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## Design for Stability using the 2010 AISC Specification

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Presented by Louis F. Geschwindner, Ph. D., P.E.  
Professor Emeritus, Architectural Engineering  
Penn State University



## AISC Webinar

# Design for Stability using the 2010 AISC Specification

July 14, 2011

Louis F. Geschwindner, Ph.D., P.E.  
Professor Emeritus, Architectural Engineering  
Penn State University



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## Outline

1. Design for Combined Forces
2. Stability Analysis and Design Requirements
3. Determination of Required Strength
  - Direct Analysis Method
  - Effective Length Method
  - First-Order Analysis
4. Summary and Simplified Method



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## Design for Combined Forces

Chapter H addresses members subject to axial force and flexure about one or both axes, with or without torsion, and to members subject to torsion only.



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## Design for Combined Forces

- H1. Doubly- and Singly-Symmetric Members Subject to Flexure and Axial Force
- H2. Unsymmetric and Other Members Subject to Flexure and Axial Force
- H3. Members Subject to Torsion and Combined Torsion, Flexure, Shear, and/or Axial Force
- H4. Rupture of Flanges with Holes Subject to Tension



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## Design for Combined Forces

2010 - Doubly and Singly Symmetric  
Members (same as 2005)

$$\frac{P_r}{P_c} \geq 0.2 \quad \frac{P_r}{P_c} + \frac{8}{9} \left[ \frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right] \leq 1.0 \quad (\text{H1-1a})$$

$$\frac{P_r}{P_c} < 0.2 \quad \frac{P_r}{2P_c} + \left[ \frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right] \leq 1.0 \quad (\text{H1-1b})$$



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## Beam-Columns

- Definitions (ASD)

$P_r$  = required compressive strength (ASD)

$P_c = P_n / \Omega_c$  = allowable compressive strength

$M_r$  = required flexural strength (ASD)

$M_c = M_n / \Omega_b$  = allowable flexural strength

$\Omega_c = 1.67$

$\Omega_b = 1.67$



Required strength from 2<sup>nd</sup> order analysis according to Chapter C

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# Beam-Columns

- Definitions (LRFD)

$P_r$  = required compressive strength (LRFD)

$P_c = \phi_c P_n$  = design compressive strength

$M_r$  = required flexural strength (LRFD)

$M_c = \phi_b M_n$  = design flexural strength

$\phi_c = 0.90$

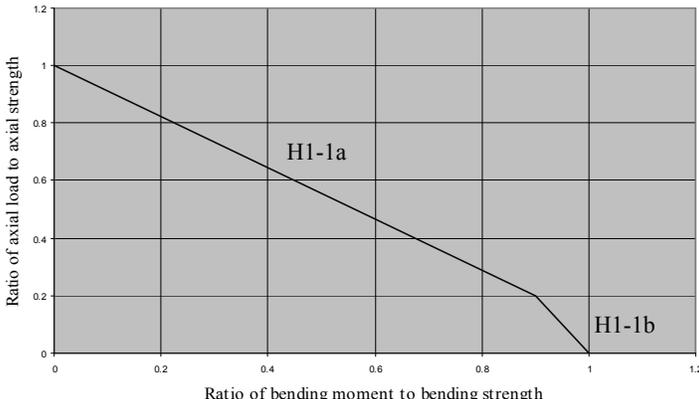
$\phi_b = 0.90$



Required strength from 2<sup>nd</sup> order analysis according to Chapter C

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# Design for Combined Forces



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**Table 6-1 (continued)**  
**Combined Flexure and Axial Force**  
**W-Shapes**  $F_y = 50$  ksi

Shape	W14												
	109		99		90								
	$p \times 10^3$	$b_x \times 10^3$	$p \times 10^3$	$b_x \times 10^3$	$p \times 10^3$	$b_x \times 10^3$							
Design		Design		Design									
$(\text{kips})^2$		$(\text{kips-ft})^2$		$(\text{kips})^2$		$(\text{kips-ft})^2$							
	0	1.04	0.694			50	5.38	3.58	3.05	2.03	5.98	3.98	3.66
<b>Other Constants and Properties:</b>													
Effective length, $KL$ (ft), with respect to least radius of gyration, $r_y$ , or tributary length, $L_b$ (ft), for X-X axis bending	12	1.36	0.774	$b_y \times 10^3, (\text{kip-ft})^{-1}$	$t_y \times 10^3, (\text{kips})^{-1}$	$t_x \times 10^3, (\text{kips})^{-1}$	$r_x/r_y$	$r_y, \text{in.}$					2
	13	1.19	0.789						3.84	2.56	4.29	0	
	14	1.21	0.855						1.04	0.694	1.15	0	
	15	1.24	0.823						1.28	0.855	1.41	0	
16	1.27	0.846					1.67			1.66			
17	1.30	0.844					3.73			3.71			
18	1.33	0.857					I shape does not meet compact limit for flexure with $F_y = 50$ ksi.						
19	1.37	0.813											
20	1.41	0.940											
22	1.51	1.00											
24	1.61	1.07											
26	1.74	1.19											
28	1.89	1.35											
30	2.06	1.37											
32	2.27	1.51											
34	2.52	1.67											
36	2.79	1.86											
38	3.11	2.07											
40	3.44	2.29											
42	3.80	2.52											
44	4.17	2.77											
46	4.55	3.03											
48	4.94	3.30											
50	5.35	3.58											
<b>Other Constants and Properties</b>													
$b_y \times 10^3, (\text{kip-ft})^{-1}$	3.84	2.56	4.29	2.85	4.90	3.26							
$t_y \times 10^3, (\text{kips})^{-1}$	1.04	0.694	1.15	0.764	1.26	0.839							
$t_x \times 10^3, (\text{kips})^{-1}$	1.28	0.855	1.41	0.940	1.55	1.03							
$r_x/r_y$	1.67		1.66		1.66								
$r_y, \text{in.}$	3.73		3.71		3.70								



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## Design Requirements

### AISC 360-10

#### B3.1 Required Strength

“The required strength of structural members and connections shall be determined by structural analysis for the appropriate load combinations as stipulated in Section B2.”

“Design by elastic, inelastic or plastic analysis is permitted.”

#### B3.5 Design for Stability

“Stability of the structure and its elements shall be determined in accordance with Chapter C.”



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## Stability Analysis and Design

AISC 360-10

C. Design For Stability

C1. General Stability Requirements

C2. Calculation of Required Strengths

C3. Calculation of Available Strengths



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## Stability Analysis and Design

AISC 360-10

C1. General Stability Requirements

“Stability shall be provided for the structure as a whole and for each of its elements. The effects of all of the following on stability of the structure and its elements shall be considered:”



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## Design for Stability

1. Flexural, shear and axial member deformations and all other deformations that contribute to displacements of the structure
2. Second-order effects (both  $P-\Delta$  and  $P-\delta$  effects)
3. Geometric imperfections
4. Stiffness reduction due to inelasticity
5. Uncertainty in stiffness and strength

“All load-dependent effects shall be calculated at a level of loading corresponding to LRFD load combinations or 1.6 times ASD load combinations.”

We will discuss  $\alpha$  in this regard later.



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## Design for Stability

### C1. General Stability Requirements

“Any rational method of design for stability that considers all of the listed effects is permitted; this includes the methods identified in Sections C1.1 and C1.2.”

#### C1.1 Direct Analysis Method of Design

#### C1.2 Alternative Methods of Design

Effective Length Method  
First-order Analysis Method } Appendix 7



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# Design for Stability

## C1. General Stability Requirements

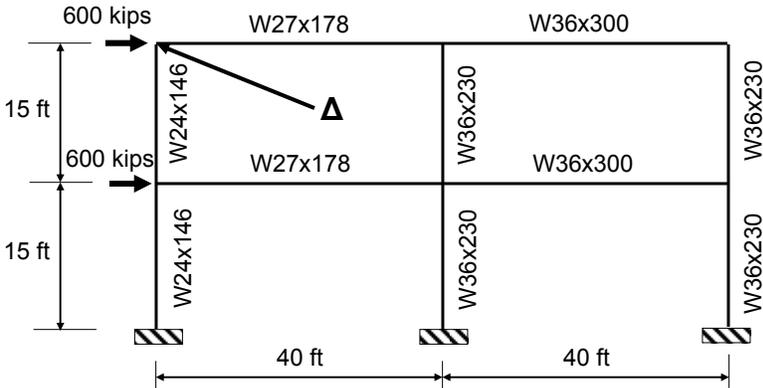
Stability shall be provided for the structure as a whole and for each of its elements. The effects of all of the following on stability of the structure and its elements shall be considered:"

1. Flexural, shear and axial member deformations and all other deformations that contribute to displacements of the structure
2. Second-order effects (both  $P-\Delta$  and  $P-\delta$  effects)
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## Elastic Analysis



$$\Delta_{flex+axial+shear} = 2.94 \text{ in.}, \Delta_{flex+axial} = 2.42 \text{ in.},$$

$$\Delta_{flex} = 2.24 \text{ in.}, \Delta_{flex+rigid\ ends} = 1.79 \text{ in.}$$



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## Elastic Analysis

With pin supports  $\Delta_{flex,axial,shear} = 7.11$  in.  
compared to  $\Delta_{flex,axial,shear} = 2.94$  in. with fixed supports



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## Design for Stability

### C1. General Stability Requirements

Stability shall be provided for the structure as a whole and for each of its elements. The effects of all of the following on stability of the structure and its elements shall be considered:"

1. Flexural, shear and axial member deformations and all other deformations that contribute to displacements of the structure
2. Second-order effects (both  $P-\Delta$  and  $P-\delta$  effects)
3. Geometric imperfections
4. Stiffness reduction due to inelasticity
5. Uncertainty in stiffness and strength

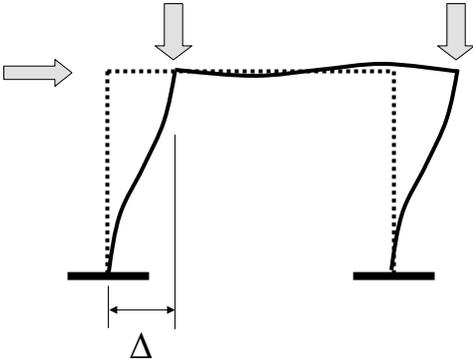


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### Second-Order Analysis

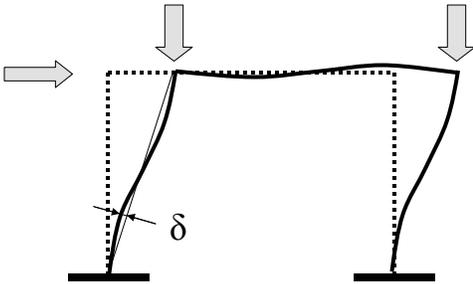
- P- $\Delta$  (sway effects)



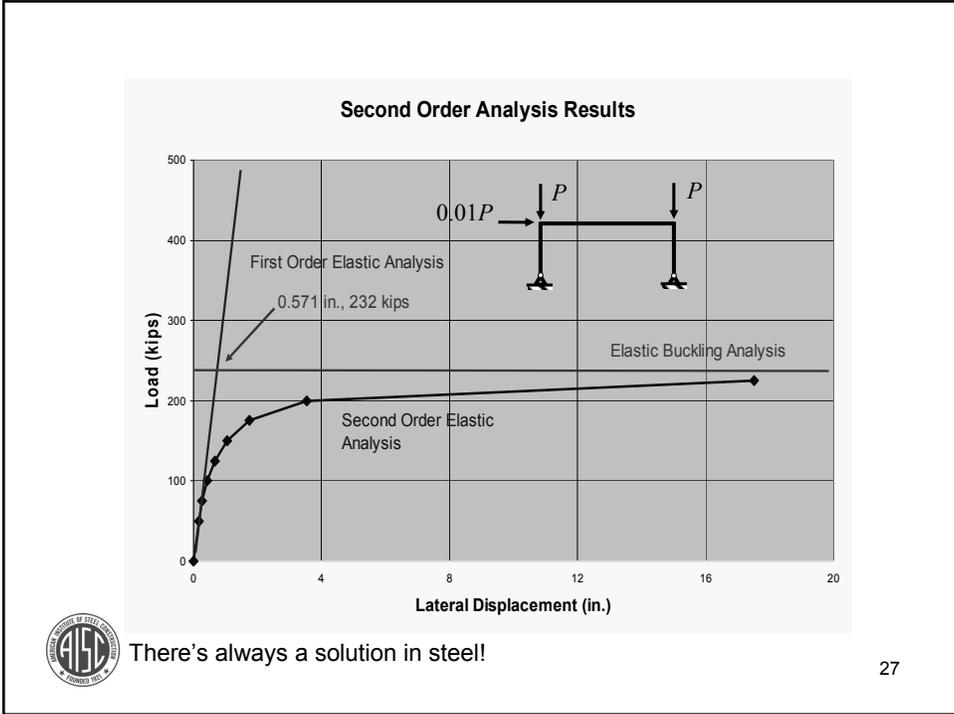
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### Second-Order Analysis

- P- $\delta$  (member effects)



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### Second-Order Analysis

Either carry out a rigorous second-order analysis, in which case the forces determined in the analysis are  $M_r$  and  $P_r$

or

Use an approximate method of second-order analysis by amplified first-order analysis as given in Appendix 8.

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## Approximate Second-Order Analysis

- App.8. Approximate Second-Order  
Analysis

### 8.2 Calculation Procedure

The required second-order flexural strength,  $M_r$ , and axial strength,  $P_r$ , of all members shall be determined as follows:

$$M_r = B_1 M_{nt} + B_2 M_{lt} \quad (\text{A-8-1})$$

$$P_r = P_{nt} + B_2 P_{lt} \quad (\text{A-8-2})$$



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## Approximate Second-Order Analysis

### Definitions

$P_{nt}$  = First-order axial force when translation is restrained

$P_{lt}$  = First-order axial force due to translation

$M_{nt}$  = First-order moment when translation is restrained

$M_{lt}$  = First-order moment due to translation

$B_1$  = Amplification for member effect

$B_2$  = Amplification for frame effect

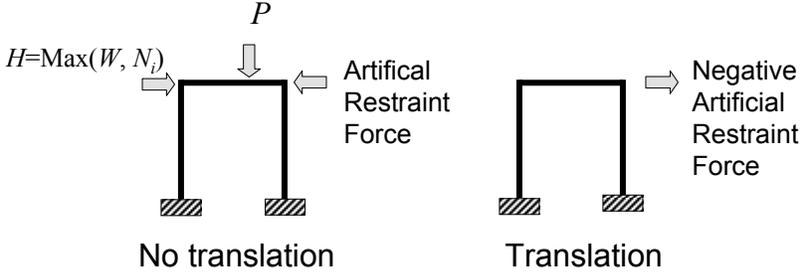


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## Approximate Second-Order Analysis

First-order analysis using  $EI$ ,  $AE$  or  $EI^*$ ,  $AE^*$  as required by method of analysis



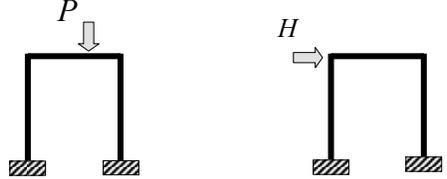
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## Approximate Second-Order Analysis

- Common design office approximation

$M_{nt}$  = gravity load moments

$M_{lt}$  = lateral load moments



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## Approximate Second-Order Analysis

- App.8.2.1 Multiplier  $B_1$  for  $P$ - $\delta$  Effects (member effects)

$$B_1 = \frac{C_m}{\left(1 - \frac{\alpha P_r}{P_{e1}}\right)} \geq 1.0 \quad (\text{A-8-3})$$

for members not loaded transversely

$$C_m = 0.6 - 0.4(M_1/M_2) \quad (\text{A-8-4})$$

$$\alpha = 1.0 \quad (\text{LRFD}) \quad \alpha = 1.6 \quad (\text{ASD})$$



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## Approximate Second-Order Analysis

- App.8.2.1 Multiplier  $B_1$  for  $P$ - $\delta$  Effects (member effects)

$P_{e1}$  is the elastic critical buckling strength in the plane of bending for the column as if it were in a braced frame

$$K_1 = 1.0 \quad P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} \quad (\text{A-8-5})$$



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## Approximate Second-Order Analysis

- App.8.2.2 Multiplier  $B_2$  for  $P$ - $\Delta$  Effects (sway effect)

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e story}}} \geq 1.0 \quad (\text{A-8-6})$$

$P_{e story}$  = elastic critical buckling strength in plane of bending  
determined for a sidesway buckling analysis

$P_{story}$  = total vertical load supported by all columns in story

$$\alpha = 1.0 \quad (\text{LRFD}) \quad \alpha = 1.6 \quad (\text{ASD})$$



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## Approximate Second-Order Analysis

- App.8.2.2 Multiplier  $B_2$  for  $P$ - $\Delta$  Effects (sway effect)

$$P_{e story} = R_M \frac{HL}{\Delta_H} \quad (\text{A-8-7})$$

$\Delta_H$  = first-order translation of the story

$H$  = story shear force producing  $\Delta_H$

$L$  = story height



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## Approximate Second-Order Analysis

- App.8.2.2 Multiplier  $B_2$  for  $P-\Delta$  Effects (sway effect)

$$R_M = 1 - 0.15 \left( \frac{P_{mf}}{P_{story}} \right) \quad (\text{A-8-8})$$

$P_{mf}$  = total vertical load in columns that are  
part of moment frames

$P_{story}$  = total vertical load supported by  
all columns in story



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## Approximate Second-Order Analysis

- App.8.2.2 Multiplier  $B_2$  for  $P-\Delta$  Effects (sway effect)

$$R_M = 1 - 0.15 \left( \frac{P_{mf}}{P_{story}} \right) \quad (\text{A-8-8})$$

- For braced frames

$$R_M = 1.0$$

- For moment frames with no gravity only columns

$$R_M = 0.85 \quad (\text{conservative in all cases})$$



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# Design for Stability

## C1. General Stability Requirements

Stability shall be provided for the structure as a whole and for each of its elements. The effects of all of the following on stability of the structure and its elements shall be considered:"

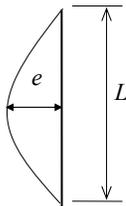
1. Flexural, shear and axial member deformations and all other deformations that contribute to displacements of the structure
2. Second-order effects (both  $P-\Delta$  and  $P-\delta$  effects)
3. Geometric imperfections
4. Stiffness reduction due to inelasticity
5. Uncertainty in stiffness and strength



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# Geometric Imperfections

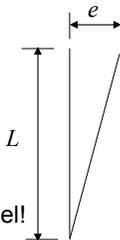
- Out-Of-Straightness



ASTM A6  
Tolerance:

$$e = L/1000$$

- Out-Of-Plumbness



Code of Standard Practice  
Tolerance:

$$e = L/500$$



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## Design for Stability

### C1. General Stability Requirements

Stability shall be provided for the structure as a whole and for each of its elements. The effects of all of the following on stability of the structure and its elements shall be considered:"

1. Flexural, shear and axial member deformations and all other deformations that contribute to displacements of the structure
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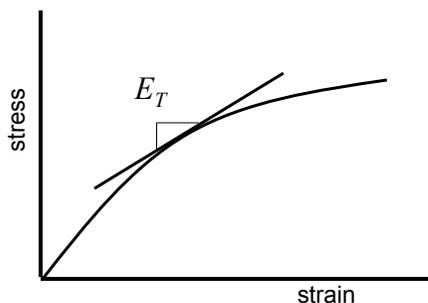


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## Stiffness Reduction

- Effects of Inelasticity
  - Stress-strain relationship no longer linear
  - Use the Tangent Modulus of Elasticity



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## Stiffness Reduction

- Depends on the level of axial stress in the member

when  $\alpha P_r/P_y \leq 0.5$ ;

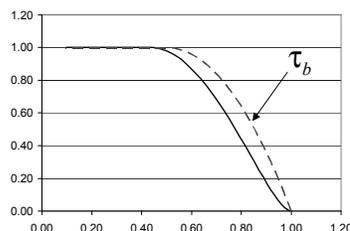
$$\tau_b = 1.0$$

when  $\alpha P_r/P_y > 0.5$ ;

$$\tau_b = 4 \left[ \frac{\alpha P_r}{P_y} \left( 1 - \frac{\alpha P_r}{P_y} \right) \right]$$

$\alpha = 1.0$  (LRFD)

$\alpha = 1.6$  (ASD)



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## Design for Stability

### C1. General Stability Requirements

Stability shall be provided for the structure as a whole and for each of its elements. The effects of all of the following on stability of the structure and its elements shall be considered:"

1. Flexural, shear and axial member deformations and all other deformations that contribute to displacements of the structure
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3. Geometric imperfections
4. Stiffness reduction due to inelasticity
5. Uncertainty in stiffness and strength



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## Uncertainty

- Included in available strength determination
  - Resistance factor,  $\phi$
  - Safety factor,  $\Omega$
- Included in stiffness reduction factor,  $\tau_b$



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## Stability Design Requirements

- So, what is really new (with 2005 and 2010 Specifications) for the engineer to consider?
  - Second-order effects
    - Not really new but...
  - Initial out-of-plumbness
    - New, but do not always need to include
  - Residual stress influence on second-order
    - Built in to second-order analysis when needed



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## Required Strength

- C2 Calculation of required strength
  - Three approaches are available
    - Section C2: Direct Analysis Method
      - This is the foundation for the other approaches presented
    - Appendix 7.2: Effective Length Method
      - Essentially what you have been used to
    - Appendix 7.3: First-order Analysis Method
      - This is the simplest approach if applicable



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## Required Strength

- What are the features of these methods
  - Direct Analysis
    - Can eliminate need to determine effective length factor,  $K$
  - Notional Loads
    - Used to account for out-of-plumbness (all three methods)
    - Used to account for second-order effects (First-order analysis method)



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## Design for Stability

- C2. Calculation of Required Strengths

“For the direct analysis method of design, the required strengths of components of the structure shall be determined from an analysis conforming to Section C2.1. The analysis shall include consideration of initial imperfections in accordance with Section C2.2 and adjustments to stiffness in accordance with Section C2.3”



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## Direct Analysis

- C2. Calculation of Required Strengths

- Applicable to all types of structures
- Does not distinguish between systems
  - Braced frames
  - Moment frames
  - Shear wall systems
  - Any combination of systems
- The bottom line,  $K = 1.0$



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## Direct Analysis

- C2.1. General Analysis Requirements

- (1) consider all deformations

- The same requirement as the general stability requirements in Section C1.(1)

- (2) conduct a second-order analysis

- The same requirement as the general stability requirements in Section C1.(2)



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## Direct Analysis

- C2.1.(2) Second-order Analysis

- Use any second-order elastic analysis that considers  $P-\Delta$  and  $P-\delta$  effects

- May ignore effect of  $P-\delta$  on the response of the structure since the exceptions in this section will likely be met.

- Options:

- Any rigorous second-order analysis method
    - Amplified first-order analysis of Appendix 8



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# Direct Analysis

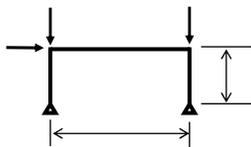
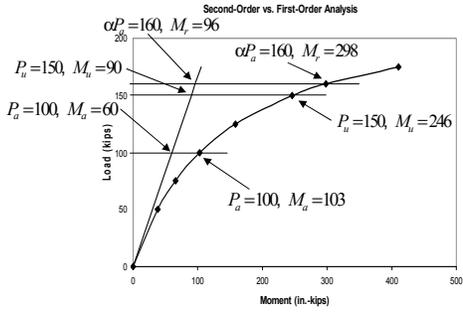
- C2.1. General Analysis Requirements
  - (1) consider all deformations
  - (2) conduct a second-order analysis
  - (3) include all gravity and other loads
    - Seems obvious
  - (4) carry out analysis for LRFD or 1.6ASD load combinations



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# Direct Analysis

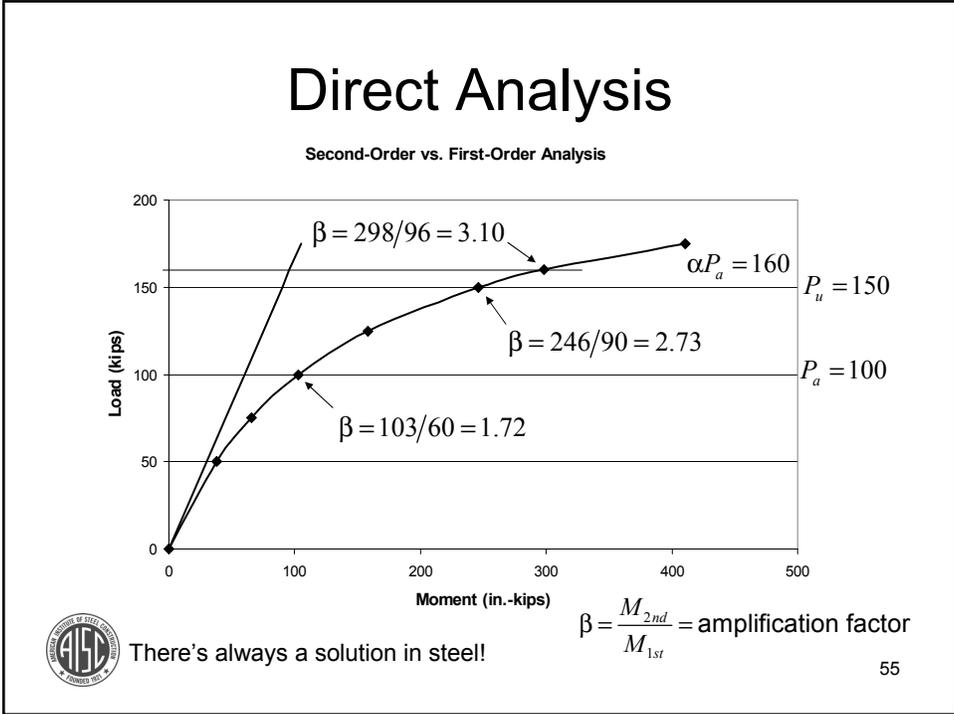
- C2.1.(4) Analysis at LRFD or 1.6ASD load combinations



Use  $\alpha$  to be sure that the analysis captures the nonlinear aspects at the ultimate strength



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### Direct Analysis

- C2.2. Consideration of Initial Imperfections
 

“The effect of initial imperfections on the stability of the structure shall be taken into account either by direct modeling of imperfections in the analysis as specified in Section C2.2a or by application of notional loads as specified in Section C2.2b.”

C2.2a. Direct modeling

C2.2b. Notional loads

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## Geometric Imperfections

### C2.2a. Direct Modeling

Top lateral displacement  $\Delta = 0.105$  in.

Support Moment  $M = 32.4$  in-kips

0.48 in.

0.24 in.

$P = 100$  kips

$P = 100$  kips

$P = 100$  kips

$P = 100$  kips

10.0 ft

10.0 ft

20.0 ft

All members W8x24  
First-Order Analysis



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## Geometric Imperfections

- C2.2b. Notional Loads  
Apply notional loads,  $N_i$ , where
 
$$N_i = 0.002\alpha Y_i$$
 $Y_i$  = the total gravity load on that story

Applies to tiered buildings and accounts for an initial out-of-plumbness at the minimum of 1/500 as defined in the COSP. If a lesser out-of-plumbness is known,  $N_i$  can be reduced



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## Geometric Imperfections

$N_i = 0.002\alpha Y_i$

Calculated independently at each level.

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## Geometric Imperfections

- C2.2b. Notional loads

All members W8x24  
First-Order Analysis

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## Geometric Imperfections

- C2.2b.(4) If the second-order effects are limited,

$$B_2 = \frac{\Delta_{2nd-order}}{\Delta_{1st-order}} < 1.7$$

these notional loads are applied in the gravity only load combinations, otherwise they are also added to the lateral loads.



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## Stiffness Reduction

- C2.3 Adjustments to Stiffness

(1) reduce all stiffnesses by 0.80

This is the unique part of design by Direct Analysis and what permits the use of  $K=1.0$  for all members

(2) reduce all flexural stiffnesses, if they contribute to stability, by  $\tau_b$



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## Stiffness Reduction

Thus,

$$EI^* = 0.8\tau_b EI$$

$$EA^* = 0.8EA$$

(3) Could use  $\tau_b = 1.0$  and apply a notional load instead.



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63

## Direct Analysis

- C3. Calculation of Available Strengths
  - Follow provisions of Chapters D through K with no further consideration of overall structure stability.
  - Take the effective length factor,  $K = 1.0$ , unless a smaller value can be justified by rational analysis
  - Bracing requirements of Appendix 6 are not applicable to bracing included as part of the overall force-resisting system.



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## Direct Analysis

- Design process
  - Perform second-order analysis
    - Use nominal geometry (not out of plumb)
    - Use reduced stiffness,  $EI^*$  and  $EA^*$
  - Apply notional loads,  $N_i = 0.002\alpha Y_i$ 
    - As a minimum lateral load if  $\frac{\Delta_{2nd-order}}{\Delta_{1st-order}} \leq 1.7$
    - As an additional lateral load if  $\frac{\Delta_{2nd-order}}{\Delta_{1st-order}} > 1.7$
  - Design members using  $K=1$  for compression



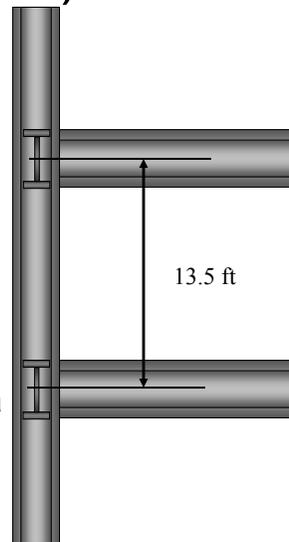
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65

## Example 1 (ASD)

- Check the adequacy of an ASTM A992 W14x99 column subjected to an axial force of 247 kips and an x-axis bending moment of 161 ft-kips at one end and 110 ft-kips at the other, from a first-order Direct Analysis (Chapter C).

The column is 13.5 ft long, is bending about the strong axis, has a length of 13.5 ft about the x- and y-axis and an unbraced length of the compression flange of 13.5 ft.



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66

## Example 1 (ASD)

- The controlling load case is  $D+(W \text{ or } 0.7E)$ 
  - The horizontal force resultant is 137 kips
  - The corresponding drift is 0.493 in. (with  $EI^*$ )
- For Direct Analysis,  $K = 1.0$
- Determine the second-order force and moment using the  $B_1 - B_2$  method



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67

## Example 1 (ASD)

- For this example,
  - The moment is a translation moment
  - The compression force is a no-translation force

$$M_{nt} = 0, \quad M_{lt} = 161, 110 \text{ ft-kips}$$

$$P_{nt} = 247 \text{ kips}, \quad P_{lt} = 0$$



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## Example 1 (ASD)

Member amplification

$$B_1 = \frac{C_m}{1 - \frac{\alpha P_r}{P_{e1}}} \geq 1.0$$

$$C_m = 0.6 - 0.4(110/161) = 0.327$$

$$\alpha = 1.6$$



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## Example 1 (ASD)

Member amplification

$$P_{e1} = \frac{\pi^2 EI^*}{(K_1 L)^2}$$

$$P_{e1} = \frac{\pi^2 (0.8)(29,000)(1,110)}{(1.0(13.5)(12))^2} = 9,600 \text{ kips}$$



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70

## Example 1 (ASD)

Member amplification

$$B_1 = \frac{C_m}{1 - \frac{\alpha P_r}{P_{e1}}} \geq 1.0$$

$$B_1 = \frac{0.327}{1 - \frac{(1.6)(247)}{9,600}} = 0.341 \neq 1.0 \therefore B_1 = 1.0$$



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## Example 1 (ASD)

Sway amplification

For the entire frame at this story

$$P_{story} = 3,750 \text{ kips}$$

$$H = 137 \text{ kips}$$

$$\Delta_H = 0.493 \text{ in.}$$

Drift using  $EI^*$



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72

## Example 1 (ASD)

Sway amplification

For a moment frame, conservatively,

$$R_M = 0.85$$

thus,

$$P_{e \text{ story}} = 0.85 \frac{HL}{\Delta_H} = 0.85 \frac{(137)(13.5(12))}{0.493} = 38,300 \text{ kips}$$

This is a measure of the frame buckling strength



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73

## Example 1 (ASD)

Sway amplification

$$B_2 = \frac{1}{1 - \frac{\alpha P_{\text{story}}}{P_{e \text{ story}}}} \geq 1.0$$

$$B_2 = \frac{1}{1 - \frac{(1.6)(3,750)}{38,300}} = 1.19$$



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## Example 1 (ASD)

Second-order moment

$$M_r = B_1 M_{nt} + B_2 M_{lt} \quad (\text{A-8-1})$$

$$M_a = 1.0(0.0) + 1.19(161) = 192 \text{ ft-kips}$$

Second-order force

$$P_r = P_{nt} + B_2 P_{lt} \quad (\text{A-8-2})$$

$$P_a = 247 + 1.19(0.0) = 247 \text{ kips}$$



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75

## Example 1 (ASD)

- Determine member strength

$$KL = 13.5 \text{ ft} \quad \frac{P_n}{\Omega_c} = 758 \text{ kips}$$

$$L_b = 13.5 \text{ ft} \quad \frac{M_n}{\Omega_b} = 430 \text{ ft-kips}$$

- Interaction Eq. H1-1a

$$\frac{247}{758} + \frac{8}{9} \left( \frac{192}{430} \right) = 0.722 < 1.0 \quad \therefore \text{ok}$$



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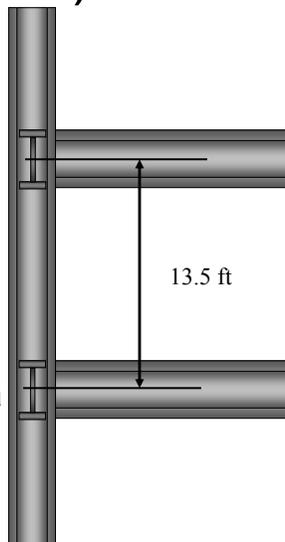
## Example 1 (LRFD)

- Check the adequacy of an ASTM A992 W14x99 column subjected to an axial force of 335 kips and an x-axis bending moment of 229 ft-kips at one end and 157 ft-kips at the other, from a first-order Direct Analysis (Chapter C).

The column is 13.5 ft long, is bending about the strong axis, has a length of 13.5 ft about the x- and y-axis and an unbraced length of the compression flange of 13.5 ft.



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## Example 1 (LRFD)

- The controlling load case is  $1.2D+1.0E+0.5L+0.2S$ 
  - The horizontal force resultant is 195 kips
  - The corresponding drift is 0.703 in. (with  $EI^*$ )
- For Direct Analysis,  $K = 1.0$
- Determine the second-order force and moment using the  $B_1 - B_2$  method



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78

## Example 1 (LRFD)

- For this example,
  - The moment is a translation moment
  - The compression force is a no-translation force

$$M_{nt} = 0, \quad M_{lt} = 229, 157 \text{ ft-kips}$$

$$P_{nt} = 335, \quad P_{lt} = 0 \text{ kips}$$



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## Example 1 (LRFD)

Member amplification

$$B_1 = \frac{C_m}{1 - \frac{\alpha P_r}{P_{e1}}} \geq 1.0$$

$$C_m = 0.6 - 0.4(157/229) = 0.326$$

$$\alpha = 1.0$$



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80

## Example 1 (LRFD)

Member amplification

$$P_{e1} = \frac{\pi^2 EI^*}{(K_1 L)^2}$$

$$P_{e1} = \frac{\pi^2 (0.8)(29,000)(1,110)}{(1.0(13.5)(12))^2} = 9,600 \text{ kips}$$



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81

## Example 1 (LRFD)

Member amplification

$$B_1 = \frac{C_m}{1 - \frac{\alpha P_r}{P_{e1}}} \geq 1.0$$

$$B_1 = \frac{0.326}{1 - \frac{(1.0)(335)}{9,680}} = 0.337 \not\geq 1.0 \therefore B_1 = 1.0$$



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## Example 1 (LRFD)

Sway amplification

For the entire frame at this story

$$P_{story} = 5,250 \text{ kips}$$

$$H = 195 \text{ kips}$$

$$\Delta_H = 0.703 \text{ in.}$$

Drift using  $EI^*$



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83

## Example 1 (LRFD)

Sway amplification

For a moment frame, conservatively,

$$R_M = 0.85$$

thus,

$$P_{e \text{ story}} = 0.85 \frac{HL}{\Delta_H} = 0.85 \frac{(195)(13.5(12))}{0.703} = 38,200 \text{ kips}$$

This is a measure of frame buckling strength



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## Example 1 (LRFD)

Sway amplification

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e story}}} \geq 1.0$$

$$B_2 = \frac{1}{1 - \frac{(1.0)(5,250)}{38,200}} = 1.16$$



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85

## Example 1 (LRFD)

Second-order moment

$$M_r = B_1 M_{nt} + B_2 M_{lt} \quad (\text{A-8-1})$$

$$M_u = 1.0(0.0) + 1.16(229) = 266 \text{ ft-kips}$$

Second-order force

$$P_r = P_{nt} + B_2 P_{lt} \quad (\text{A-8-2})$$

$$P_u = (335) + 1.16(0.0) = 335 \text{ kips}$$



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## Example 1 (LRFD)

- Determine member strength

$$KL = 13.5 \text{ ft} \quad \phi_c P_n = 1140 \text{ kips}$$

$$L_b = 13.5 \text{ ft} \quad \phi_b M_n = 646 \text{ ft-kips}$$

- Interaction Eq. H1-1a

$$\frac{335}{1140} + \frac{8}{9} \left( \frac{266}{646} \right) = 0.66 < 1.0 \quad \therefore \text{ok}$$



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87

## Direct Analysis

- Determination of notional loads

- To account for initial out-of-plumbness

$$N_i = 0.002\alpha Y_i$$

- $Y_i$  is based on applicable load combination

- For ASD, use  $\alpha = 1.6$  times the applicable load combination if rigorous second-order analysis is used (do not use 1.6 if  $B_1$ - $B_2$  method is used since it is already included in  $B_1$ - $B_2$ )

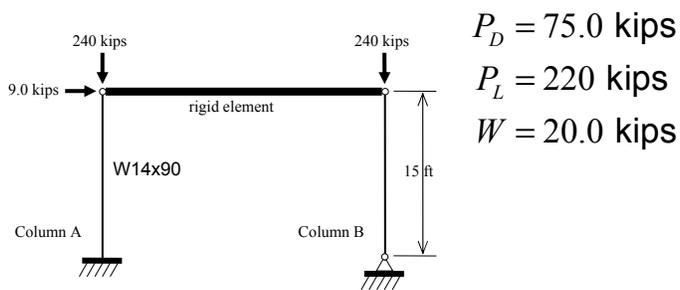


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## Example 2 (ASD)

- Determination of notional loads



$$D + 0.75L + 0.75(0.6W)$$



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## Example 2 (ASD)

- Determination of notional loads

- Notional load

$$Y_i = (240 + 240) = 480 \text{ kips}$$

$$N_i = 0.002(480) = 0.960 \text{ kips} < 9.00 \text{ kips}$$

- Assume

$$B_2 \leq 1.7$$

- It is permissible to apply the notional load in the gravity only combinations only.

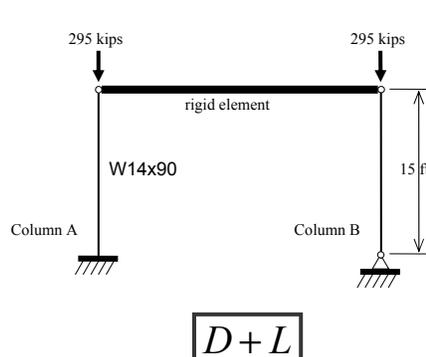


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## Example 2 (ASD)

- Determination of notional loads



$$P_D = 75.0 \text{ kips}$$

$$P_L = 220 \text{ kips}$$

$$W = 20.0 \text{ kips}$$



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## Example 2 (ASD)

- Determination of notional loads

– Notional load

$$Y_i = (295 + 295) = 590 \text{ kips}$$

$$N_i = 0.002(590) = 1.18 \text{ kips}$$

– Since there is no lateral load, the notional load must be applied.

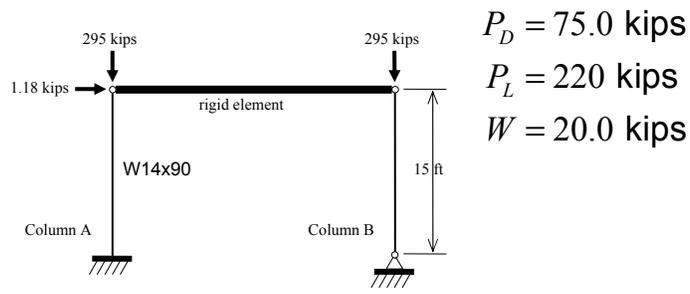


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## Example 2 (ASD)

- Determination of notional loads



$P_D = 75.0$  kips  
 $P_L = 220$  kips  
 $W = 20.0$  kips

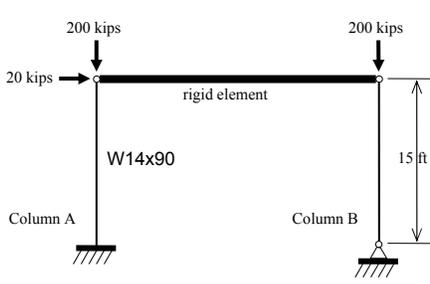
$D + L$



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## Example 2 (LRFD)

- Determination of notional loads



$P_D = 75.0$  kips  
 $P_L = 220$  kips  
 $W = 20.0$  kips

$1.2D + 0.5L + 1.0W$



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## Example 2 (LRFD)

- Determination of notional loads
- Notional load

$$Y_i = (200 + 200) = 400 \text{ kips}$$

$$N_i = 0.002(400) = 0.8 \text{ kips} < 20.0 \text{ kips}$$

– Assume

$$B_2 \leq 1.7$$

– It is permissible to apply the notional load in the gravity only combinations only.

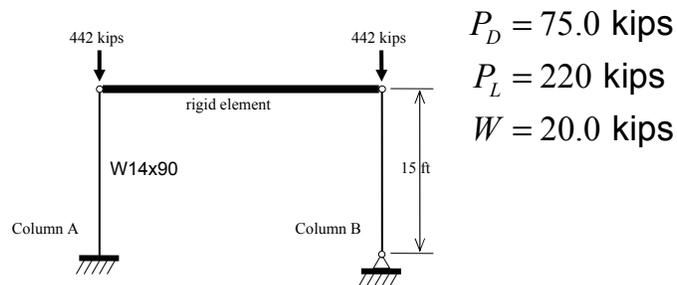


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## Example 2 (LRFD)

- Determination of notional loads



$$P_D = 75.0 \text{ kips}$$

$$P_L = 220 \text{ kips}$$

$$W = 20.0 \text{ kips}$$

$$1.2D + 1.6L$$



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## Example 2 (LRFD)

- Determination of notional loads
  - Notional load

$$Y_i = (442 + 442) = 884 \text{ kips}$$
$$N_i = 0.002(884) = 1.77 \text{ kips}$$

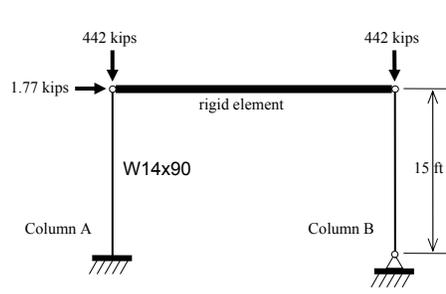
- Since there is no lateral load, the notional load must be applied.



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## Example 2 (LRFD)

- Determination of notional loads



$$P_D = 75.0 \text{ kips}$$
$$P_L = 220 \text{ kips}$$
$$W = 20.0 \text{ kips}$$

$$1.2D + 1.6L$$



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## Direct Analysis

- Why use the Direct Analysis Method
  1. You want to eliminate the need to calculate  $K$
  2. Or it must be used because

$$B_2 = \frac{\Delta_{2nd-order}}{\Delta_{1st-order}} > 1.5$$

when using  $EI$  and  $AE$

$$B_2 = \frac{\Delta_{2nd-order}}{\Delta_{1st-order}} > 1.7$$

when using  $EI^*$  and  $AE^*$



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## Other Analysis Methods

- However, if
$$B_2 = \frac{\Delta_{2nd-order}}{\Delta_{1st-order}} \leq 1.5$$
- There are 3 choices. You may use
  - Chapter C: Direct Analysis Method
  - Appendix 7.2: Effective Length Method
  - Appendix 7.3: First-order Analysis Method



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## Effective Length Method

- Appendix 7.2 Effective Length Method
  - Apply notional loads,  $N_i = 0.002\alpha Y_i$  in gravity only load cases
  - Perform a second-order elastic analysis
    - Use nominal geometry
    - Use nominal stiffness
  - Determine  $K$  from a sidesway buckling analysis

$$\text{If } \Delta_{2nd\text{-order}} / \Delta_{1st\text{-order}} \leq 1.1 \text{ then } K = 1.0$$



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## Effective Length Method

- Appendix 7.2 Effective Length Method
  - How does this differ from the Direct Analysis Method?
    - Do not use the reduced stiffness,  $EI^*$  and  $EA^*$ .
    - Must determine  $K$ .
  - How does this differ from what you should have been doing all along?
    - Must consider initial out-of-plumbness (notional loads) (as with Direct Analysis only in the gravity-only load case).



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## Effective Length Method

- The problem: how to determine  $K$ 
  - With alignment chart, many of our assumptions are not valid, such as:
    - Behavior is purely elastic.
    - rotations at opposite ends of restraining beams are equal producing reverse curvature
    - Stiffness parameter,  $L\sqrt{P/EI}$ , of all columns is equal
    - All columns buckle simultaneously

Consider the “gravity-only” columns



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## Gravity-Only Columns

- What are gravity-only columns?
  - Columns that do not contribute to the lateral load resistance of the structure
  - They rely on the remaining portion of the structure to provide their lateral restraint
  - Also called “leaning columns”
  - Designed with  $K = 1.0$

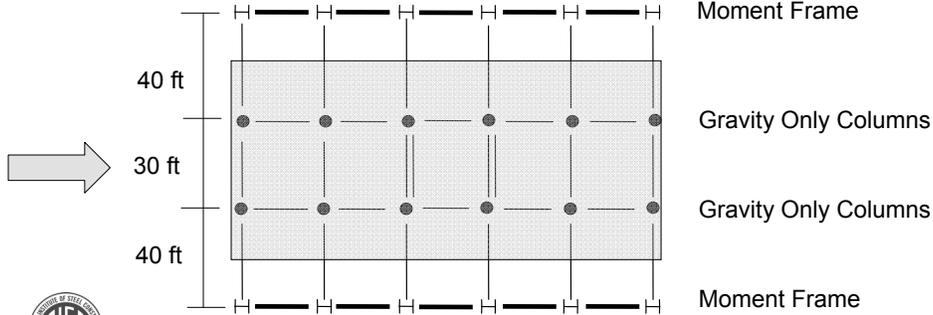


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## Gravity-Only Columns

In the left to right direction, this structure shows 12 gravity-only columns, 50%, but the relationship of loading, based on tributary area, is more like 64% on the gravity-only columns



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## Example 3 (ASD)

Check a W14x120 exterior column as part of a moment frame by the effective length method using an amplified first-order analysis

$$\text{Load Combination} = D + 0.75L + 0.75(0.6W)$$

$$P_{nt} + P_{lt} = 378 + 46 = 424 \text{ kips}$$

$$M_{nt} + M_{lt} = 87.8 + 72.0 = 160 \text{ ft-kips}$$



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## Example 3 (ASD)

Determine second-order moments

No translation,  $M_{nt}$

If the moment at one end is  $\frac{1}{2}$  that at other  
end, then  $M_1 = 0.5M_2$  and

$$C_m = 0.6 - 0.4(M_1/M_2)$$

$$C_m = 0.6 - 0.4(43.9/87.8) = 0.4$$



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## Example 3 (ASD)

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2}$$

$$P_{e1} = \frac{\pi^2 (29,000)(1,380)}{(1.0(12.5)(12))^2} = 17,550 \text{ kips}$$



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### Example 3 (ASD)

$$B_1 = \frac{C_m}{1 - \frac{\alpha P_r}{P_{e1}}} \geq 1.0$$

$$B_1 = \frac{0.4}{1 - \frac{1.6(424)}{17,550}} = 0.42 \not\geq 1.0 \therefore B_1 = 1.0$$



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### Example 3 (ASD)

Determine second-order moments

Translation,  $M_{lt}$ .

Use the story stiffness approach

For design, the frame deflection will be limited to

$$\Delta_H = L/400$$

with  $\sum H = 150$  kips (service load to cause drift limit)



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110

## Example 3 (ASD)

- For a moment frame system where 36% of the gravity load is carried by the moment frames, as illustrated in an earlier framing plan,

$$R_M = 1 - 0.15(0.36) = 0.946$$



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111

## Example 3 (ASD)

thus,

$$P_{e \text{ story}} = 0.946 \frac{HL}{\Delta_H} = 0.946(150)(400) = 56,800 \text{ kips}$$

This is a measure of the frame sway  
buckling strength



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## Example 3 (ASD)

For the entire frame at this story

$$P_{story} = 2,270 \text{ kips (total gravity load)}$$

- Sway amplification factor

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e story}}} = \frac{1}{1 - \frac{1.6(2,270)}{56,800}} = 1.07$$



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113

## Example 3 (ASD)

Second-order moment

$$M_r = B_1 M_{nt} + B_2 M_{lt}$$

$$M_a = 1.0(87.8) + 1.07(72.0) = 165 \text{ ft-kips}$$

Second-order force

$$P_r = P_{nt} + B_2 P_{lt}$$

$$P_a = (378) + 1.07(46.0) = 427 \text{ kips}$$



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### Example 3 (ASD)

Effective length,  $K_x$

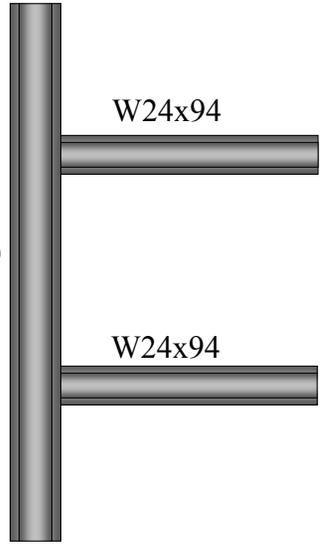
$$G_T = G_B = \frac{2 \left( \frac{1380}{12.5} \right)}{\left( \frac{3270}{30.0} \right)} = 2.03$$

$$K_x = 1.60$$

W14x120

Effective length,  $K_y$ ,  
assumed braced frame

$$K_y = 1.0$$



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### Example 3 (ASD)

- If the load on the gravity-only columns is equal to 64% of the total load on all columns

$$K = K_o \sqrt{1 + \frac{\text{gravity-only column load}}{\text{restraining column load}}} = 1.60 \sqrt{1 + \frac{0.64(2270)}{0.36(2270)}} = 2.67$$

Approximation found in the Commentary to the 2<sup>nd</sup> Edition LRFD, 1993



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## Example 3 (ASD)

Nominal compressive strength

In-plane effective length

$$\frac{KL_x}{r_x/r_y} = \frac{2.67(12.5)}{1.67} = 20.0 \quad \star$$

$$KL_y = 12.5$$

$$\frac{P_n}{\Omega_c} = 782 \text{ kips}$$



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## Example 3 (ASD)

Determine which interaction equation to use

$$\frac{P_a}{P_n/\Omega_c} = \frac{428}{782} = 0.55 > 0.2 \therefore \text{ use H1-1a}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9} \left( \frac{M_{ax}}{M_{nx}/\Omega_b} \right) \leq 1.0$$



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## Example 3 (ASD)

- Moment Strength

$$\frac{M_n}{\Omega} = 529 \text{ ft-kips}$$

- Interaction Equation H1-1a

$$0.55 + \frac{8}{9} \left( \frac{166}{529} \right) = 0.83 \leq 1.0$$



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## Example 3 (ASD)

Thus, the W14x120,  $F_y = 50$  will work for this loading combination

Now it should be checked for any other load combination, such as D + L plus the minimum lateral notional load



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## Example 3 (LRFD)

Check a W14x120 exterior column as part of  
a moment frame by the effective length  
method

Load Combination =  $1.2D + 0.5L + 1.0W$

$$P_{nt} + P_{lt} = 408 + 98 = 506 \text{ kips}$$

$$M_{nt} + M_{lt} = 94.5 + 154.5 = 249 \text{ ft-kips}$$



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## Example 3 (LRFD)

Determine second-order moments

No translation,  $M_{nt}$

If the moment at one end  $\frac{1}{2}$  that at other  
end, then  $M_1 = 0.5M_2$  and

$$C_m = 0.6 - 0.4(M_1/M_2)$$

$$C_m = 0.6 - 0.4(47.25/94.5) = 0.4$$



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### Example 3 (LRFD)

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2}$$

$$P_{e1} = \frac{\pi^2 (29,000)(1,380)}{(1.0(12.5)(12))^2} = 17,550 \text{ kips}$$



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123

### Example 3 (LRFD)

$$B_1 = \frac{C_m}{1 - \frac{\alpha P_r}{P_{e1}}} \geq 1.0$$

$$B_1 = \frac{0.4}{1 - \frac{506}{17,550}} = 0.41 \not\geq 1.0 \therefore B_1 = 1.0$$



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124

## Example 3 (LRFD)

Determine second-order moments

Translation,  $M_{lt}$ .

Use the story stiffness approach

For design the frame deflection will be limited to

$$\Delta_H = L/400$$

with  $\sum H = 150$  kips (service load to cause drift limit)



There's always a solution in steel!

125

## Example 3 (LRFD)

- For a moment frame system where 36% of the gravity load is carried by the moment frames, as illustrated in an earlier framing plan,

$$R_M = 1 - 0.15(0.36) = 0.946$$



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126

## Example 3 (LRFD)

thus,

$$P_{e \text{ story}} = 0.946 \frac{HL}{\Delta_H} = 0.946(150)(400) = 56,800 \text{ kips}$$

This is a measure of the frame sway  
buckling strength



There's always a solution in steel!

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## Example 3 (LRFD)

For the entire frame at this story

$$P_{\text{story}} = 2445 \text{ kips (total gravity load)}$$

- Sway amplification factor

$$B_2 = \frac{1}{1 - \frac{\alpha P_{\text{story}}}{P_{e \text{ story}}}} = \frac{1}{1 - \frac{1.0(2445)}{56,800}} = 1.04$$



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### Example 3 (LRFD)

Second-order moment

$$M_r = B_1 M_{nt} + B_2 M_{lt}$$

$$M_u = 1.0(94.5) + 1.04(154.5) = 255 \text{ ft-kips}$$

Second-order force

$$P_r = P_{nt} + B_2 P_{lt}$$

$$P_u = (408) + 1.04(98) = 510 \text{ kips}$$



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### Example 3 (LRFD)

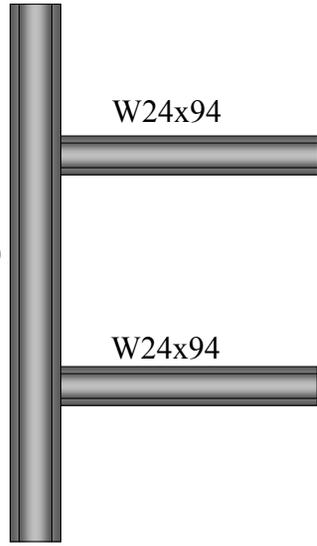
Effective length,  $K_x$

$$G_T = G_B = \frac{2 \left( \frac{1380}{12.5} \right)}{\left( \frac{3270}{30.0} \right)} = 2.03$$

$$K_x = 1.60 \quad \text{W14x120}$$

Effective length,  $K_y$ ,  
assumed braced frame

$$K_y = 1.0$$



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### Example 3 (LRFD)

- If the load on the gravity-only columns is equal to 64% of the total load on all columns

$$K = K_o \sqrt{1 + \frac{\text{gravity-only column load}}{\text{restraining column load}}} = 1.60 \sqrt{1 + \frac{0.64(2445)}{0.36(2445)}} = 2.67$$

Approximation found in the Commentary to the 2<sup>nd</sup> Edition LRFD, 1993



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### Example 3 (LRFD)

Nominal compressive strength

In-plane effective length

$$\frac{KL_x}{r_x/r_y} = \frac{2.67(12.5)}{1.67} = 20.0 \quad \star$$

$$KL_y = 12.5$$

$$\phi_c P_n = 1,180 \text{ kips}$$



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## Example 3 (LRFD)

Determine which interaction equation to use

$$\frac{P_u}{\phi_c P_n} = \frac{511}{1180} = 0.43 > 0.2$$

∴ use H1-1a

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} \right) \leq 1.0$$



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## Example 3 (LRFD)

- Moment Strength

$$\phi M_n = 795 \text{ ft-kips}$$

- Interaction Equation H1-1a

$$0.43 + \frac{8}{9} \left( \frac{257}{795} \right) = 0.72 \leq 1.0$$



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## Example 3 (LRFD)

Thus, the W14x120,  $F_y = 50$  will work for this loading combination.

Now it should be checked for any other load combination, such as 1.2D+1.6L plus the minimum lateral notional load



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## Other Analysis Methods

- However, if

$$B_2 = \frac{\Delta_{2nd-order}}{\Delta_{1st-order}} \leq 1.5$$

- There are 3 choices. You may use
  - Chapter C: Direct Analysis Method
  - Appendix 7.2: Effective Length Method
  - Appendix 7.3: First-order Analysis Method



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## First-Order Analysis Method

### App.7.3.1. Limitations

- (1) Structure supports gravity loads primarily through nominally vertical columns, walls or frames
- (2) Second-order effects must be limited

$$B_2 = \frac{\Delta_{2nd-order}}{\Delta_{1st-order}} \leq 1.5$$

- (3) Inelasticity must not be significant

$$\frac{\alpha P_r}{P_y} \leq 0.5$$



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## First-Order Analysis Method

### App.7.3.2. Required Strengths

Shall be determined from a first-order analysis with the following additional requirements:

- (1) All load combinations shall include an additional lateral load

$$N_i = 2.1\alpha(\Delta/L)Y_i \geq 0.0042Y_i \quad (\text{A-7-2})$$

- (2) Nonsway second-order effects must be included through the amplification factor,  $B_1$ .



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## First-Order Analysis Method

### App.7.3.3. Available Strengths

Shall be calculated in accordance with the provisions of Chapters D through K.

Effective length factors shall be taken as  $K = 1.0$  for all members

Bracing shall have sufficient stiffness. (Appendix 6)



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## First-Order Analysis Method

- Design process
  - Perform a first-order analysis
    - Use nominal geometry
    - Use nominal stiffness
  - Apply additional notional loads,  
$$N_i = 2.1\alpha(\Delta/L)Y_i \geq 0.0042Y_i$$
  - Apply  $B_1$  multiplier to moment in beam-columns
  - Use  $K=1.0$



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## First-Order Analysis Method

- How does this differ from Direct Analysis?
  - Does not use the reduced stiffness,  $EI^*$  and  $EA^*$ .
  - Notional load always applied
  - No need to do a second-order analysis except for  $B_1$ .



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## First-Order Analysis Method

- How does this differ from Effective Length Method?
  - Notional load always applied
  - Don't need to do a second-order analysis except for  $B_1$ .
  - $K$  always = 1.0



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# Stability Analysis and Design

**Table 2-2  
Summary Comparison of Methods  
for Stability Analysis and Design**

	Direct Analysis Method	Effective Length Method	First-Order Analysis Method
Limitations on Use <sup>a</sup>	None	$\Delta_{2nd}/\Delta_{1st} \leq 1.5$	$\Delta_{2nd}/\Delta_{1st} \leq 1.5$ $\alpha B_1/B_2 \leq 0.5$
Analysis Type	Second-order elastic <sup>b</sup>		First-order elastic
Geometry of Structure	All three methods use the undeformed geometry in the analysis.		
Minimum or Additional Lateral Loads Required in the Analysis	Minimum; 0.2% of the story gravity load	Minimum; 0.2% of the story gravity load	Additive; at least 0.42% of the story gravity load
Member Stiffnesses Used in the Analysis	Reduced EA and EI	Nominal EA and EI	
Design of Columns	K = 1 for all frames	K = 1 for braced frames. For moment frames, determine K from sideways buckling analysis <sup>c</sup>	K = 1 for all frames <sup>d</sup>
Specification Reference for Method	Chapter C	Appendix Section 7.2	Appendix Section 7.3

<sup>a</sup>  $\Delta_{2nd}/\Delta_{1st}$  is the ratio of second-order drift to first-order drift, which can be taken to be equal to  $B_2$  calculated per Appendix 8.  $\Delta_{2nd}/\Delta_{1st}$  is determined using LRFD load combinations or a multiple of 1.6 times ASD load combinations.  
<sup>b</sup> Either a general second-order analysis method or second-order analysis by amplified first-order analysis (the "B<sub>1</sub>-B<sub>2</sub> method" described in Appendix 8) can be used.  
<sup>c</sup> This notional load is additive if  $\Delta_{2nd}/\Delta_{1st} > 1.5$ .  
<sup>d</sup> K = 1 is permitted for moment frames when  $\Delta_{2nd}/\Delta_{1st} \leq 1.1$ .  
<sup>e</sup> An additional amplification for member curvature effects is required for columns in moment frames.



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# Basic Design Values

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**Analysis and Design**

**Simplified Method (see Note 1)**

Step 1. Perform first-order analysis. Use 0.2% of total story gravity load as minimum lateral load in all load combinations.  
 Step 2. Establish the design story drift limit and determine the lateral load required to produce it.  
 Step 3. Determine the ratio of the total story gravity load to the lateral load determined in Step 2. For ASD, multiply by 1.6.  
 Step 4. Multiply first-order results by the tabular value,  $R_m$ , except for moment frames when the tabular value is greater than 1.1.

Design Story Drift Limit	RFD		
	80	100	120
H/100	1	1.1	
H/200	1	1.1	
H/300	1	1.1	
H/400	1	1.1	
H/500	1	1.1	

Summary

Elastic Methods (for plastic design, see Appendix 1)	Effective Length	Forces and Moments	Limitations	Reference
First-order analysis method – second-order effects captured from effects of additional lateral load	K = 1 for all frames (see Note 2)	From analysis	$\Delta_{2nd}/\Delta_{1st} \leq 1.5$ ; Axial load limited	Appendix 7.3
Effective length method – second-order analysis with 0.2% of total story gravity load as minimum lateral load in all load combinations (see Note 3)	K = 1, except for moment frames with $\Delta_{2nd}/\Delta_{1st} > 1.1$	From analysis (see Note 3)	$\Delta_{2nd}/\Delta_{1st} \leq 1.5$	Appendix 7.2
Direct analysis method – second-order analysis with notional lateral load and reduced EI and AE (see Note 3)	K = 1 for all frames	From analysis (see Note 3)	None	Chapter C

**Notes:**

- Derived from the effective length method, using the B<sub>1</sub>-B<sub>2</sub> approximation with B<sub>2</sub> taken equal to B<sub>1</sub>.
- An additional amplification for member curvature effects is required for columns in moment frames.
- The B<sub>1</sub>-B<sub>2</sub> approximation (Appendix 8) can be used to accomplish a second-order analysis within the limitation that B<sub>2</sub> ≤ 1.5. Also, B<sub>1</sub> and B<sub>2</sub> can be taken equal to the multiplier tabulated for the simplified method above.
- $\Delta_{2nd}/\Delta_{1st}$  is the ratio of second-order drift to first-order drift, which is also represented by B<sub>2</sub>.

$R_m = 0.85/1.0$



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# A Simplified Method

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**Analysis and Design**

**Simplified Method (see Note 1)**

Step 1. Perform first-order analysis. Use 0.2% of total story gravity load as minimum lateral load in all load combinations.  
 Step 2. Establish the design story drift limit and determine the lateral load required to produce it.  
 Step 3. Determine the ratio of the total story gravity load to the lateral load determined in Step 2. For ASD, multiply by 1.6.  
 Step 4. Multiply first-order results by the tabular value,  $K=1$ , except for moment frames when the tabular value is greater than 1.1.

Design Story Drift Limit	Ratio from Step 3 (times 1.6 for ASD, 1.0 for LRFD)										
	0	5	10	20	30	40	50	60	80	100	120
H/100	1	1.1	1.1	1.3	1.5/1.4						
H/200	1	1	1.1	1.1	1.2	1.3	1.4/1.3	1.5/1.4			
H/300	1	1	1	1.1	1.1	1.2	1.2	1.3	1.5/1.4		
H/400	1	1	1	1.1	1.1	1.1	1.2	1.2	1.3	1.4/1.3	1.5/1.4
H/500	1	1	1	1	1.1	1.1	1.1	1.2	1.2	1.3	1.4/1.3

When ratio exceeds 1.5, simplified method requires a stiffer structure.

Elastic Methods (for plastic design, see Appendix 1)	Effective Length	Forces and Moments	Limitations	Reference
First-order analysis method – second-order effects captured from effects of additional lateral load	$K = 1$ for all frames (see Note 2)	From analysis	$\Delta_{2nd}/\Delta_{1st} \leq 1.5$ ; Axial load limited	Appendix 7.3
Effective length method – second-order analysis with 0.2% of total story gravity load as minimum lateral load in all load combinations (see Note 3)	$K = 1$ , except for moment frames with $\Delta_{2nd}/\Delta_{1st} > 1.1$	From analysis (see Note 3)	$\Delta_{2nd}/\Delta_{1st} \leq 1.5$	Appendix 7.2
Direct analysis method – second-order analysis with notional lateral load and reduced $EI$ and $AE$ (see Note 3)	$K = 1$ for all frames	From analysis (see Note 3)	None	Chapter C

**Notes:**

- Derived from the effective length method, using the  $B_1$ - $B_2$  approximation with  $B_1$  taken equal to  $B_2$ .
- An additional amplification for member curvature effects is required for columns in moment frames.
- The  $B_1$ - $B_2$  approximation (Appendix 8) can be used to accomplish a second-order analysis within the limitation that  $B_2 \leq 1.5$ . Also,  $B_1$  and  $B_2$  can be taken equal to the multiplier tabulated for the simplified method above.
- $\Delta_{2nd}/\Delta_{1st}$  is the ratio of second-order drift to first-order drift, which is also represented by  $B_2$ .

$R_m = 0.85/1.0$



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# Second-Order Analysis

The simplification: Using one amplification factor for both moments

$$M_r = B_2 (M_{nt} + M_{lt}) = B_2 M$$

Valid as long as  $B_2 > B_1$



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## Example 4 (ASD)

Check the column of Example 3 using  
the simplified approach from the Basic  
Design Value cards

1. Results of a first-order analysis

$$P_a = 424 \text{ kips}$$

$$M_a = 160 \text{ ft-kips}$$

2. Design story drift

$$W = 150 \text{ kips} \quad \Delta_{\max} = \frac{L}{400}$$



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## Example 4 (ASD)

3. Ratio of story gravity to lateral load from  
step 2

$$\frac{1.6(2,270)}{150} = 24.0$$

4. Enter table with 24.0 and  $L/400$



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# Example 4 (ASD)

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**Simplified Method (see Note 1)**

Step 1. Perform first-order analysis. Use 0.2% of total story gravity load as minimum lateral load in all load combinations.  
 Step 2. Establish the design story drift limit and determine the lateral load required to produce it.  
 Step 3. Determine the ratio of the total story gravity load to the lateral load determined in Step 2. For ASD, multiply by 1.6.  
 Step 4. Multiply first-order results by the tabular value,  $K=1$ , except for moment frames when the tabular value is greater than 1.1.

Design Story Drift Limit	Ratio from Step 3 (times 1.6 for ASD, 1.0 for LRFD)										
	0	5	10	20	30	40	50	60	80	100	120
H/100	1	1.1	1.1	1.3	1.5/1.4						
H/200	1	1	1.1	1.1	1.2	1.3	1.4/1.3	1.5/1.4			
H/300	1	1	1	1.1	1.1	1.2	1.2	1.3	1.5/1.4		
H/400	1	1	1	1.1	1.1	1.1	1.2	1.2	1.3	1.4/1.3	1.5/1.4
H/500	1	1	1	1	1.1	1.1	1.1	1.2	1.2	1.3	1.4/1.3

When ratio exceeds 1.5, simplified method requires a stiffer structure.

Elastic Methods (for plastic design, see Appendix 1)	Effective Length	Forces and Moments	Limitations	Reference
First-order analysis method – second-order effects captured from effects of additional lateral load	$K = 1$ for all frames (see Note 2)	From analysis	$\Delta_{2nd}/\Delta_{1st} \leq 1.5$ ; Axial load limited	Appendix 7.3
Effective length method – second-order analysis with 0.2% of total story gravity load as minimum lateral load in all load combinations (see Note 3)	$K = 1$ , except for moment frames with $\Delta_{2nd}/\Delta_{1st} > 1.1$	From analysis (see Note 3)	$\Delta_{2nd}/\Delta_{1st} \leq 1.5$	Appendix 7.2
Direct analysis method – second-order analysis with notional lateral load and reduced $EI$ and $AE$ (see Note 3)	$K = 1$ for all frames	From analysis (see Note 3)	None	Chapter C

**Notes:**

- Derived from the effective length method, using the  $B_1$ - $B_2$  approximation with  $B_1$  taken equal to  $B_2$ .
- An additional amplification for member curvature effects is required for columns in moment frames.
- The  $B_1$ - $B_2$  approximation (Appendix 8) can be used to accomplish a second-order analysis within the limitation that  $B_2 \leq 1.5$ . Also,  $B_1$  and  $B_2$  can be taken equal to the multiplier tabulated for the simplified method above.
- $\Delta_{2nd}/\Delta_{1st}$  is the ratio of second-order drift to first-order drift, which is also represented by  $B_2$ .

$R_m = 0.85/1.0$



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# Example 4 (ASD)

- Amplified load and moment

$$P_a = 1.1(424) = 466 \text{ kips}$$

$$M_a = 1.1(160) = 176 \text{ ft-kips}$$

$$K_x = 1.0 \text{ therefore, } KL_x = 12.5 \text{ ft and } KL_y = 12.5 \text{ ft}$$

$$\frac{P_n}{\Omega_c} = 940 \text{ kips}$$



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## Example 4 (ASD)

- Moment strength is the same as in Example 4

$$\frac{M_n}{\Omega_b} = 529 \text{ ft-kips}$$

- Interaction Eq. H1-1a

$$\frac{466}{940} + \frac{8}{9} \left( \frac{176}{529} \right) = 0.50 + 0.30 = 0.80 < 1.0$$



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## Example 4 (LRFD)

Check the column of Example 3 using the simplified approach from the Basic Design Value cards

1. Results of a first-order analysis

$$P_u = 506 \text{ kips}$$

$$M_u = 249 \text{ ft-kips}$$

2. Design story drift

$$W = 150 \text{ kips} \quad \Delta_{\max} = \frac{L}{400}$$



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## Example 4 (LRFD)

3. Ratio of story gravity to lateral load from step 2

$$\frac{(2,445)}{150} = 16.3$$

4. Enter table with 16.3 and  $L/400$



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## Example 4 (LRFD)

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**Simplified Method (see Note 1)**

Step 1. Perform first-order analysis. Use 0.2% of total story gravity load as minimum lateral load in all load combinations.  
 Step 2. Establish the design story drift limit and determine the lateral load required to produce it.  
 Step 3. Determine the ratio of the total story gravity load to the lateral load determined in Step 2. For ASD, multiply by 1.6.  
 Step 4. Multiply first-order results by the tabular value.  $K=1$ , except for moment frames when the tabular value is greater than 1.1.

Design Story Drift Limit	Ratio from Step 3 (times 1.6 for ASD, 1.0 for LRFD)										
	0	5	10	20	30	40	50	60	80	100	120
H/100	1	1.1	1.1	1.3	1.5/1.4	1.5	1.5	1.5	1.5	1.5	1.5
H/200	1	1	1.1	1.1	1.2	1.3	1.4/1.3	1.5/1.4	When ratio exceeds 1.5, simplified method requires a stiffer structure.		
H/300	1	1	1	1.1	1.1	1.2	1.2	1.3	1.5/1.4		
H/400	1	1	1	1.1	1.1	1.1	1.2	1.2	1.3	1.4/1.3	1.5/1.4
H/500	1	1	1	1	1.1	1.1	1.1	1.2	1.2	1.3	1.4/1.3

Elastic Methods (for plastic design, see Appendix 1)	Effective Length	Forces and Moments	Limitations	Reference
First-order analysis method – second-order effects captured from effects of additional lateral load	$K=1$ for all frames (see Note 2)	From analysis	$\Delta_{2nd}/\Delta_{1st} \leq 1.5$ ; Axial load limited	Appendix 7.3
Effective length method – second-order analysis with 0.2% of total story gravity load as minimum lateral load in all load combinations (see Note 3)	$K=1$ , except for moment frames with $\Delta_{2nd}/\Delta_{1st} > 1.1$	From analysis (see Note 3)	$\Delta_{2nd}/\Delta_{1st} \leq 1.5$	Appendix 7.2
Direct analysis method – second-order analysis with notional lateral load and reduced $EI$ and $AE$ (see Note 3)	$K=1$ for all frames	From analysis (see Note 3)	None	Chapter C

**Notes:**

- Derived from the effective length method, using the  $B_1$ - $B_2$  approximation with  $B_1$  taken equal to  $B_2$ .  $R_m = 0.85/1.0$
- An additional amplification for member curvature effects is required for columns in moment frames.
- The  $B_1$ - $B_2$  approximation (Appendix 8) can be used to accomplish a second-order analysis within the limitation that  $B_2 \leq 1.5$ . Also,  $B_1$  and  $B_2$  can be taken equal to the multiplier tabulated for the simplified method above.
- $\Delta_{2nd}/\Delta_{1st}$  is the ratio of second-order drift to first-order drift, which is also represented by  $B_2$ .



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## Example 4 (LRFD)

- Amplified load and moment

$$P_u = 1.1(506) = 557 \text{ kips}$$

$$M_u = 1.1(249) = 274 \text{ ft-kips}$$

$K_x = 1.0$  therefore,  $KL_x = 12.5 \text{ ft}$  and  $KL_y = 12.5 \text{ ft}$

$$\phi_c P_n = 1,415 \text{ kips}$$



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## Example 4 (LRFD)

- Moment strength is the same as in Example 4

$$\phi_b M_n = 795 \text{ ft-kips}$$

- Interaction Eq. H1-1a

$$\frac{557}{1,415} + \frac{8}{9} \left( \frac{274}{795} \right) = 0.39 + 0.31 = 0.70 < 1.0$$



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## Conclusions

- What's really new
  - Must consider initial-out-of-plumbness
  - Direct Analysis removes need to determine  $K$
  - Notional loads used to account for other effects
  - 3 approaches provided for required strength



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# Thank You

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