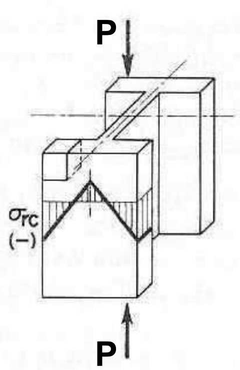
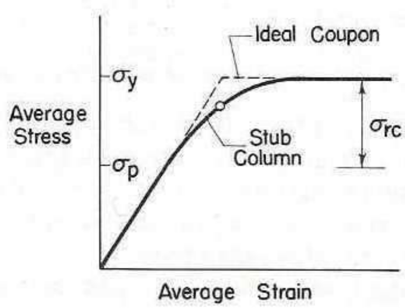
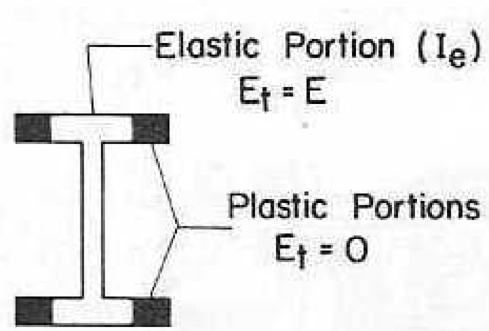
RESIDUAL STRESS EFFECT



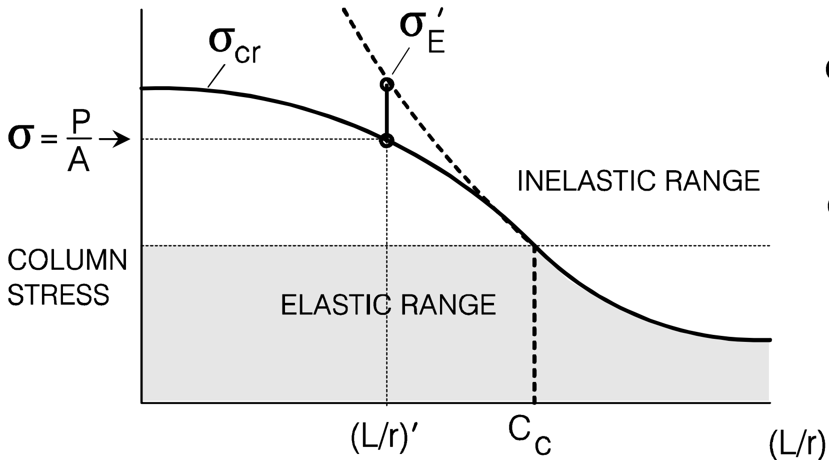




Loss of stiffness

64

STIFFNESS MODIFICATION FACTOR, τ



$$\sigma_{cr} = \frac{\pi^2 E_T}{(L/r)^2}$$

$$\sigma_E = \frac{\pi^2 E}{(L/r)^2}$$

$$\tau = \frac{E_T}{E} = \frac{(P/A)}{\sigma_E @ (L/r)'} = \frac{\sigma}{\sigma'_E}$$

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Table 4–21
Stiffness Reduction Factor τ_a

2005 AISC MANUAL

29 ksi

0.771

ASD P_a/A_g / LRFD P_u/A_g	F_y = 35 ksi ASD	35 LRFD	36 ASD	36 LRFD	42 ASD	42 LRFD	46 ASD	46 LRFD	50 ASD	50 LRFD
45	–	–	–	–	–	–	–	–	–	–
44	–	–	–	–	–	–	–	–	–	0.0599
43	–	–	–	–	–	–	–	–	–	0.118
42	–	–	–	–	–	–	–	–	–	0.175
41	–	–	–	–	–	–	–	0.0262	–	0.231
40	–	–	–	–	–	–	–	0.0905	–	0.285
39	–	–	–	–	–	–	–	0.153	–	0.338
38	–	–	–	–	–	–	–	0.214	–	0.389
37	–	–	–	–	–	0.0570	–	0.274	–	0.438
36	–	–	–	–	–	0.127	–	0.331	–	0.486
35	–	–	–	–	–	0.194	–	0.387	–	0.532
34	–	–	–	–	–	0.260	–	0.441	–	0.577
33	–	–	–	–	–	0.323	–	0.492	–	0.620
32	–	–	–	0.0334	–	0.384	–	0.542	–	0.660
31	–	0.0429	–	0.115	–	0.443	–	0.590	–	0.699
30	–	0.127	–	0.194	–	0.500	–	0.636	–	0.736
29	–	0.207	–	0.270	–	0.554	–	0.679	0.0842	0.771
28	–	0.285	–	0.344	–	0.606	–	0.720	0.171	0.804
27	–	0.360	–	0.414	–	0.655	0.0534	0.759	0.254	0.835
26	–	0.431	–	0.481	–	0.701	0.148	0.796	0.334	0.863

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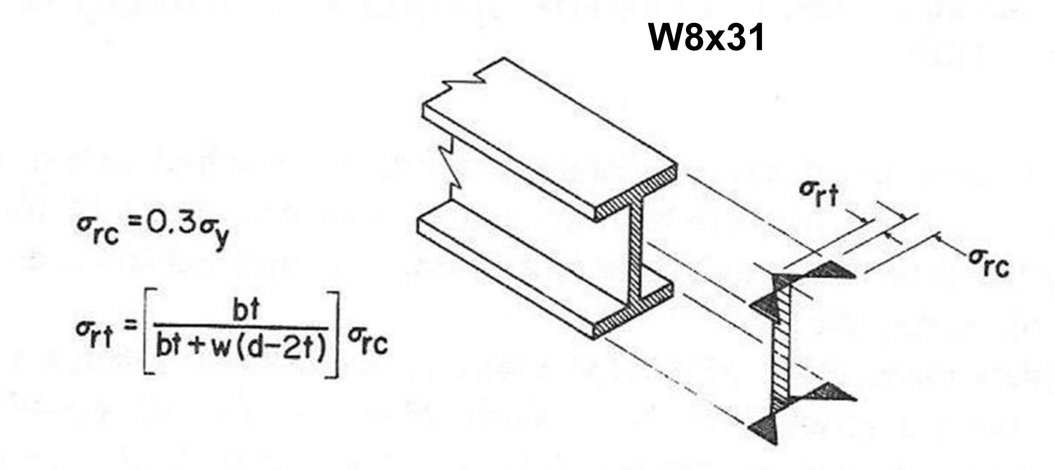
KETTER ASSUMED RESIDUAL STRESSES: 1952 - Present

W8x31



Used in all beam-column and sway frame 2nd order inelastic studies for AISC Spec

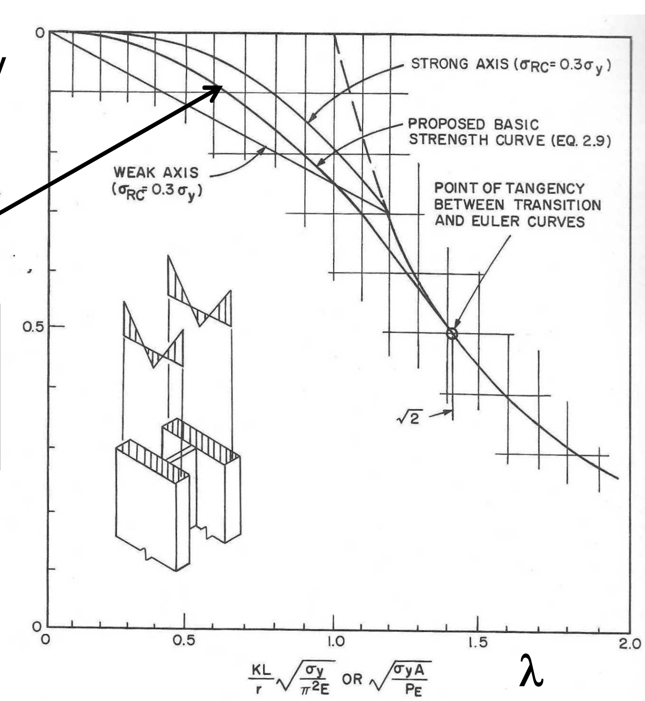
67

SSRC

$\frac{\mathbf{P}}{\mathbf{P_y}}$

For a cross section two column curves because of residual stresses

Johnson Parabola (CRC) single curve compromise for design



λ

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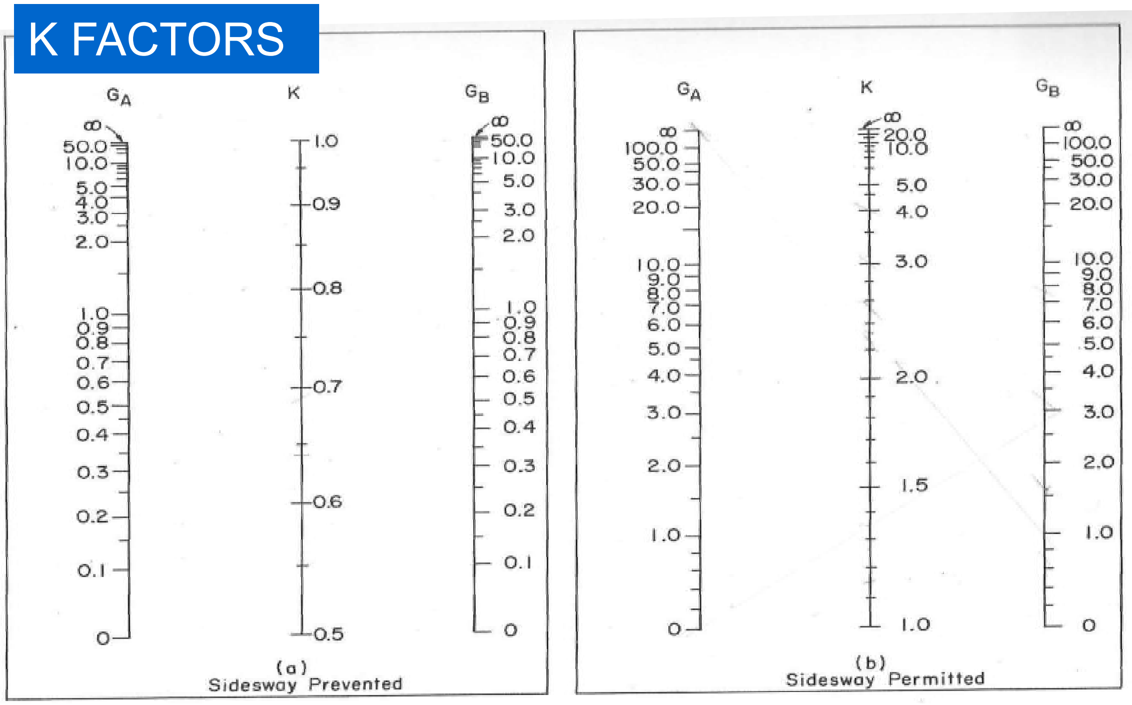
K FACTORS



$$G = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}}$$

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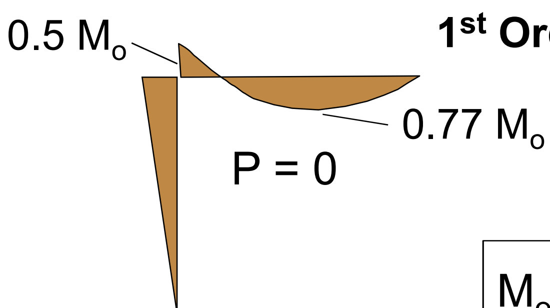
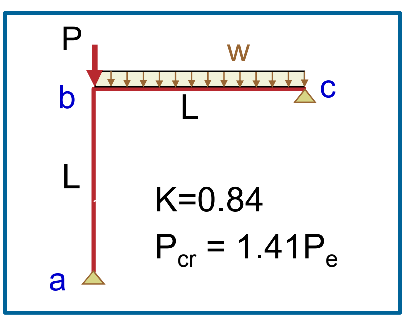
EFFECT OF AXIAL LOAD ON FRAME MOMENTS

1^{st} Order Elastic Structural Analysis



$M_o \equiv wL^2/8$



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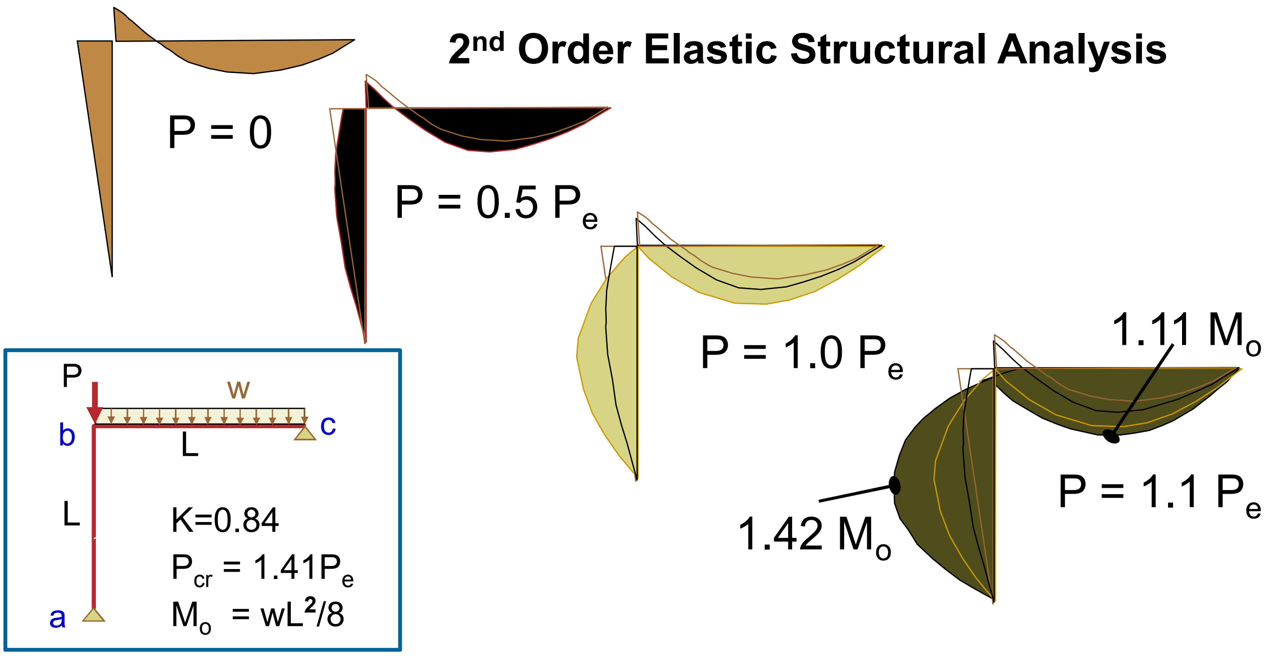
EFFECT OF AXIAL LOAD ON FRAME MOMENTS



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1963 AISC INTERACTION EQUATION

$$\frac{P}{P_{K=1.0}} + \frac{M}{M_y} = 1.0$$ Previous

$$\boldsymbol{\frac{P}{P_{crK}} + \frac{C_m M}{\left(1 - \frac{P}{P_{eK}}\right) 1.1 M_y} = 1.0}$$

Not derivable - a fit to many single member loading cases with exact 2nd order elastic analysis

72

1963 AISC INTERACTION EQUATION

Amplification factor
≈ 2^{nd} order elastic analysis

$$\frac{P}{P_{crK}} + \frac{C_m}{\left(1 - \frac{P}{P_{eK}}\right)} \frac{M}{1.1M_y} = 1.0$$

Column Eq. with KL
Safety Factor 1.67-1.92

≈ M_p for x-x

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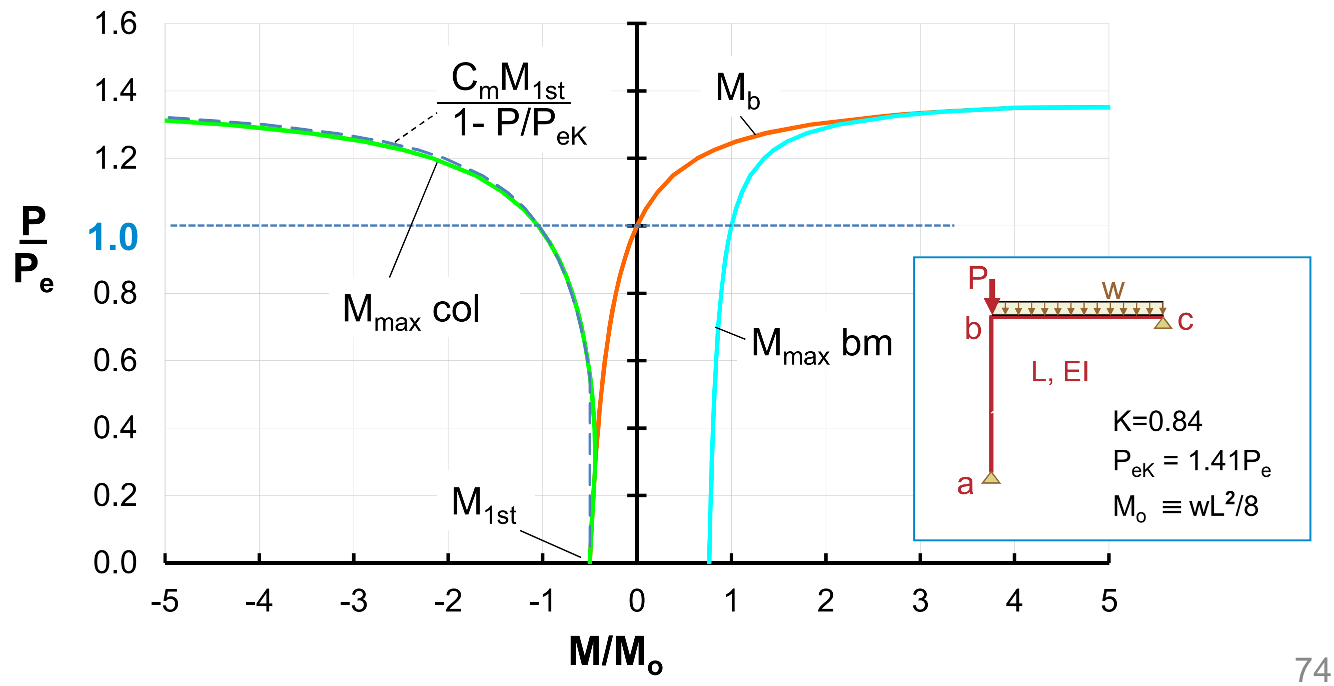
EFFECT OF COLUMN STIFFNESS ON FRAME MOMENTS



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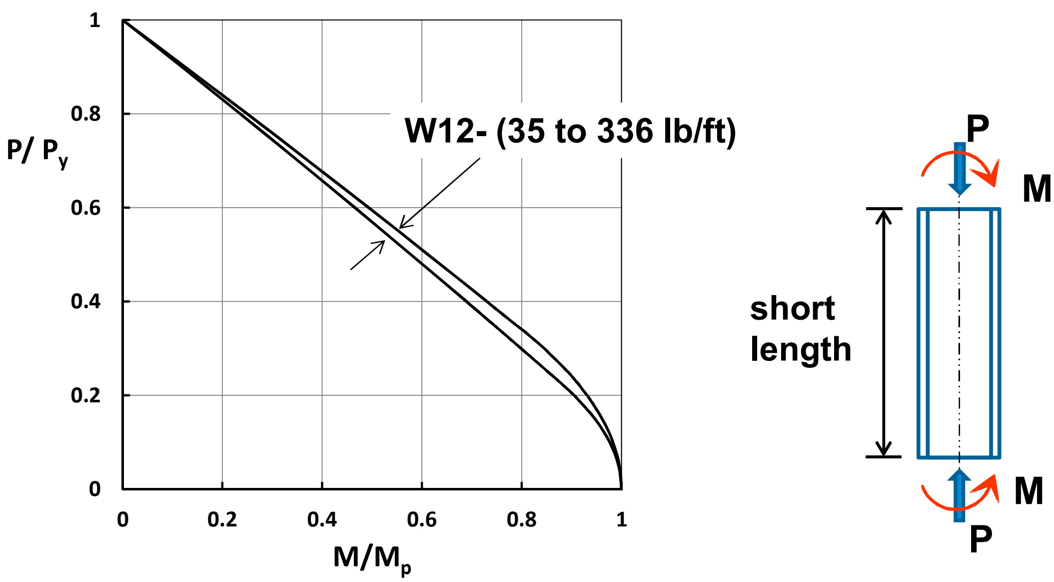
PLASTIC DESIGN - ULTIMATE STRENGTH



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OUTLINE: (KEY WORDS)

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6. 1970 -1985 (frames, multiple column curves, LRFD)
7. 1985-2018 (K =1.0, 2nd order modified analysis)
8. Future

76

FRAME STABILITY: ΣP CONCEPT

- **FOR SWAY BUCKLING OF A STORY**

$$\Sigma P_{\text{story column loads}} \leq \underbrace{\Sigma P_{cri}}_{\text{Sway buckling load of each column using alignment chart K}}$$

- **EACH COLUMN MUST SUPPORT ITS OWN AXIAL LOAD IN THE NO SWAY MODE (i.e. K = 1.0)**

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HAND CALCULATOR - 1970

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AVAILABLE TOOLS 1970-1985

Engineering Practice

- Hand calculator
- Programs for 1st order structural analysis

University Researchers

- Main frame computers
- Programs for 2nd order inelastic **ultimate strength analysis**
  1. strength of columns including the effects of out-of-straightness and residual stresses (Buckling approach abandoned for column curve)
  2. braced and unbraced frames with residual stresses (Beam-column interaction equation)

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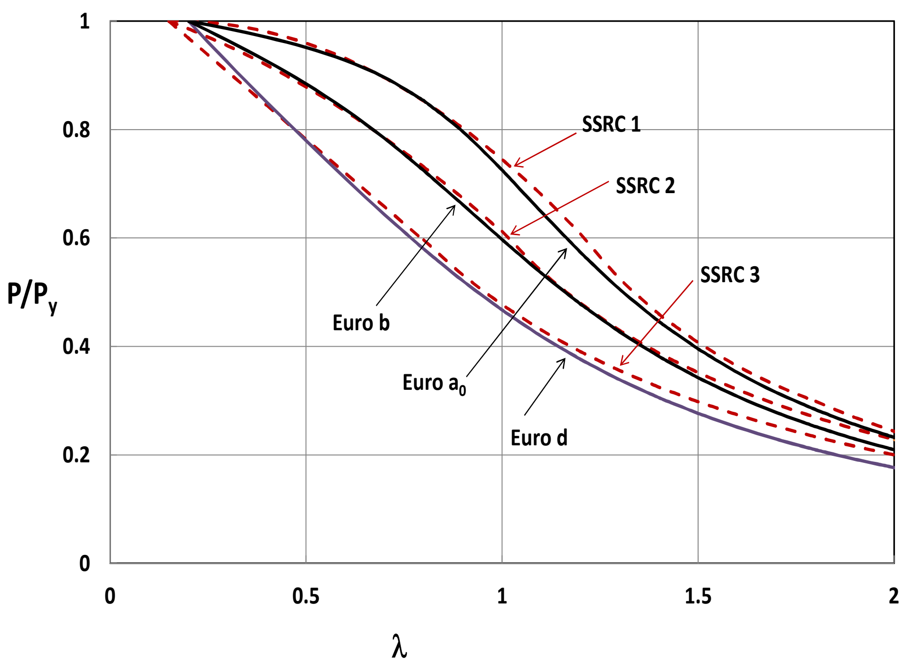
MULTIPLE COLUMN CURVES: 1970 - PRESENT



Euro curves are lower bound fits to 1067 pinned end column test results supplemented by numerical simulations.

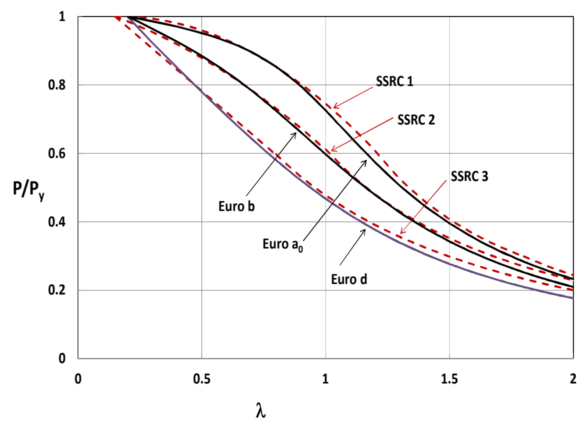
80

MULTIPLE COLUMN CURVES: 1970 - PRESENT

BS EN 1993-1-1:2005
EN 1993-1-1:2005 (E)

Table 6.2: Selection of buckling curve for a cross-section

Cross section		Limits		Buckling about axis	Buckling curve S 235 S 275 S 355 S 420	S 460
Rolled sections		$h/b > 1{,}2$	$t_f \le 40$ mm	y – y	a	a_0
				z – z	b	a_0
			40 mm $< t_f \le 100$	y – y	b	a
				z – z	c	a
		$h/b \le 1{,}2$	$t_f \le 100$ mm	y – y	b	a
				z – z	c	a
			$t_f > 100$ mm	y – y	d	c
				z – z	d	c
Welded I-sections			$t_f \le 40$ mm	y – y	b	b
				z – z	c	c
			$t_f > 40$ mm	y – y	c	c
				z – z	d	d
Hollow sections			hot finished	any	a	a_0
			cold formed	any	c	c



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MULTIPLE COLUMN CURVES: 1970 - PRESENT

Adopted by ECCS (Euro Code) in 1978

Concept not considered by AISC Specification

- **Added complexity not practically justified**
- Curves based on effect of residual stresses in columns with pinned ends
- Selection table incomplete and based on limited data
- End restraint greatly diminished the difference between x-x and y-y column response
- **Research on columns in frames (combined axial and bending interaction equations) was based only on a single column curve**

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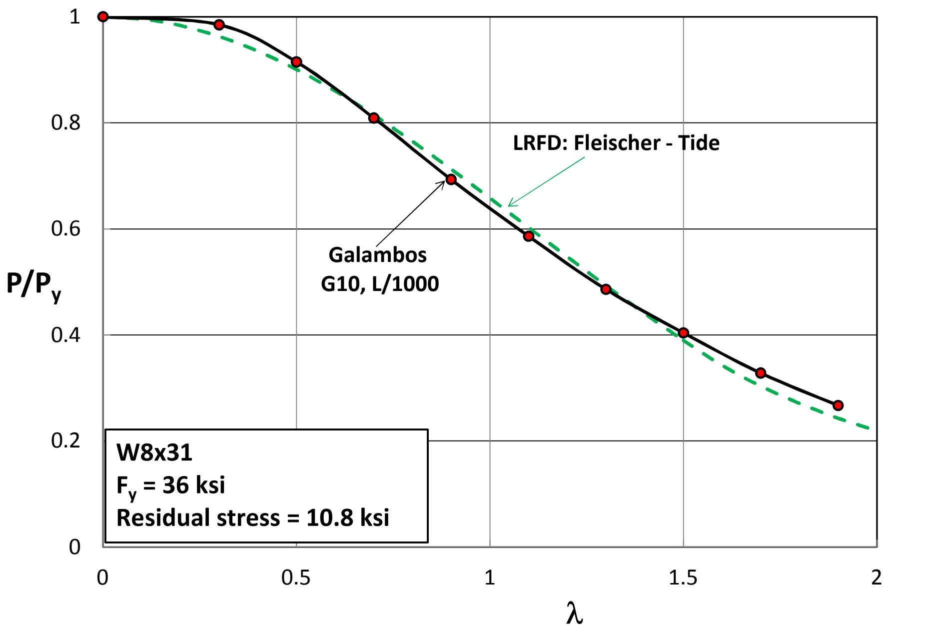
1985 LRFD COLUMN

Same as current formula



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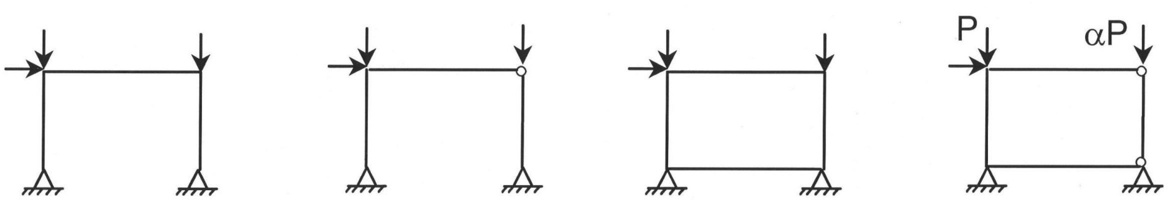
1985 LRFD INTERACTION EQUATION

- **AISC Stability Task Group spent 8 years in development**
- **Over 2000 unbraced frames were analyzed for maximum strength**
- **Columns were initially straight but contained residual stresses**
- **Three full size two-bay unbraced frames were tested**
- **Three laterally loaded biaxial restrained beam-columns were tested**



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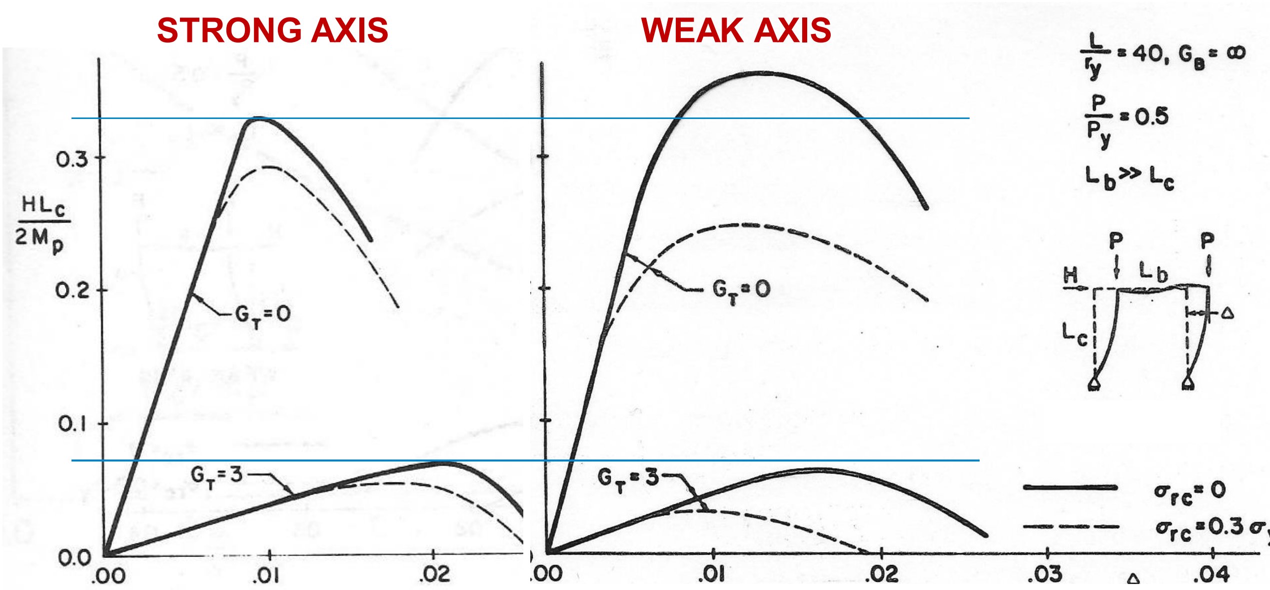
1985 LRFD INTERACTION EQUATION

Frame Results



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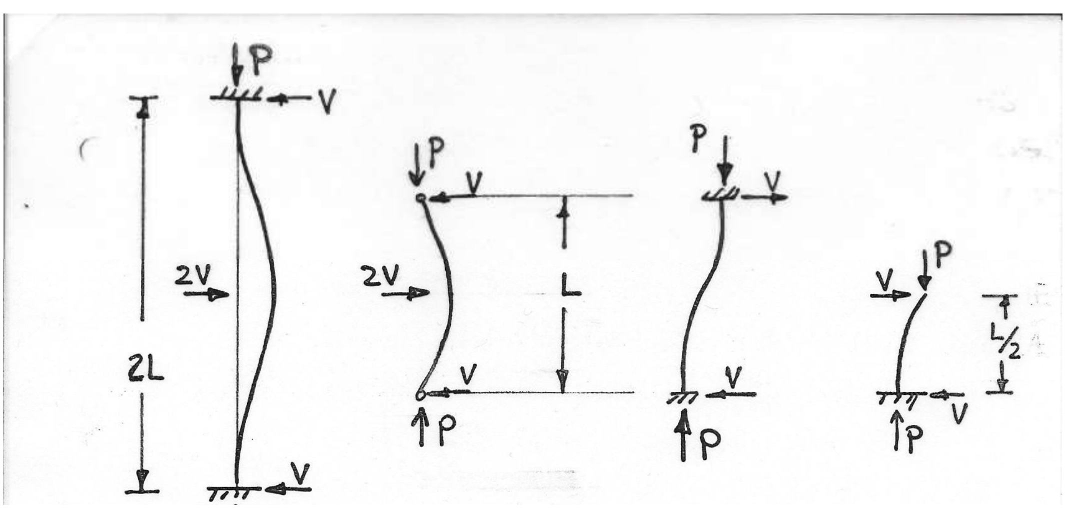
1985 LRFD INTERACTION EQUATION

Similar 2nd order Analysis Problems

Need $\frac{P}{P_y}$ or $\frac{P}{P_{crK}}$ in the first term to get similar answers, not $\frac{P}{P_{crK=1.0}}$



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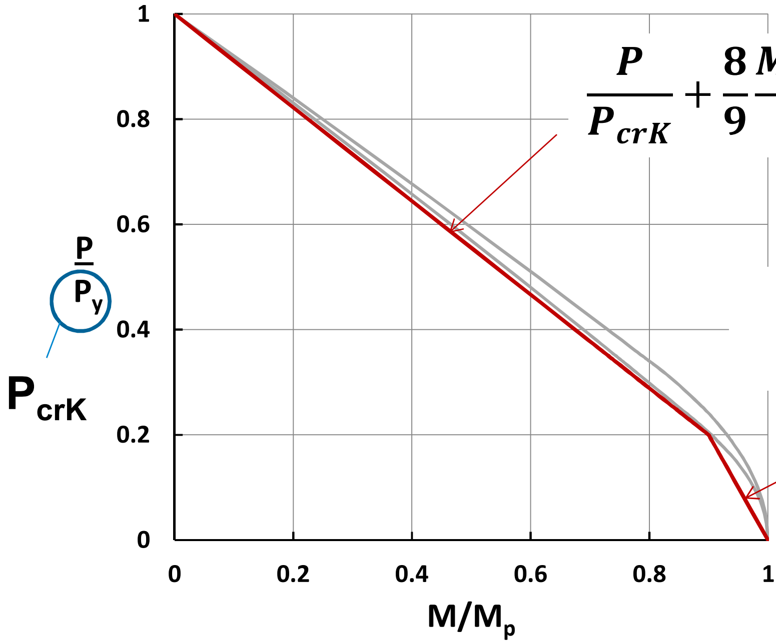
1985 LRFD INTERACTION EQUATION



$$\frac{P}{P_{crK}} + \frac{8}{9}\frac{M_{2^{nd}elastic}}{M_p} = 1.0$$

$$\frac{P}{2P_{crK}} + \frac{M_{2^{nd}elastic}}{M_p} = 1.0$$

87

1985 LRFD INTERACTION EQUATION

$M_{2^{nd}elastic}$ ------ **Must include Pδ and PΔ effects**

Can approximate by using $M_{1st\ order}$
with B_1 and B_2 amplification factors

88

1985 PREDICTION

WHERE ARE WE HEADED

- USE INELASTIC 2nd ORDER ANAL. for $P\Delta_i$
  - USE τI in elastic anal. programs to get Δ_i
- THERE WILL BE AN INTERACTION EQ.

$$1.0 = \frac{P}{P_{yield}} + \frac{M_{sway\ inelastic\ 2nd\ order}}{M_u} + \frac{M_{no\ sway}}{M_u}\frac{C_m}{1-\frac{P}{\tau P_E}}$$

B_{2_i}: $M_{1st\ order}$ use τ

strong M_p
weak $< M_p$

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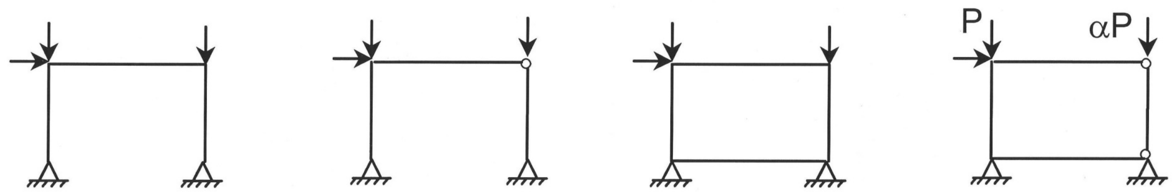
8. Future

90

2005 LRFD INTERACTION EQUATION

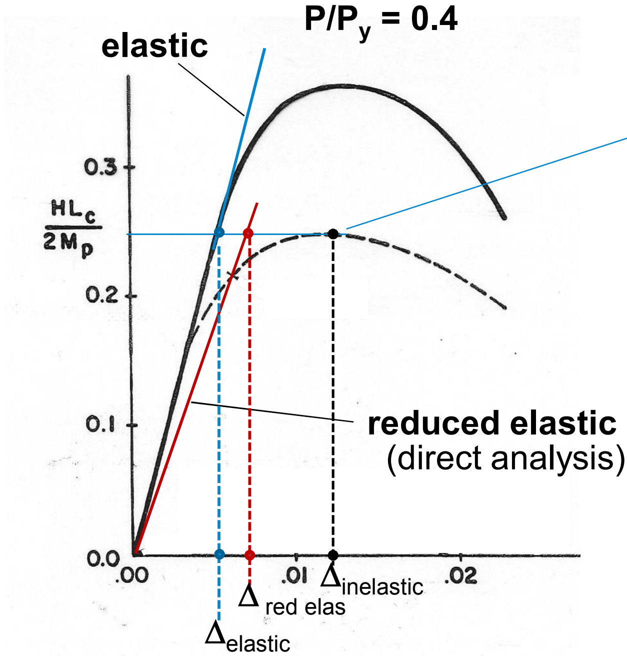
- Same frames in the 1985 LRFD study were reanalyzed but with an initial column out of straightness = L / 1000.
- Take advantage of available structural analysis programs that perform correct elastic 2nd order analyses.
- Develop a design approach that eliminates effective length



2005 LRFD INTERACTION EQUATION

Frame Result

2nd order inelastic analysis



Maximum Moment

$\frac{HL}{2} + P\Delta_{inelastic}$ (exact)

$\frac{HL}{2} + P\Delta_{reduced\ elastic}$ (use K= 1) (direct analysis)

$\frac{HL}{2} + P\Delta_{elastic}$ (use KL)

no residual stress ———

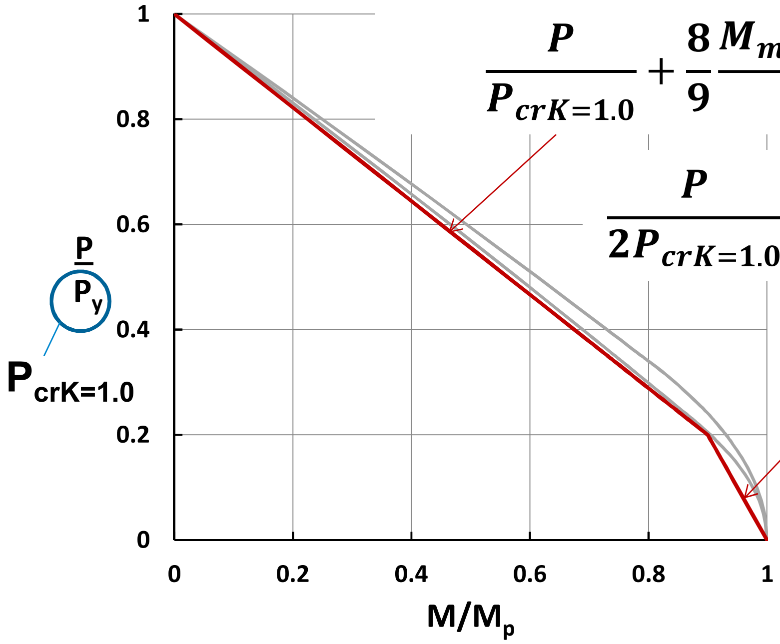
residual stress = 0.3F_y - - -

2005 LRFD INTERACTION EQUATION

Direct Analysis

$$\frac{P}{P_{crK=1.0}} + \frac{8}{9}\frac{M_{modified\ 2nd\ elastic}}{M_p} = 1.0$$

$$\frac{P}{2P_{crK=1.0}} + \frac{M_{modified\ 2^{nd} elastic}}{M_p} = 1.0$$



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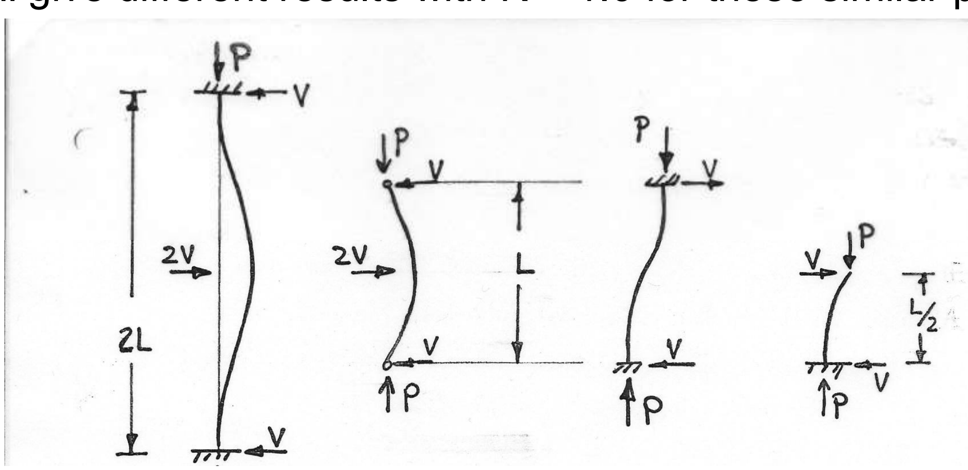
2005 LRFD INTERACTION EQUATION

K = 1.0

Direct Analysis reduces the elastic stiffness and uses τ to produce an approximate second order inelastic analysis

Will give different results with K = 1.0 for these similar problems



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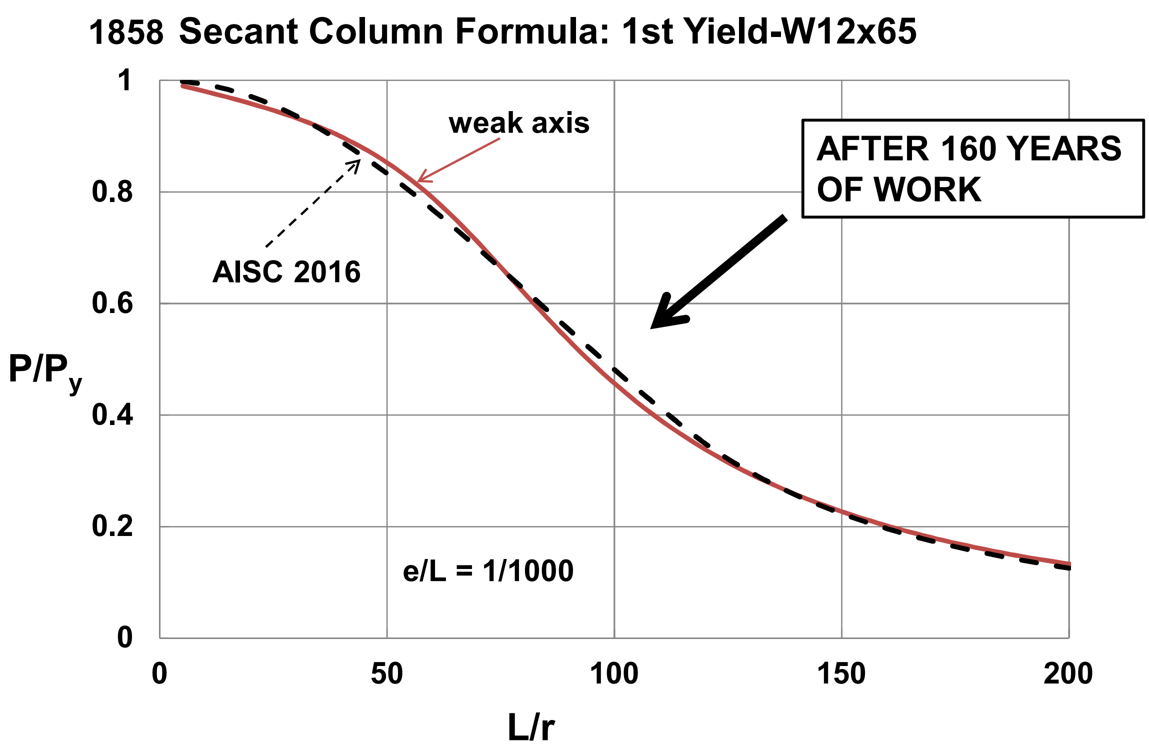

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CURRENT COLUMN DESIGN

- Based on ultimate strength from a 2nd order inelastic analysis that considers the effects of 1880's vintage column out-of- straightness (L/1000) and residual stresses from the 1960's (0.3F_y).
- Steel production and manufacturing processes have changed:
    1. Continuous casting in a dog-bone shape
    2. Continuous rolling operation
    3. Cold rotary straightening of most sections

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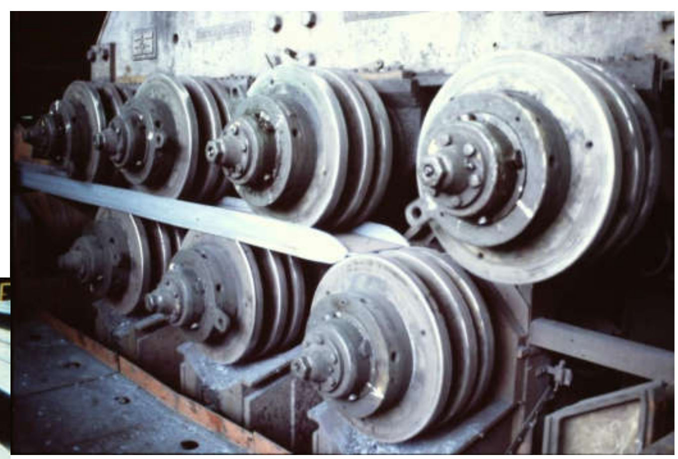
RESIDUAL STRESSES

Rotary straightening equipment



Before straightening

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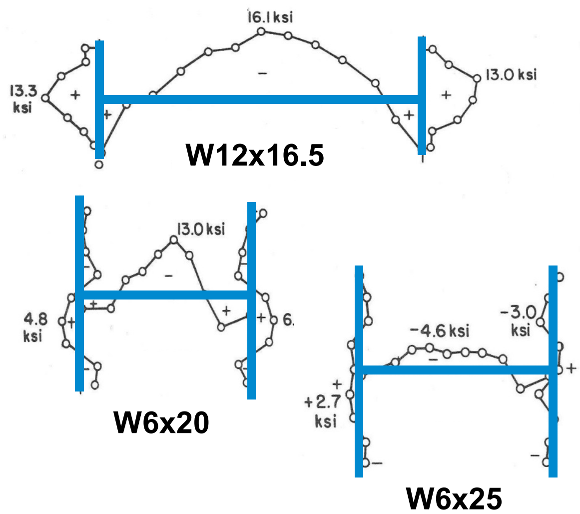
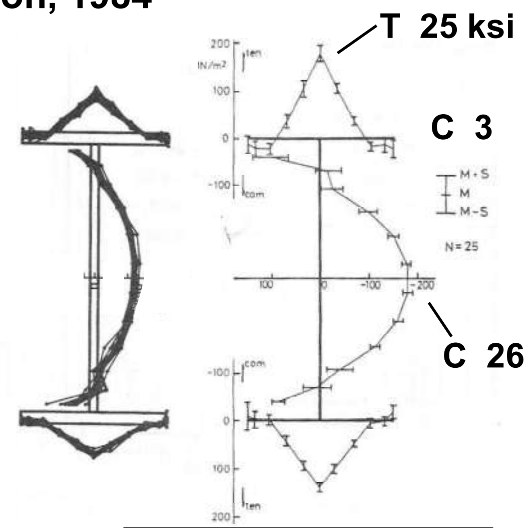
RESIDUAL STRESS

Rotary-Straightened Members

Yura, 1964



Itoh, 1984



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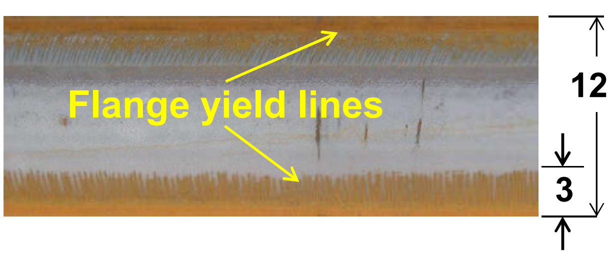
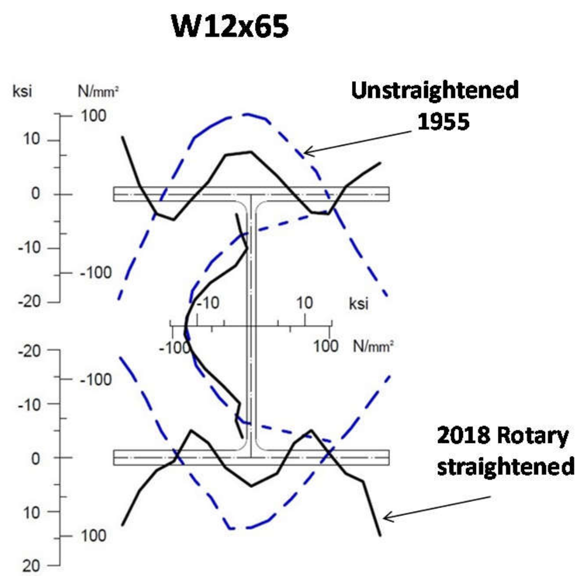
ROTARY STRAIGHTENED 2018

Sections up to W24x370





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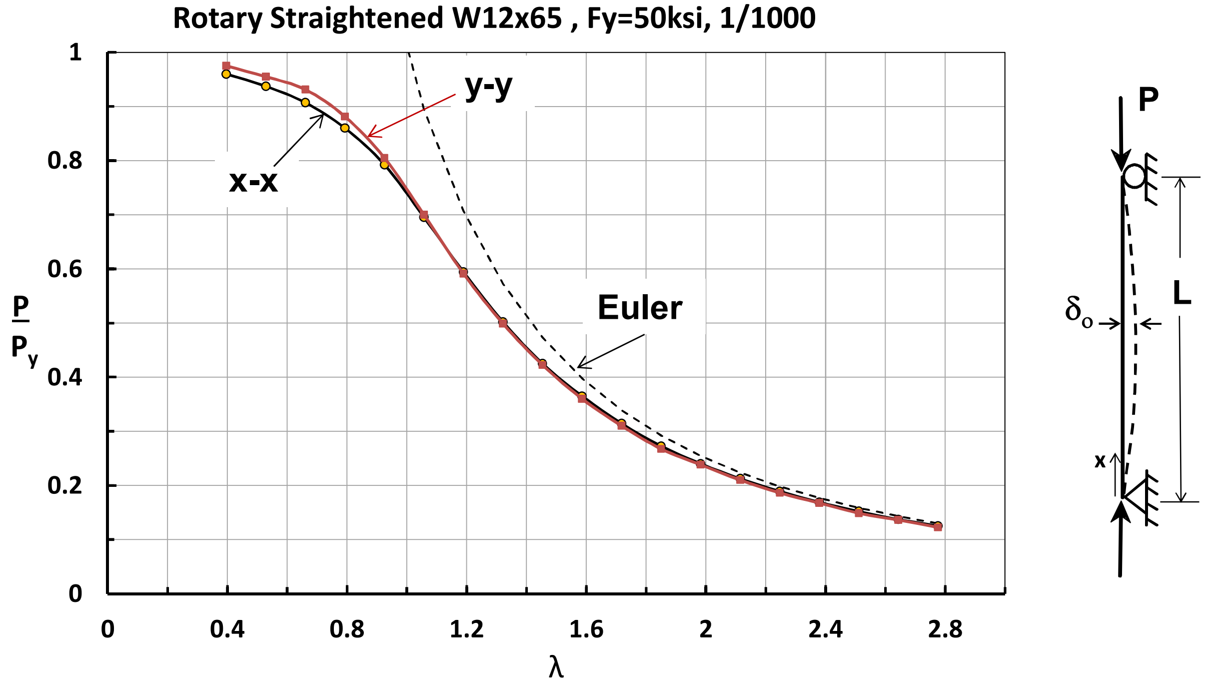
COLUMN CURVES

Ultimate Strength -No End Restraint



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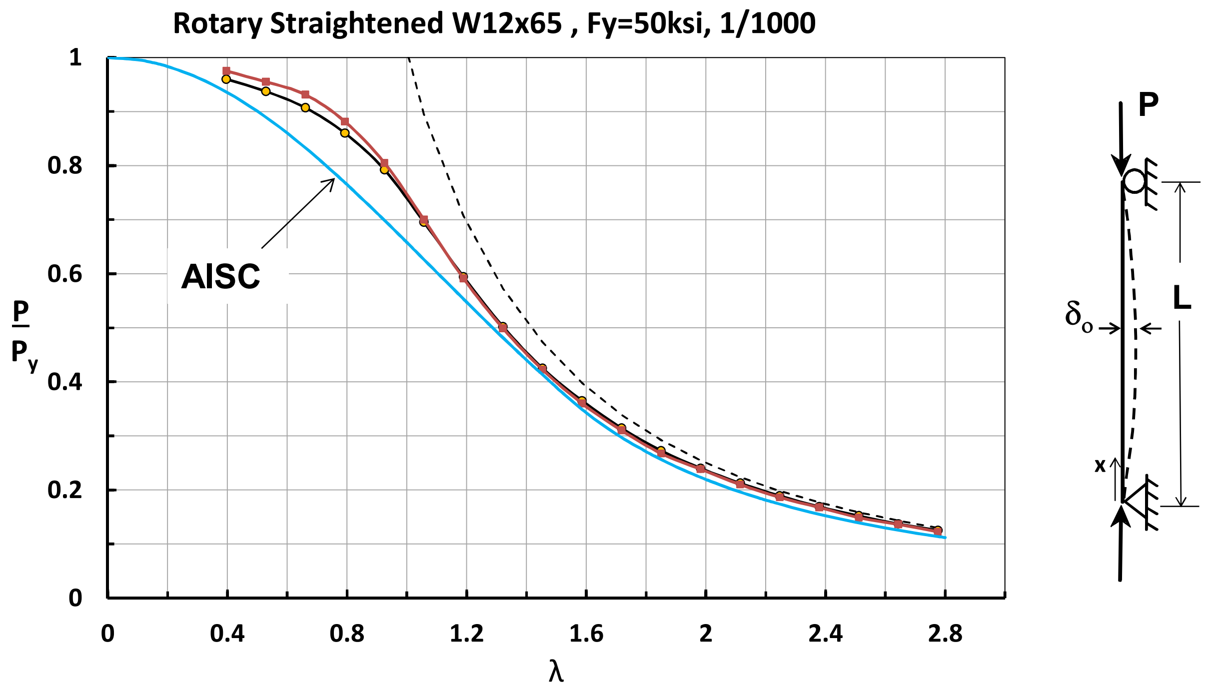
COLUMN CURVES



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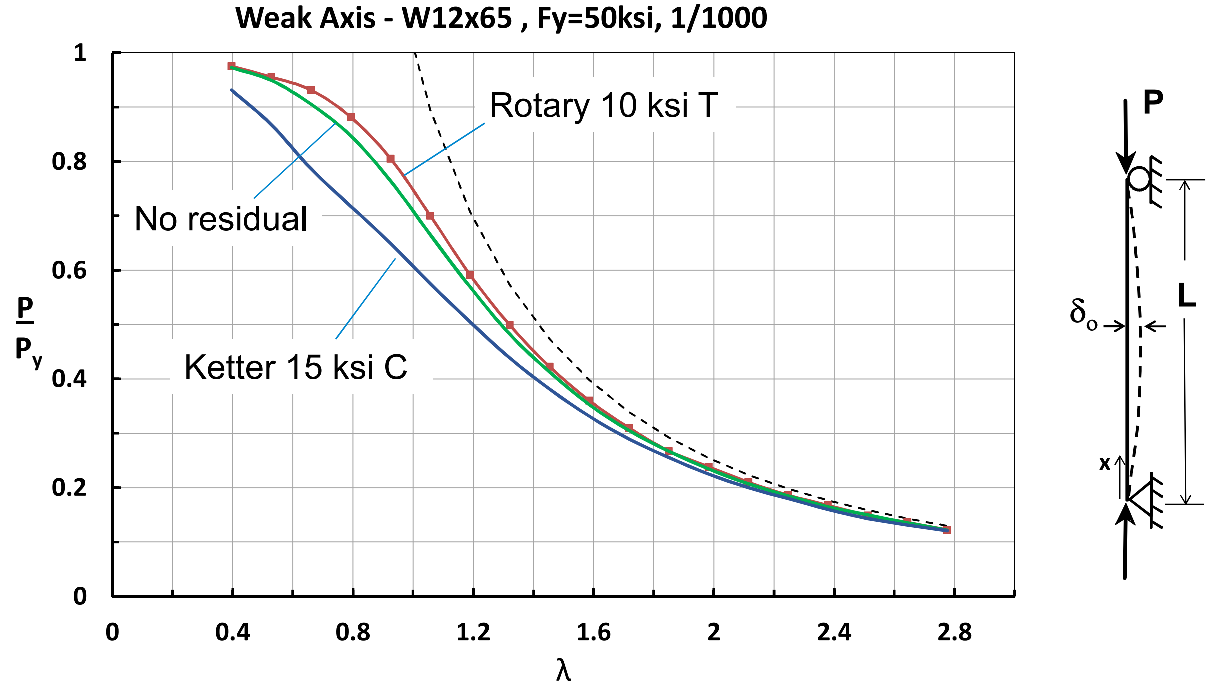
COLUMN CURVES

Residual Stress Effect



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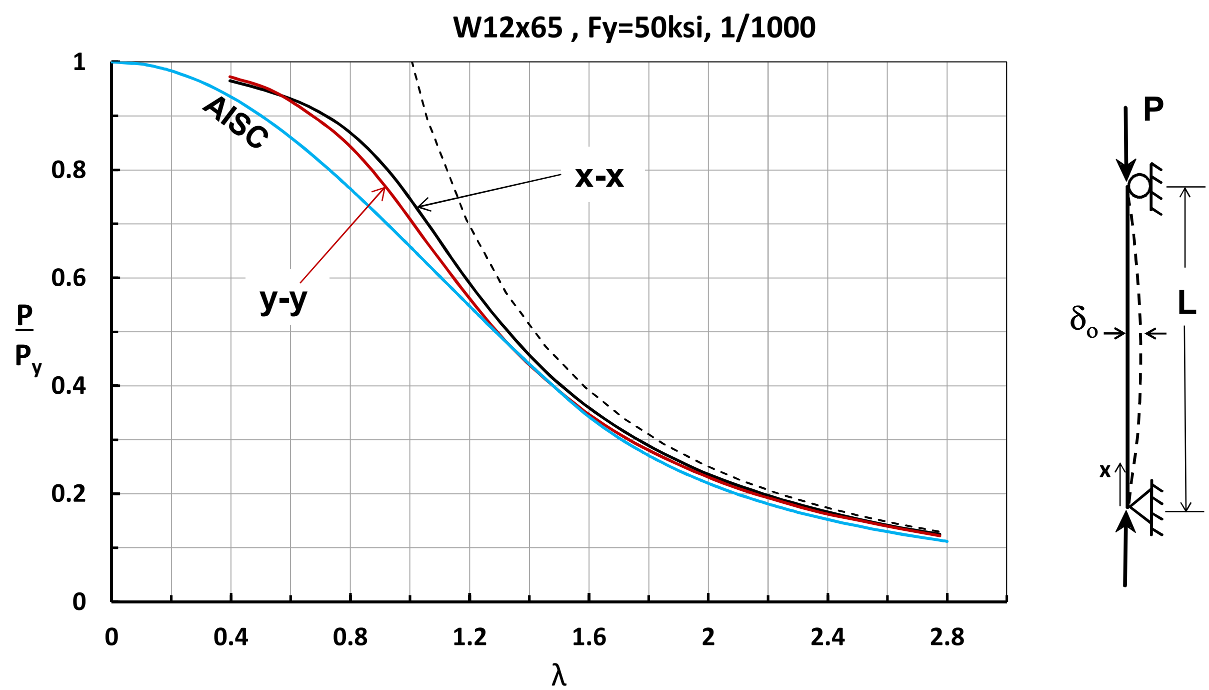
COLUMN CURVES

No Residual Stresses



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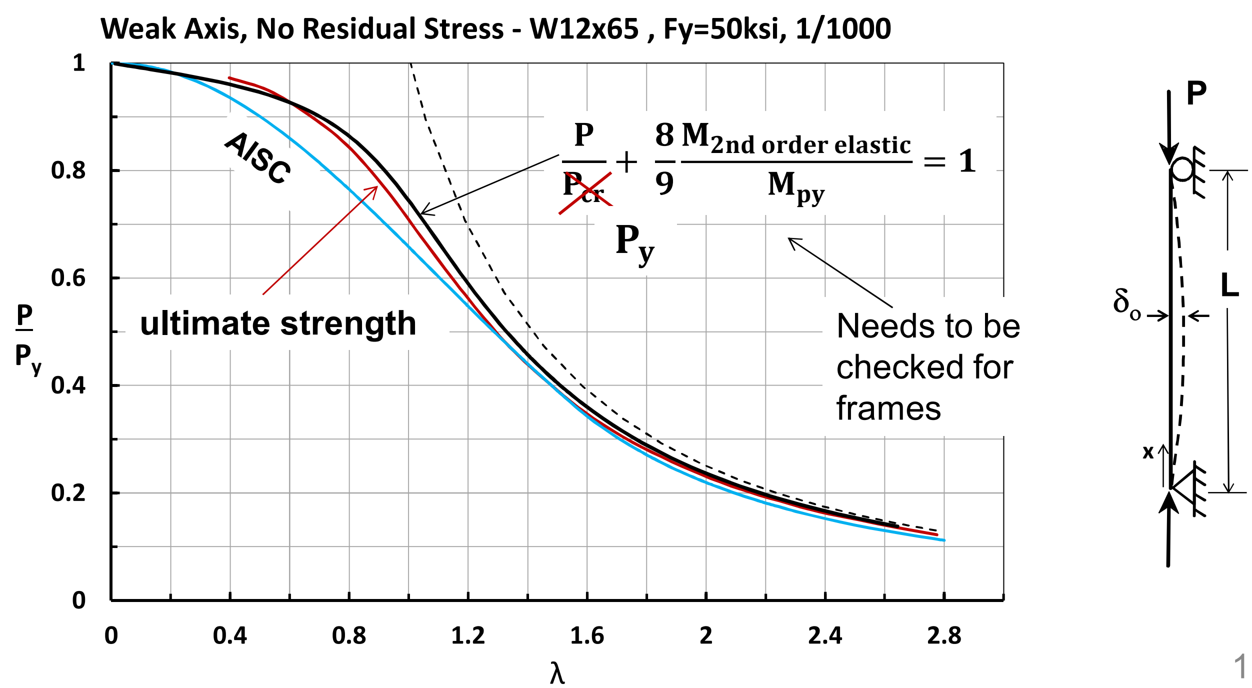
COLUMN CURVES

Interaction Equation



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SUMMARY

- **The column curve itself has changed very little over the past 200 years.**
- **The basis of the column curve has transitioned from:**
  1. **Eccentrically loaded with elastic limit (1820-1961)**
  2. **Straight member (buckling) with residual stresses (1961-1985)**
  3. **Ultimate strength with initial out-of-straightness, residual stress and small restraint (1985-present)**
  4. **Plastic strength with initial eccentricity, no residual stress (maybe)**

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SUMMARY **Interaction Equations**

< 1936	(Axial + Bending) Stresses ≤ Column Strength	
1936-1961	$\frac{P}{P_{crK=1.0}} + \frac{M_{1st}}{M_y} = 1$	(Not Derivable)
1961-1985	$\frac{P}{P_{crK}} + \frac{C_m M_{1st}}{\left(1 - \frac{P}{P_{eK}}\right) 1.1 M_y} = 1.0$	(Not Derivable)
1985-2005	$\frac{P}{P_{crK}} + \frac{8}{9}\frac{M_{2nd}}{M_p} = 1.0$	(Not Derivable)
2005-2018	$\frac{P}{P_{crK=1.0}} + \frac{8}{9}\frac{M_{modified\ 2nd}}{M_p} = 1.0$	(Not Derivable)

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SUMMARY **Interaction Equations**

Future ? $\frac{P}{P_y} + \frac{8}{9}\frac{M_{2nd\ order\ elastic}}{M_p} = 1$ (Derivable)

THE END

Thank You for Attending

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